

# FFLO state in thin superconducting films

A. Buzdin<sup>1</sup>, Y. Matsuda<sup>2,3</sup>, T. Shibauchi<sup>2</sup>

<sup>1</sup> *Institut Universitaire de France and Université Bordeaux I, CPMOH, UMR 5798, 33405 Talence, France*

<sup>2</sup> *Department of Physics, Kyoto University, Sakyo-ku, Kyoto 606-8502, Japan*

<sup>3</sup> *Institute for Solid State Physics, University of Tokyo, Kashiwa, Chiba 277-8581, Japan*

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We present the analysis of the inhomogeneous Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) superconducting state in thin superconducting films in the parallel magnetic field. For the tetragonal crystal symmetry (relevant to CeCoIn<sub>5</sub> - the most probable candidate for the FFLO state formation) we predict a very peculiar in-plane angular dependence of the FFLO critical field due to the orbital effect. In the uniform superconducting state the critical field should be isotropic. The magnetic field pins also the direction of the FFLO modulation permitting thus to study the critical current anisotropy. Our calculations reveal a strong critical current anisotropy in the FFLO state in sharp contrast with the usual superconducting state. The predicted characteristic anisotropies of the critical field and critical current may provide an unambiguous probe of the FFLO phase formation.

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## I. INTRODUCTION

Recently the strong experimental evidence has been obtained in favor of the existence of the inhomogeneous superconducting state in the heavy fermion compound CeCoIn<sub>5</sub> (see ref. 1 and references cited therein). This state has been predicted a long time ago by Larkin and Ovchinnikov<sup>2</sup>, and Fulde and Ferrell<sup>3</sup> who demonstrated that in a pure ferromagnetic superconductor at low temperature the superconductivity may be non-uniform (for a review see refs. 1 and 4). The conditions of such FFLO state formation are rather stringent. In CeCoIn<sub>5</sub> the FFLO state exists owing to the paramagnetic pair-breaking effect which dominates the orbital one. Moreover the superconductivity of CeCoIn<sub>5</sub> corresponds to the clean limit. Note also that the magnetic-field-induced superconductivity has been observed in the quasi two-dimensional organic conductor  $\lambda$ -(BETS)<sub>2</sub>FeCl<sub>4</sub>, which may be another good candidate for the FFLO state realization<sup>5</sup>.

Although the upper critical field in CeCoIn<sub>5</sub> is dominated by the paramagnetic effect, the orbital effect also plays an important role in this compound leading to the vortex state formation. According to the theoretical works<sup>6,7,8,9,10</sup> in the FFLO state the vortex lattices may be very special and this circumstance complicates an unambiguous identification of the FFLO phase. The ideal system to study FFLO state without masking effect of the orbital field may be a two dimensional superconductor in a parallel magnetic field. It is a challenging experimental task to fabricate thin films of CeCoIn<sub>5</sub> with thickness smaller than superconducting coherence length. *There are strong experimental evidences that CeCoIn<sub>5</sub> is a d-wave superconductor<sup>4</sup>. The potential scattering is harmful for both d-wave superconductivity<sup>12</sup> and FFLO phase<sup>4</sup>. Therefore the films must have an atomically smooth surface and be in the clean limit.* However due to the recent progress of the multilayer fabrication technology (in particular due to the epitaxial technique) such systems would be created in a near future. In the present work we study theoretically the properties of the FFLO state in superconducting film with the thickness  $d$  smaller than the superconducting coherence length  $\xi$ . To be more specific, we assume the CeCoIn<sub>5</sub> multilayered structure with the tetragonal crystal symmetry and  $c$  axis perpendicular to the superconducting layers. Our approach is based on the modified Ginzburg-Landau (MGL) theory<sup>11</sup> which adequately describes the FFLO state with a long wave length modulation - near the tricritical point (TCP). This TCP is the meeting point of three transition lines separating the normal, the uniform superconducting and the FFLO states. However qualitatively our results may be applied to the whole region of the FFLO state existence. The orbital effect for the magnetic field parallel to the film is weak, and comparing with the bulk superconductor its contribution to the upper critical field is reduced by the factor  $(d/\xi_c)^2 \ll 1$ , where  $\xi_c$  is the superconducting coherence length along  $c$  axis. Then we may expect a much larger FFLO region on the  $(H, T)$  phase diagram. In the present work, we demonstrate that small but finite orbital effect leads to a characteristic in-plane anisotropy of the upper critical field which may serve as a clear indication of the FFLO state formation. Also the magnetic field lifts the degeneracy of the direction of the FFLO modulation and permits to create a monodomain FFLO state. Moreover our analysis shows that we may expect a pronounced anisotropy of the superconducting critical current in FFLO state.

