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Vibrating Microcantilevers : Tools for Microrheology

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Abstract. The aim of this paper is to demonstrate that vibrating microcantilevers can be used to quantify fluid properties such as density and viscosity. Contrary to classical rheological measurements using microcantilevers, the development of the proposed microrheometer is based on the measurement of fluid properties over a range of vibration frequencies, without necessarily being restricted to resonant phenomena. To this end, an analytical model is implemented and, when combined with measurements, allows the determination of the viscosity as a function of frequency. The preliminary results are encouraging for the development of a useful microrheometer on a silicon chip for microfluidic applications.

Keywords: Microrheology, microcantilever, viscosity, density

PACS : 8785-gf

INTRODUCTION

Simultaneous measurements of both phase and amplitude of a vibrating microcantilever in a fluid can lead to the rapid determination of both unknown fluid parameters (viscosity and density). First, a brief review of the theory of fluid-microcantilever interaction is made to explain the proposed method of fluid property extraction from the beam's dynamic response. This is then followed by a discussion of our first measurements and fluid parameter determination for Newtonian silicon oils. To finish, the same approach is presented for the determination of viscoelastic properties of complex fluids.

VISCOSIMETRY

Under the incompressible and Newtonian assumption, viscosity and density are intrinsic constants. When a microcantilever vibrates in a viscous fluid, the fluid offers resistance to the motion of the beam. The fluid loading can be interpreted as the sum of two forces: an inertial force that is proportional to the cantilever acceleration, and a viscous force that is proportional to the cantilever velocity. Thus, a viscous fluid exerts on the beam surface the drag force per unit length^{1,2,3}:

$$F_{fluid} = -g_1(f) \frac{\partial w(t)}{\partial t} - g_2(f) \frac{\partial^2 w(t)}{\partial t^2} \quad (1)$$

with w the deflection at the end of the beam, f the vibration frequency, and t is time.

An analogy between beam dynamics in viscous fluids and second order low-pass band allows the simplification of the mechanical transfer function. The assumption that the contribution of higher modes in the transfer function value is negligible has been made, i.e., the first mode is considered dominant. Referring to the differential equation governing the microcantilever deflection, the amplitude and phase measurements ($|H|, \phi$) may be converted to the drag force components at each frequency:

$$g_1 = -2\pi m_L \left| \frac{H_0}{H} \right| \frac{f_{0,vac}^2}{f} \sin \phi \quad (2)$$

$$g_2 = m_L \left(\left(1 - \left| \frac{H_0}{H} \right| \cos \phi \right) \left(\frac{f_{0,vac}}{f} \right)^2 - 1 \right) \quad (3)$$

where m_L is the cantilever mass per unit length, $f_{0,vac}$ the resonant frequency in vacuum, H_0 the static value of the mechanical transfer function H , and f the frequency.

Thus, the determination of the two parts of the drag force allows one to extract analytically at each frequency the viscosity and density, i.e., the fluid's rheological properties:

$$\rho = \frac{2\pi f g_2 (2a_1 b_2 - a_2 b_1) + a_2 \left(g_1 a_2 - \sqrt{(2\pi f g_2 b_1 - a_2 g_1)^2 + 8\pi f g_2 g_1 a_1 b_2} \right)}{\pi^2 b^2 f a_1 (a_1 b_2 - a_2 b_1)} \quad (4)$$

$$\eta = \frac{a_1 \left(2\pi f g_2 b_1 + g_1 a_2 - \sqrt{(2\pi f g_2 b_1 - a_2 g_1)^2 + 8\pi f g_2 g_1 a_1 b_2} \right)^2}{\pi (a_1 b_2 - a_2 b_1) \left(2\pi f g_2 (2a_1 b_2 - a_2 b_1) + a_2 \left(g_1 a_2 - \sqrt{(2\pi f g_2 b_1 - a_2 g_1)^2 + 8\pi f g_2 g_1 a_1 b_2} \right) \right)} \quad (5)$$

where $a_1=1.0553$, $a_2=3.7997$, $b_1=3.8018$ and $b_2=2.7364$ are Maali's parameters¹ and b is the microcantilever width.

