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Conditions for stability and instability of retrial queueing systems with general retrial times

Tewfik Kernane

Department of Probability and Statistics, Faculty of Mathematics, University of Sciences and Technology USTHB, Algiers, Algeria

E-mail: tkernane@gmail.com

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Abstract

We study the stability of single server retrial queues under general distribution for retrial times and stationary ergodic service times, for three main retrial policies studied in the literature: classical linear, constant and control policies. The approach used is the renovating events approach to obtain sufficient stability conditions by strong coupling convergence of the process modeling the dynamics of the system to a unique stationary ergodic regime. We also obtain instability conditions by convergence in distribution to improper limiting sequences.

Key words: Retrial queues; Stability; instability; Stochastic recursive sequence; Renovation events theory; Linear retrial policy; Constant retrial policy; Control retrial policy; Strong coupling convergence

Introduction

The analysis of stability in queueing systems is the first step in studying such models. The steady state solutions and performance characteristics of the system do not exist if it is not stable. The efficiency of a queueing system is related closely to its stability and is considered as inefficient if it is unstable. Retrial queues have the characteristic that an arriving customer who finds all waiting positions and service zones occupied must join a group of "blocked" customers in an additional queue called "orbit" and reapplies for getting served after random time intervals according to a specific retrial policy. They arise in many practical situations. The classical example can be found in telephone traffic theory where subscribers redial after receiving a busy signal. For computer and communication applications, peripherals in computer systems may make

retrials to receive service from a central processor. Another example can be adopted from the aviation where an aircraft is directed into the waiting zone, if the runway is found busy, from which the demand of landing is repeated at random periods of time. Retrial queueing models are generally more complicated than traditional ones especially when dealing with general distribution for retrial times. The existence of this supplementary flow from the orbit and the random access to the server (as for the linear policies that depend on the number of customers in orbit) make the system more congested and difficult to model by simple random processes like Markovian ones, which have properties that allow to derive easily conditions for stability, especially when we do not assume an exponential distribution (which has the memorylless property) allowing to obtain Markovian processes modeling the system. Furthermore, it has been observed in telecommunication systems that the exponential law is not a good estimator for the distribution of retrial times (see Yang et al., 1994).

The subject of this paper is to analyze the stability of single server retrial queues under general distribution for retrial times and stationary ergodic service times (without independence assumption), for three main retrial policies studied in the literature: classical linear, constant and control policies. Stability results for such models with general retrial times are rare and generally reduced to Markovian assumptions. For the linear retrial policy, Koba and Kovalenko (2004) obtained a sufficient stability condition (arrival rate is less than the service rate) for an M/G/1 system with non-lattice distribution for retrial times satisfying an additional estimate condition, with i.i.d service times. For the constant retrial policy, Koba (2002) derived a stability condition for a GI/G/1 retrial system with a FIFO discipline for the access from the orbit to the server and a general distribution for orbit time in latticed and non-latticed cases with i.i.d service times. For the control policy, Gomez-Corral (1999) studied extensively an M/G/1 retrial queue with general retrial times where he derived the stability condition for i.i.d service times and a FIFO discipline. For non-independent service times, Altman and Borovkov (1997) obtained a sufficient condition for the stability of a linear retrial queue under general stationary ergodic service times and independent and exponentially distributed interarrival and retrial times using the method of renovation events. Kernane and Aïssani (2006) obtained sufficient conditions for the stability of various retrial queues with versatile retrial policy which incorporates the constant and linear retrial policies under general stationary ergodic service times and independent and exponentially distributed interarrival and retrial times.

The main approach used in this paper is the method of renovation events originated in the work of Akhmarov and Leont'eva (1976) and developed by Borovkov (1984) in the stationary ergodic setting. In the following section, we derive stability and instability conditions for the classical linear retrial policy with general retrial times, stationary ergodic service times and Poisson

arrivals. In Section 3, we obtain a stability condition and an instability one for the constant retrial policy system with general retrial times, stationary ergodic service times and Poisson arrivals. With the later assumptions, we derive in Section 4, stability and instability conditions for the control policy retrial model.

1 Linear Retrial Policy

We begin by considering the classical single server retrial system with linear retrial policy. Customers arrive from outside according to a Poisson process with rate λ. If an arriving customer finds the server busy, he joins the orbit and repeats his attempt to get served after random time intervals. We consider the linear retrial policy where each customer in orbit attempts to get served independently of other customers and we assume that the sequence of inter-retrial times of a single customer is an independent sequence with general distribution $R(\cdot)$, density function $r(\cdot)$ and Laplace transform $r^*(z)$, $z > 0$. The successive service times $\{\sigma_n\}$ are assumed to form a stationary (in the strict sense) and ergodic (which essentially means that time averages converge to constants $a.s$) sequence with $0 < \mathbb{E}\sigma_n < \infty$. The inter-arrival, inter-retrial and service times are assumed to be mutually independent.