## II. GENERALIZED GINZBURG-LANDAU FUNCTIONAL

The long period FFLO modulation near the superconducting transition can be described by the MGL functional<sup>11</sup> which in addition to the usual gradient terms contains the higher derivatives of the superconducting order parameter  $\Psi$ . The necessity to add the higher derivatives is related to a special behavior of the coefficient on the usual gradient term which goes through zero and becomes negative in the region of FFLO phase. This circumstance makes the MGL theory qualitatively different from the standard Ginzburg-Landau approach and is responsible for the peculiar properties of the FFLO state. As an example, we note that in the Ginzburg-Landau functional the higher derivatives describe the weak non-local effects (see, for example, ref. 13) which are related to the details of the Fermi surface. In the FFLO phase these non-local terms are of the primary importance and then making the properties of the FFLO phase ultimately dependent on the details of the electronic spectrum.

The MGL functional provides an adequate description of the FFLO state near the tricritical point where the wave-vector of the FFLO modulation is small but obtained results could be extrapolated qualitatively on the whole region of the FFLO phase. In the case of pure paramagnetic effect this TCP corresponds to  $T^* = 0.56T_{c0}$ ,  $H^* = H(T^*) = 0.61\Delta_0/\mu_B = 1.05T_{c0}/\mu_B$ , where  $T_{c0}$  is the critical temperature in the absence of the paramagnetic effect. The orbital effect decreases  $T^*$  and in CeCoIn<sub>5</sub>  $T^*$  seems to be  $(0.2 - 0.3)T_{c0}$  (ref. 1) depending on the field orientation. In the bulk superconductors, the relative contribution of the paramagnetic and orbital effects may be characterized by the Maki parameter  $\alpha = \sqrt{2} \frac{H_{c2}^{orb}}{H_p}$ , where  $H_{c2}^{orb}$  is the orbital critical field extrapolated to  $T = 0$  from the slope of  $H_{c2}(T)$  near  $T_c$  and  $H_p$  is a paramagnetic limit at  $T = 0$ . In CeCoIn<sub>5</sub> the Maki parameter is large  $\alpha \approx 5$  ensuring the condition for the FFLO phase formation. The particularity of CeCoIn<sub>5</sub> is that the superconducting transition is slightly first order<sup>14</sup> below  $\sim 0.4T_{c0}$  for the field  $\mathbf{H} \parallel \mathbf{ab}$  and below  $\sim 0.3T_{c0}$  for  $\mathbf{H} \perp \mathbf{ab}$ , and it remains first order at FFLO transition<sup>1</sup>. This intriguing behavior may be related with rather pronounced magnetic fluctuations in this compound which is expected to be at the vicinity of quantum critical point<sup>15</sup>. More generally the internal field  $h$  acting on the electron spins have the contribution coming from the Ce band, in addition to Zeeman's term  $\tilde{\mu}H$  (where  $\tilde{\mu}$  is effective electron magnetic moment). If the exchange integral describing the interaction between this band and the superconducting electrons is  $I$ , the contribution from the polarized Ce atoms being  $SI$ , where  $S$  is their relative magnetization. Owing to the interband interaction, it would be a contribution to the Ce polarizability from the electron susceptibility  $\chi_e$ , which changes at the superconducting transition  $\chi_e \sim \chi_e^0 \left(1 - c' \frac{|\Psi|^2}{T_c^2}\right)$ , where the constant  $c' \sim 1$ <sup>16</sup>. In the result the internal exchange field  $h$  will acquire a correction  $\sim -IH|\Psi|^2$ . If the exchange integral is positive it will give a negative  $\sim |\Psi|^4$  contribution in the Ginzburg-Landau functional. Note that for the negative exchange integral we may have an inverse situation and then the superconducting transition could be the second order one at all temperature. Therefore depending on the sign of the exchange integral, the Ce band could favor the first or second order superconducting transition.