The deflection amplitude and phase have been measured as functions of frequency on a range of 10 to 10000 Hz using an optical read-out scheme. An alternating current circulates on the beam, which, in presence of a magnet, generates the beam vibration by electromagnetic actuation. Measurements are done in a thermo-regulated room in which temperature is controlled at 20°C. The beam has been immersed in oils of known viscosities ranging from 10 cP to 30000 cP. In Fig. 1.a, no resonant peaks occur for viscosities above 100cP because of the excessive damping. Using Eqs. 4 and 5, viscosity and density can be calculated after extraction of the drag force components (Eqs. 2 and 3). Preliminary measurements (Fig. 1) are in good agreement with the rheological behaviour of the silicon oils for this frequency range⁴. As is implicit in the Newtonian and incompressible model, we observe viscosity and density as frequency-independent constants. The model works well for high viscosities despite the absence of a resonant peak. For that reason, this represents an alternative to classical microcantilever-based rheometer^{5,6}. The viscosity measurements are more accurate than the density values. This shows that, for those cases in which the density is known, the proposed method may be used to accurately quantify the dynamic viscosity. The accuracy of both, the beam geometry and the static value of the transfer function determines the quality of the measurements.

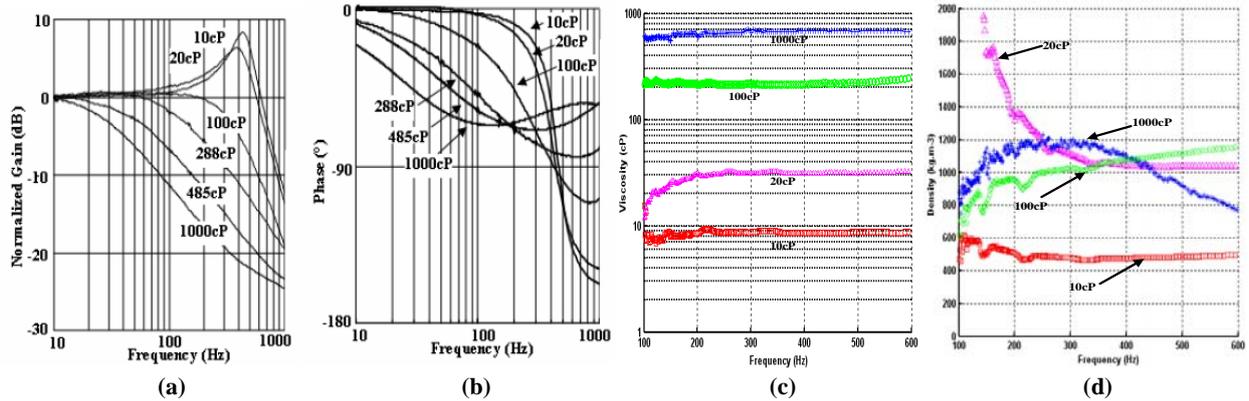


FIGURE 1. Measurements of amplitude (a) and phase (b) for a silicon cantilever (18 μ m x 600 μ m x 4mm) immersed in various silicon oils: 10cP, 20cP, 100cP, 288cP, 485cP and 1000cP. Oil densities are 1000kg.m⁻³. Determination of viscosity (c) and density (d) corresponding to the spectra presented in (a) and (b).

VISCOELASTICITY

The significant frequency-dependence of the properties of viscoelastic fluids motivates us to consider the use of microcantilevers to measure their properties over a broad frequency range, i.e., to serve as an improved rheometer. The objective is to extract several fluid properties from the beam response and to identify viscous/elastic or vitreous transitions. In the previous section, we have seen that a microcantilever can be used to determine a fundamental Newtonian fluid property (viscosity). A similar approach is possible for viscoelastic fluids. To achieve such a goal, we introduce the imaginary shear modulus $G = j\omega\eta$ into the simplified expression of the complex hydrodynamic function $\Gamma' + j\Gamma''$ proposed by Maali&al¹ for the case of a viscous fluid. Then, the obtained expression of $\Gamma' + j\Gamma''$ as a function of the imaginary shear modulus is used for the study of a viscoelastic fluid (complex shear modulus) by replacing G by $G'+jG''$. This can be done because the cantilever is larger than the mesh size of the rheological network⁷. By extracting the real and imaginary parts of the resulting hydrodynamic function, the expressions for a viscoelastic fluid are as follows:

$$g_1 = \frac{\pi b_2 G''}{2 \omega} + \frac{\pi}{4\sqrt{2}} \sqrt{\rho_f} b \left[(b_1 - a_2) \sqrt{\sqrt{G'^2 + G''^2} + G'} + (a_2 + b_1) \sqrt{\sqrt{G'^2 + G''^2} - G'} \right] \quad (6)$$

$$g_2 = \frac{\pi}{4} a_1 \rho_f b^2 + \frac{\pi b_2 G'}{2 \omega^2} + \frac{\pi}{4\sqrt{2}} \frac{\sqrt{\rho_f} b}{\omega} \left[(a_2 + b_1) \sqrt{\sqrt{G'^2 + G''^2} + G'} + (a_2 - b_1) \sqrt{\sqrt{G'^2 + G''^2} - G'} \right] \quad (7)$$

These analytical expressions (6-7) have been used in Fig.2 to simulate the special case of a Maxwellian viscoelastic fluid. An example of the simulation is plotted in Fig.2 where it can be seen that resonant phenomena clearly occur as a result of the low viscosities associated with high frequencies (Fig.2a), even if the fluid is a ‘compact gel’ in its static state. The characteristics of the resonance (frequency and phase slope) depend on the plateau elastic modulus G_0 . In Fig. 2b, the asymptotic values of amplitude depend only on G_0 while the microcantilever deflection depends also on the relaxation time τ , at frequencies near or less than $1/\tau$. Another set of experimental data was generated and analyzed for three different fluids: dodecane, EHDT-silicon gel and air. EHDT-silicon gel results from a mixture of powder in dodecane. In Fig.2c and Fig.2d, we see that the EHDT and dodecane have different mechanical properties. This change in dynamic behavior reflects the new mechanical behavior of the fluid.

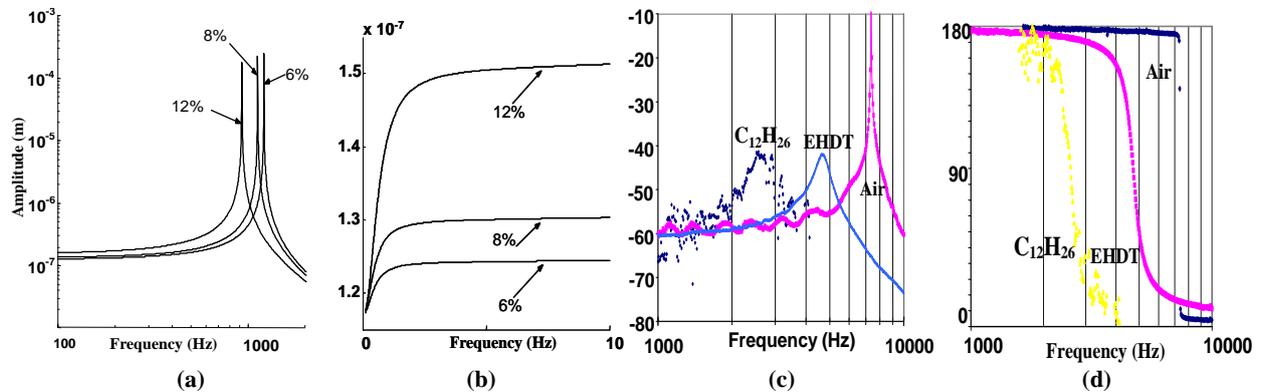


FIGURE.2. Theoretical amplitude (a and b) of deflection for a silicon cantilever ($18\mu\text{m} \times 600\mu\text{m} \times 4000\mu\text{m}$) oscillating in a CpCl solution at various mass concentration in salted water: 12% ($G_0=235\text{Pa}$, $\tau=1.02\text{s}$), 8% ($G_0=106\text{Pa}$, $\tau=1.07\text{s}$), 6% ($G_0=62\text{Pa}$, $\tau=0.99\text{s}$). Experimental amplitude (c) and phase (d) of deflection for a silicon cantilever oscillating in dodecane, EHDT, and air.

CONCLUSIONS

The results are very encouraging as they demonstrate the potential for beam-based microstructures to serve as useful tools for rheological measurements at fixed frequency over a wide frequency range. The theoretical model developed in the present paper describes the dynamics of the beam-fluid interaction quite well. However, future work will be required to refine the model for viscoelastic fluids, to perform additional experiments on other complex fluids, and to assess in a more systematic manner the potential for microcantilever-based rheometers to serve as competitive devices for determining rheological properties of complex fluids.

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