Let $Q(t)$ be the number of customers in orbit at time t and denote by s_n the instant when the nth service time ends. Consider the embedded process $Q_n = Q(s_n+)$ of the number of customers in orbit just after the end of the nth service duration. Denote by $N_\lambda(t)$ the counting Poisson process with parameter λ which counts the number of arriving customers during a time interval $(0, t]$. If $Q_n = k$, then we denote by $\pi_1(n), ..., \pi_k(n)$ the residual retrial times (forward recurrence times) of the customers in orbit just after the instant s_n and by γ_n the residual external arrival time at the same instant.

It is easy to see that the process Q_n satisfies the following recurrence relation:

$$Q_{n+1} = (Q_n + \xi_n)^+, \tag{1}$$

where $x^+ = \max[0, x]$ and

$$\xi_n = N_\lambda(\sigma_n) - \mathbb{I}\left\{\min(\pi_1(n), ..., \pi_{Q_n}(n)) < \gamma_n\right\}, \tag{2}$$

We have then expressed Q_n as a Stochastic Recursive Sequence (SRS) (for the definition see Borovkov, 1998).

We introduce the $\sigma-$algebra $\mathcal{F}_n^\sigma$ generated by the set of random variables $\{\sigma_k : k \leq n\}$ and $\mathcal{F}^\sigma$ generated by the entire sequence $\{\sigma_n : -\infty < n < +\infty\}$ and for which any independent sequence not depending on $\{\sigma_n\}$ is $\mathcal{F}^\sigma$-measurable

(see Borovkov (1976) p.14). Let U be the measure preserving shift transformation of $\mathcal{F}^\sigma$-measurable random variables, that is $U\sigma_k = \sigma_{k+1}$, and if $\eta \in \mathcal{F}^\sigma$ then the sequence $\{\eta_n = U^n\eta : -\infty < n < +\infty\}$ is a stationary ergodic sequence where U^n is the nth iteration of U and U^{-n} is the inverse transformation of U^n $n \in \mathbb{Z}$. We shall denote by T the corresponding transformation of events in $\mathcal{F}^\sigma$, that is for any $\mathcal{F}^\sigma$-measurable sequence η_n :

$$T\left\{\omega : (\eta_0(\omega), ..., \eta_k(\omega)) \in (B_0, ..., B_k)\right\} = \left\{\omega : (\eta_1(\omega), ..., \eta_{k+1}(\omega)) \in (B_0, ..., B_k)\right\}, \tag{3}$$

where the events $B_i \in \mathcal{F}^\sigma$, $i = 0, ..., k$.

An event $A \in \mathcal{F}^\xi_{n+m}$, $m \geq 0$, is a renovation event for the SRS $\{Q_n\}$ on the segment $[n, n+m]$ if there exists a measurable function g such that on the set A

$$Q_{n+m+1} = g(\xi_n, ..., \xi_{n+m}). \tag{4}$$

The sequence A_n, $A_n \in \mathcal{F}^\xi_{n+m}$, is a *renovating sequence of events* for the SRS $\{Q_n\}$ if there exists an integer n_0 such that (4) holds true for $n \geq n_0$ with a common function g for all n.

We say that the SRS $\{Q_n\}$ is *coupling convergent* to a stationary sequence $\{Q^n = U^nQ^0\}$ if

$$\lim_{n\to\infty} \mathbb{P}\left\{Q_k = Q^k;\ \forall\ k \geq n\right\} = 1. \tag{5}$$

Set $\nu_k = \min\{n \geq -k : U^{-k}Q_{n+k} = Q^n\}$ and $\nu = \sup_{k\geq 0} \nu_k$.

A SRS $\{Q_n\}$ is *strong coupling convergent* to a stationary sequence $\{Q^n = U^nQ^0\}$ if $\nu < \infty$ with probability 1.

Theorem 1 *Assume that $\lambda\mathbb{E}\sigma_1 < 1$. Then the process $\{Q_n\}$ is strong coupling convergent to a unique stationary ergodic regime.*
If $\lambda\mathbb{E}\sigma_1 > 1$, then the process $\{Q_n\}$ converges in distribution to an improper limiting sequence.