Taking CeCoIn<sub>5</sub> in mind, we assume here that the superconducting transition is weakly first order and then the MGL functional reads:

$$F_G = a(H, T) |\Psi|^2 - \alpha \left( |\Pi_x \Psi|^2 + |\Pi_y \Psi|^2 \right) + \beta \left( |\Pi_x^2 \Psi|^2 + |\Pi_y^2 \Psi|^2 + |\Pi_y \Pi_x \Psi|^2 + |\Pi_x \Pi_y \Psi|^2 \right) \quad (1)$$

$$+ \varepsilon \left( \Pi_x^2 \Psi (\Pi_y^2 \Psi)^* + c.c. \right) - \frac{2b}{3} |\Psi|^4 + \frac{8\lambda}{15} |\Psi|^6,$$

where  $\Pi_x = \frac{\partial}{\partial x} - i2eA_x$  and  $\Pi_y = \frac{\partial}{\partial y} - i2eA_y$ , the superconducting film in  $xy$  plane and  $z$  axis perpendicular to the film. In the FFLO region the coefficients  $\alpha, \beta > 0$  and the choice  $b, \lambda > 0$  ensures the first order transition. Magnetic field is directed along the film and makes the angle  $\theta$  with  $x$  axis and then

$$H_y = H \sin \theta, \quad H_x = H \cos \theta,$$

$$A_y = -zH \cos \theta, \quad A_x = zH \sin \theta.$$

The coefficient  $a(H, T)$  vanishes at the line of the second order transition to the uniform superconducting state and at fixed  $H$  it may be written as  $a = a_0(T - T_{cu}(H))$ , where  $T_{cu}(H)$  is the second order transition temperature into the uniform state. Due to the small thickness of the film  $d \ll \xi_c$ , the superconducting order parameter is constant over its thickness and then in (1) we have omitted the derivatives on  $z$ . The higher derivatives terms with the coefficient  $\varepsilon$  describe explicitly the difference between isotropic  $s$ -wave pairing model and the real situation realized in the tetragonal crystals and/or with  $d$ -wave pairing. *In isotropic  $s$ -wave superconductor there is a degeneracy over the direction of the FFLO modulation. For  $d$ -wave superconductor this degeneracy is lifted and in 3D case the modulation vector is always directed along the order parameter nodes<sup>17</sup>. In 2D  $d$ -wave superconductor at low temperature  $T \lesssim 0.06T_c$  a first order re-orientational transition occurs to the state with a modulation vector along the order parameter lobes<sup>18</sup>.* Note that in general  $\varepsilon \sim \beta$  and the effect of anisotropy for the FFLO state cannot be expected to be small. This is an

important difference with the standard Ginzburg-Landau theory where only the first rank tensors enter as a gradient terms and then the cubic crystal structure (or tetragonal in  $ab$  plane) is equivalent to the isotropic one. Then the form of the Fermi-surface *and the type of the superconducting pairing are both equally important* in determining the wave-vectors of FFLO modulation. *For the FFLO transition the interplay between the Fermi surface structure and the type of the superconductivity has been studied in refs. 19 and 20 on the basis of the tight binding model and a very rich variety of the scenarios of the FFLO transition was revealed.* In general it may be demonstrated<sup>21</sup> that the effective mass approximation can be reduced to the isotropic model by scaling transformation. However there are namely the deviations from the elliptical Fermi-surface which are crucial to the adequate FFLO description. Note that the tensor coefficients on the second derivatives terms in MGL are given by the expression<sup>22</sup>  $\beta_{ij} \sim \langle v_i^2 v_j^2 |\psi(\mathbf{k})|^2 \rangle$ , where  $v_i$  are the components of the Fermi velocity,  $\psi(\mathbf{k})$  is the gap function and the averaging is performed over the Fermi surface.

### III. ORIENTATIONAL EFFECT OF THE IN-PLANE FIELD. IN-PLANE ANISOTROPY OF THE CRITICAL FIELD.

Let us consider first the quadratic terms in (1) which depend on the orientation of the FFLO modulation. If the transition is of the second order (or weakly first order) the solution for the order parameter is of the form  $\Psi(\mathbf{r}) = f \cos(\mathbf{q}\mathbf{r})$ .

Without orbital effect

$$\delta F(\varphi) \sim \left[ -\alpha q^2 + \beta q^4 + \frac{\varepsilon}{2} q^4 \sin^2 2\varphi \right] |\Psi_q|^2, \quad (2)$$

where  $\varphi$  is the angle between the FFLO modulation vector and  $\mathbf{x}$ -axis. For  $\varepsilon > 0$  the minimum energy (and maximum critical temperature) corresponds to  $\varphi = 0, \pm\pi/2, \pi$  directions, i.e. along the  $x, y$  axis and the wave-vector  $q_0 = \sqrt{\frac{\alpha}{2\beta}}$ . For  $\varepsilon < 0$  the minimum energy corresponds to  $\varphi = \pm 3\pi/4, \pm\pi/4$  directions, i.e. along the diagonals.

In the case of a thin superconducting film in a parallel field it is easy to take into account the orbital effect - it is simply needed to average the functional (1) over the film thickness. The angular dependent part is

$$\begin{aligned} \delta F(\varphi, \theta) \sim |\Psi_q|^2 \left\{ -\alpha \left( q^2 + (2eH)^2 \frac{d^2}{12} \right) + \beta \left( q^4 + (2eH)^4 \frac{d^4}{80} \right) + \frac{\varepsilon}{2} \left( q^4 \sin^2 2\varphi + (2eH)^4 \frac{d^4}{80} \sin^2 2\theta \right) + \right. \\ \left. + 2q^2 (2eH)^2 \frac{d^2}{12} \left[ 2\beta + \frac{\varepsilon}{2} - \left( \beta + \frac{\varepsilon}{4} \right) \cos(2\theta - 2\varphi) + \frac{3\varepsilon}{4} \cos(2\theta + 2\varphi) \right] \right\}. \end{aligned} \quad (3)$$

Let us consider first the isotropic case  $\varepsilon = 0$ . Naturally the properties of the superconducting system are not depending on the field orientation itself and the angular dependent part has the form  $-\beta |\Psi_q|^2 q^2 d^2 (2eH)^2 \cos(2\theta - 2\varphi)$  and the minimum of the energy is attended at  $\theta = \varphi$ , i.e. when the FFLO modulation is directed along the magnetic field. So the magnetic field provides an orientational effect on the FFLO phase. In the absence of other sources of anisotropy the resulting field dependent contribution to the energy is isotropic in  $xy$  plane. The quadratic over  $H$  contribution vanishes for the wave-vector of the FFLO modulation  $q = q_0 = \sqrt{\frac{\alpha}{2\beta}}$ . Then there is no linear diamagnetic response in the FFLO phase  $\delta F = \beta |\Psi_q|^2 q^2 \frac{d^2}{6} (2eH)^2 - \alpha (2eH)^2 \frac{d^2}{12} |\Psi_q|^2 = 0$ . The resulting orbital field contribution  $\sim |\Psi_q|^2 \beta (2eH)^4 \frac{d^4}{180}$  is quartic over  $H$  and then the diamagnetic moment is pretty small and proportional to  $H^3$ .

In the more realistic case the anisotropy (crystalline and/or Cooper pairing) plays a very important role and pins the orientation of the FFLO modulation. Let us suppose for example that  $\varepsilon > 0$  and then in the absence of the orbital effect the FFLO modulation vectors are along the  $x, y$  axis. *Note that in the case  $\varepsilon < 0$  the rotation of the  $xy$  axis by  $45^\circ$  provides us the same functional (1) with renormalized coefficients  $\tilde{\varepsilon}$  and  $\tilde{\beta}$  but with  $\tilde{\varepsilon} > 0$ . Therefore our analysis presented below may be directly adapted to this case.* If  $\beta > \frac{\varepsilon}{2}$  and the angle of the magnetic field  $|\theta| < \pi/4$  then the direction of the wave vector  $q$  will be close to the  $x$  axis while for  $3\pi/4 > |\theta| > \pi/4$  the modulation will be along  $y$  axis. For  $\beta < \frac{\varepsilon}{2}$  the situation is inverse and the system chooses the modulation along the axis ( $x$  or  $y$ ) making the largest angle with field direction. The deviation of the modulation direction from the principal axis  $x, y$  is small and for  $\beta > \frac{\varepsilon}{2}$  (and  $|\theta| < \pi/4$ ) the equilibrium angle  $\varphi$  is

$$\varphi \approx \frac{\beta + \varepsilon}{\varepsilon q^2} (2eH)^2 \frac{d^2}{12} \sin 2\theta \approx \beta \frac{\beta + \varepsilon}{\varepsilon \alpha} (2eH)^2 \frac{d^2}{6} \sin 2\theta \ll 1. \quad (4)$$

The diamagnetic moment of the FFLO state is strongly angular dependent. For  $\beta > \frac{\varepsilon}{2}$

$$M \sim -|\Psi_q|^2 \frac{\alpha}{\beta} H d^2 \left[ \beta (1 - |\cos 2\theta|) + \frac{\varepsilon}{2} (1 + |\cos 2\theta|) \right], \quad (5)$$

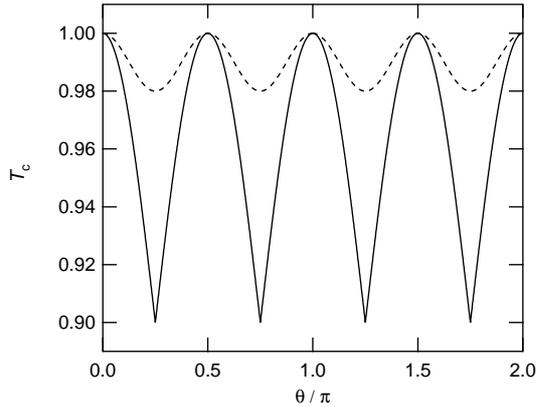


FIG. 1: Schematic presentation of the angular dependence of the in-plane critical field (or critical temperature) in FFLO state (solid line) and in the usual superconducting state (dotted line).

Note that in the usual uniform superconducting phase there is no linear angular dependent contribution to the magnetic moment.

The angular dependence of the critical temperature (critical field) for the FFLO state is

$$a(h, T) = \frac{\alpha^2}{2\beta} - \frac{\alpha}{\beta}(2eH)^2 \frac{d^2}{12} \left[ \beta + \frac{\varepsilon}{4} - \frac{1}{2} \left( \beta - \frac{\varepsilon}{2} \right) |\cos 2\theta| \right], \quad (6)$$

This angular dependence is presented in Fig. 1. In the uniform phase the angular dependence appears only due to the anisotropy term and it is proportional to  $(2eH)^4$ :

$$a(h, T) = \alpha(2eH)^2 \frac{d^2}{12} - \beta(2eH)^4 \frac{d^4}{80} - \frac{\varepsilon}{2}(2eH)^4 \frac{d^4}{80} \frac{1}{2}(1 - \cos 4\theta). \quad (7)$$

Comparing the angular dependence of the critical field in FFLO phase (Eq. 6 with that in the uniform state (Eq. 7), we see that the latter is much weaker ( $\sim H^4$ ) and has a different form - see Fig. 1.

Therefore the experimental studies of the in plane anisotropy of the critical field above and below  $T^*$  may provide a conclusive test of the FFLO state formation. As it has been already noted, the critical field in  $\text{CeCoIn}_5$  is mainly determined by the paramagnetic limit  $H \sim H_p$  (the Maki parameter is large  $\alpha_M \approx 5$ ).

The orbital effect sufficiently far away from the tricritical point provides the relative contribution to critical field of the order of  $\sim \left( \frac{Hd\xi_0}{\Phi_0} \right)^2$ , where  $\xi_0$  is the superconducting correlation length in plane. We may rewrite this as  $\left( \frac{Hd\xi_0}{\Phi_0} \right)^2 \sim \left( \frac{H_p}{H_{orb}} \right)^2 \left( \frac{d}{\xi_0} \right)^2 \sim \frac{1}{(\alpha_M)^2} \left( \frac{d}{\xi_0} \right)^2$ , with  $H_{orb} \sim \frac{\Phi_0}{\xi_0^2}$ . Near the tricritical point the usual orbital effect weakens as  $\frac{\alpha}{\alpha_0} \left( \frac{H_p}{H_{orb}} \right)^2 \left( \frac{d}{\xi_0} \right)^2 \sim \frac{\alpha}{\alpha_0} \frac{1}{(\alpha_M)^2} \left( \frac{d}{\xi_0} \right)^2$ , here  $\alpha_0$  is the gradient term coefficient near  $T_{c0}$  (i.e. far away from the FFLO transition). The higher derivatives terms also contribute to the in-plane anisotropy of the critical field through the so called non-local corrections to the GL theory<sup>13</sup>. Their contribution is of the order of  $\left( \frac{H_p}{H_{orb}} \right)^4 \left( \frac{d}{\xi_0} \right)^4 \sim \frac{1}{(\alpha_M)^4} \left( \frac{d}{\xi_0} \right)^4$ . Therefore the condition of the domination of the special FFLO behavior being  $\frac{\alpha}{\alpha_0} > \frac{1}{(\alpha_M)^2} \left( \frac{d}{\xi_0} \right)^2$ . Then even for  $d \lesssim \xi_0$  the characteristic FFLO regime would be observed everywhere except a tiny vicinity of the tricritical point.

*In this section we supposed that the FFLO transition is a second order transition. In the case of the first order transition (like in  $\text{CeCoIn}_5$ ) the performed calculations give a field (or temperature) of the over cooling of the normal phase. The actual field of the first order transition will be somewhat higher. However in the case of a weak first order transition the corresponding field (critical temperature) is obtained by the simple shift of  $a(h, T)$  - see next section. Therefore in this case also we may expect a peculiar angular dependence (fig. 1) of the critical field.*

#### IV. CURRENT IN THE FFLO STATE. ANISOTROPY OF THE CRITICAL CURRENT.

As it has been discussed in the previous section the orbital effect of the parallel magnetic field is small but permits to orient the wave-vector of the FFLO modulation in combination with crystalline/pairing anisotropy and then the critical current of the film will be anisotropic. Near the transition into the FFLO state the minimum energy is achieved for the one dimensional cos-like modulation of the order parameter<sup>2,10,11</sup>. Assuming  $\beta > \frac{\varepsilon}{2}$  and the magnetic field oriented along the  $x$ -axis we have

$$\Psi(\mathbf{r}) = f \cos(q_0 x). \quad (8)$$

Apart from the choice of the direction of the FFLO modulation, the orbital effect of the parallel magnetic field leads only to the small renormalization of the coefficients of the (1). Therefore the FFLO in the thin film opens the possibility to study the critical current and its anisotropy.

Naturally the ground state  $\Psi(\mathbf{r}) = f \cos(q_0 x)$  has no current and to describe the current carrying states we choose the order parameter in the form

$$\Psi(\mathbf{r}) = f \cos(q_0 x) \exp(i\varphi(\mathbf{r})). \quad (9)$$

To calculate the in-plane current we need to introduce the parallel components of the vector-potential  $\mathbf{A}_{\parallel} = (A_x, A_y)$  and the part of the functional describing the interaction with  $A_{\parallel}$  being

$$2\delta F_{\mathbf{A}} = a(H, T)f^2 - \alpha f^2 \left( (\nabla\varphi - 2e\mathbf{A})^2 + q_0^2 \right) + \quad (10)$$

$$+ \beta \left( q_0^4 + 6q_0^2 \left( \frac{\partial\varphi}{\partial x} - 2eA_x \right)^2 + 2q_0^2 \left( \frac{\partial\varphi}{\partial y} - 2eA_y \right)^2 \right) f^2 + 2\varepsilon q_0^2 \left( \frac{\partial\varphi}{\partial y} - 2eA_y \right)^2 f^2.$$

In this expression we have retained only the leading gradient terms assuming  $|\vec{\nabla}\varphi| \ll q_0$  and performed the averaging over the FFLO modulation. Note that for the current calculation we need to use its most general definition  $\mathbf{j} = -\frac{\delta F}{\delta \mathbf{A}}$  and not the formula from the Ginzburg-Landau theory. This is a consequence of the fact that the electrodynamics of the MGL theory is in fact very different from the standard GL one<sup>23</sup>. Using the relation  $q_0^2 = \frac{\alpha}{2\beta}$  we obtain the following expressions for the current

$$j_x = 4e\alpha f^2 \left( \frac{\partial\varphi}{\partial x} \right), \quad j_y = 2e\alpha \frac{\varepsilon}{\beta} f^2 \left( \frac{\partial\varphi}{\partial y} \right). \quad (11)$$

For  $\varphi = kx$  the state with the uniform current along the  $x$  axis is realized. It is interesting to note that the perpendicular current along the  $y$  axis (with  $\varphi = ky$ ) is proportional to the anisotropy parameter  $\varepsilon$  and vanishes in the idealized isotropic model.

Now we calculate the critical current in the FFLO state following the approach similar to the standard GL theory (see, for example, ref. 24) but taking into account the first order character of the FFLO transition. In the absence of the current the order parameter has the form (8) and putting it into (1) we obtain the averaged free energy density

$$F_G = (a - a_2)(H, T)f^2 - \frac{b}{2}f^4 + \frac{\lambda}{3}f^6,$$

where  $a = a_2 = \frac{\alpha^2}{4\beta}$  corresponds to the second order transition into FFLO state. The first order transition occurs at higher temperature/magnetic field and its ‘‘temperature’’  $a_1 = a_2 + \frac{3b^2}{16\lambda}$  and amplitude of the order parameter  $f_0^2 = \frac{3b}{4\lambda}$  may be easily found from the conditions

$$F_G(a_1, f_0) = (a_1 - a_2)f_0^2 - \frac{b}{2}f_0^4 + \frac{\lambda}{3}f_0^6 = 0, \quad (12)$$

$$\frac{\partial F_G(a_1, f_0)}{\partial f_0^2} = (a_1 - a_2) - bf_0^2 + \lambda f_0^4 = 0. \quad (13)$$

Note that the ‘‘critical temperature’’  $a_1$  of the first order transition is simply obtained from the ‘‘critical temperature’’ of the second order transition  $a_2$  by the shift on  $\frac{3b^2}{16\lambda}$ .

It is convenient to introduce the normalized ‘‘temperature’’ and order parameter:  $t = (a - a_2) / (a_1 - a_2)$ ,  $\tilde{f} = f / f_0$ . Without the current the ‘‘temperature’’ dependence of the order parameter is given by  $\tilde{f}_t^2 = (2 + \sqrt{4 - 3t}) / 3$ . In a

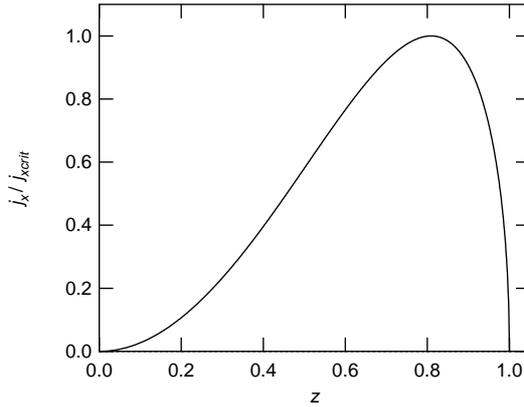


FIG. 2: Superconducting current versus the normalized amplitude of the order parameter  $z = \tilde{f}/\tilde{f}_t$ . Maximum current corresponds to the critical current of the FFLO state. For both orientations the maximum of the current is achieved at the same value  $z \approx 0.81$ .

presence of the current along the  $x$  axis, taking into account the equations (10), (11), and (13) we have the following relationship between the current and the amplitude of the order parameter.

$$j_x^2 = \frac{3\alpha e^2 b^2}{\lambda} \tilde{f}_t^8 f_0^4 z^4 (1 - z^2)(7 - z^2), \quad (14)$$

where  $z = \tilde{f}/\tilde{f}_t$ . The condition of the applicability of our approach  $|\vec{\nabla}\varphi| \ll q_0$  reads  $b^2 \ll \alpha\lambda/\sqrt{\beta}$ , i.e. the FFLO transition must be weakly first order. The plot  $j_x(z)$  (see Fig. 2) have a maximum at  $z^2 = (3\sqrt{2} - \sqrt{11})/\sqrt{2} \approx 0.65$  which gives us the critical current along the  $x$  axis

$$j_{xcrit}^2 \approx \frac{2.85\alpha e^2 b^2}{\lambda} \tilde{f}_t^8 f_0^4. \quad (15)$$

Analogously we find that the critical current in the direction perpendicular to the FFLO modulation is

$$j_{ycrit}^2 \approx \frac{1}{4} \left(\frac{\varepsilon}{\beta}\right)^2 \frac{2.85\alpha e^2 b^2}{\lambda} \tilde{f}_t^8 f_0^4. \quad (16)$$

The coefficient 1/4 is coming from the averaging on the order parameter modulation along the  $x$  axis. Finally the anisotropy of the critical current in the FFLO phase is very pronounced

$$\frac{j_{xcrit}}{j_{ycrit}} = \frac{2\beta}{\varepsilon}.$$

and in the isotropic model the critical current along the  $y$  is even vanishing. In real system the ratio  $\frac{2\beta}{\varepsilon}$  is expected to be of the order of one and the measurements of the critical current can permit to determine directly this parameter. As the critical current in the uniform state is isotropic far away from the tricritical point, then the experimental observation of the anisotropy of the critical current may serve as a clear indication of the FFLO phase formation.

## V. CONCLUSIONS

To summarize, we have investigated the properties of the FFLO phase in a thin film at parallel magnetic field. The orbital effect (even though it is small) leads to the orientation of the FFLO modulation through the whole film providing the monodomain FFLO state. Moreover in the FFLO state a peculiar angular dependence of the in-plane upper critical field must be observed. This conclusion is quite general and holds for both first and second order FFLO transitions. We predict also an important anisotropy of the in-plane critical current in the FFLO phase depending on the current direction with respect to the FFLO modulation. Such characteristic anisotropies of the critical field

and critical current may be considered as a smoking gun of the FFLO phase formation. Note that in<sup>25</sup> it has been demonstrated that the superconducting fluctuational regime changes drastically near the FFLO TCP. However in the case of the first order FFLO transition the fluctuational regime could be inaccessible on experiment. Our analysis was based on the very general MGL functional approach which is valid for both *s*-wave and *d*-wave superconductors. This approach is fully justified near the tricritical point and in the case of the weakly first order phase transition. Nevertheless qualitatively the obtained results could be extrapolated to the whole region of the FFLO phase existence and may be relevant for the CeCoIn<sub>5</sub> thin film experiments. In conclusion we stress that the predicted anisotropy of the of the in plane critical field and critical current must be also observed in *s*-wave tetragonal FFLO superconductor.

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- <sup>1</sup> Matsuda Y. and Shimahara H. *J. Phys. Soc. Jpn.* **76** (2007) 051005.  
<sup>2</sup> Larkin, A. I., and Ovchinnikov Yu. N. *Sov. Phys. JETP* **20** (1965) 762.  
<sup>3</sup> Fulde, P., and Ferrell R. A. *Phys. Rev.* **135** (1964) A550.  
<sup>4</sup> Casalbuoni, R., and Nardulli G. *Rev. Mod. Phys.* **76** (2004) 263.  
<sup>5</sup> Uji S. et al. *Phys. Rev. Lett.* **97** (2006) 157001.  
<sup>6</sup> Gruenberg L. W. and Gunther L. *Phys. Rev. Lett.* **16** (1966) 996.  
<sup>7</sup> Bulaevskii L. N. *Sov. Phys. JETP* **38** (1974) 634.  
<sup>8</sup> Buzdin, A. I. and Brison J.-P. *Phys. Lett. A* **218** (1996) 359.  
<sup>9</sup> Buzdin, A. I. and Brison J.-P. *Europhys. Lett.* **35** (1996) 707.  
<sup>10</sup> Shimahara H. and Rainer D. *J. Phys. Soc. Jpn.* **66** (1997) 3591.  
<sup>11</sup> Buzdin, A. I. and Kachkachi H. *Phys. Lett. A* **225** (1997) 341.  
<sup>12</sup> Mineev V. P. and Samokhin K. V. *Introduction to unconventional superconductivity of Metals and Alloys* (1999) Gordon and Breach.  
<sup>13</sup> Kogan V. G. et al. *Phys. Rev. B* **62** (2000) 9077.  
<sup>14</sup> Izawa, K. et al. *Phys. Rev. Lett.* **87** (2001) 057002.  
<sup>15</sup> Bianchi, A. et al. *Phys. Rev. Lett.* **91** (2003) 257001.  
<sup>16</sup> Bulaevskii L. N. et al. *Advances in Physics* **34** (1985) 175.  
<sup>17</sup> Samokhin K. V. *Physica C* **274** (1997) 156.  
<sup>18</sup> Yang K. and Sondhi S. L. *Phys. Rev. B* **57** (1998) 8566.  
<sup>19</sup> Shimahara H. *J. Phys. Soc. Jpn.* **66** (1997) 541.  
<sup>20</sup> Shimahara H. *J. Phys. Soc. Jpn.* **68** (1999) 3069.  
<sup>21</sup> Brison J. P. et al. *Physica C* **250** (1995) 128.  
<sup>22</sup> Houzet, M. and Mineev V. P. *Phys. Rev. B* **74** (2006) 144522.  
<sup>23</sup> Houzet, M. and Buzdin A. *Phys. Rev. B* **63** (2001) 184521.  
<sup>24</sup> De Gennes, P. G. *Superconductivity of Metals and Alloys* (1966) New York : Benjamin.  
<sup>25</sup> Korschelle, F. et al. *Europhys. Lett.* **79** (2007) 67001.