PROOF. Since the driving sequence ξ_n depend on Q_n, we will proceed first by considering an auxiliary sequence Q^*_n which majorizes Q_n and having a driving sequence ξ^*_n independent of Q_n and it has the following form:

$$Q^*_0 = Q_0, \quad Q^*_{n+1} = \max(C, Q^*_n + \xi^*_n), \tag{6}$$

where

$$\xi^*_n = N_\lambda(\sigma_n) - \mathbb{I}\left\{\min(\pi_1(n), ..., \pi_C(n)) < \gamma_n\right\}. \tag{7}$$

The constant integer C will be chosen later appropriately, and if $Q^*_n > C$ the C customers for which we consider the forward recurrence times $\pi_1(n), ..., \pi_C(n)$ are chosen randomly by an urn scheme without repetition. Following the procedure used in Altman and Borovkov (1997) and later in Kernane and Aïssani

(2006), we will construct stationary renovation events with strictly positive probability for Q_n from those of Q_n^*, and applying an ergodic theorem (Theorem 11.4 in Borovkov, 1998) which states that an SRS is strong coupling convergent to a unique stationary regime, satisfying the same recursion, if there exist stationary renovating events of strictly positive probability.
The stationarity and ergodicity of ξ_n^* follows from the fact that ξ_n^* is $\mathcal{F}^\sigma$-measurable (for more details on the ergodicity and stationarity of ξ_n^* see Kernane and Aïssani, 2006). We have

$$\begin{aligned}
\mathbb{E}\xi_n^* &= \lambda\mathbb{E}\sigma_1 - \mathbb{P}\left(\min(\pi_1(n), ..., \pi_C(n)) < \gamma_n\right) && (8)\\
&= \lambda\mathbb{E}\sigma_1 - \left[1 - \mathbb{P}\left(\pi_1(n) \geq \gamma_n, ..., \pi_C(n) \geq \gamma_n\right)\right] && (9)\\
&= \lambda\mathbb{E}\sigma_1 - \left[1 - \int_0^\infty \lambda e^{-\lambda t}\ \mathbb{P}\left(\pi_1(n) \geq t, ..., \pi_C(n) \geq t\right)\ dt\right] && (10)\\
&= \lambda\mathbb{E}\sigma_1 - \left[1 - \int_0^\infty \lambda e^{-\lambda t} \prod_{i=1}^{C} \mathbb{P}(\pi_i(n) \geq t)\ dt\right]. && (11)
\end{aligned}$$

Since $\lim_{C\to\infty} \prod_{i=1}^{C} \mathbb{P}(\pi_i(n) \geq t) = 0$, then by dominated convergence theorem

$$\lim_{C\to\infty} \int_0^\infty \lambda e^{-\lambda t} \prod_{i=1}^{C} \mathbb{P}(\pi_i(n) \geq t)\ dt = 0. \tag{12}$$

If the condition $\lambda\mathbb{E}\sigma_1 < 1$ is satisfied, we can choose the constant C such that $\mathbb{E}\xi_n^* < 0$. It follows from example 11.1 in Borovkov (1998) that there exists a stationary renovating sequence of events with positive probability for Q_n^*, from which we deduce those of Q_n (see Altman and Borovkov, 1997). Applying the ergodic theorem (Theorem 11.4 in Borovkov, 1998) we obtain that the sequence Q_n is strong coupling convergent to a unique stationary process $\widetilde{Q}_n = U^n\widetilde{Q}_0$, with $\widetilde{Q}_0$ $\mathcal{F}^\sigma$-measurable and since $\widetilde{Q}_n$ is an $U-$shifted $\mathcal{F}^\sigma$-measurable sequence then it is ergodic.
For the instability condition, consider the auxiliary process Q_n^S corresponding to a simple single server queue without retrials, that is

$$Q_0^S = Q_0, \quad Q_{n+1}^S = (Q_n^S + \xi_n^S)^+, \tag{13}$$

where

$$\xi_n^S = N_\lambda(\sigma_n) - 1. \tag{14}$$

Clearly $Q_n^S \leq_{st} Q_n$ and it is well known that if $\lambda\mathbb{E}\sigma_1 > 1$ then $\lim_{n\to\infty} Q_n^S = +\infty$ *a.s.* (see Theorem 1.7 in Borovkov, 1976). Thus, the process $\{Q_n\}$ converges in distribution to an improper limiting sequence.

2 Constant Retrial Policy

Consider now a single server retrial queue governed by the constant retrial policy which is described as follows. After a random time generally distributed (which we will call the orbit retrial time), one customer from the orbit (at the head of the queue or a randomly chosen one if any) take his service if the server is free, so an orbit time can be in progress even though the server is busy, this may happen in system where the orbit has no information about the state of the server. The sequence of orbit cycle times $\{r_i\}$ is assumed to be i.i.d, having $R(\cdot)$ as cdf, $r(\cdot)$ as density function with mean $\mathbb{E}r_1$ and Laplace transform $r^*(z)$, $z > 0$. Let $\pi(n)$ be the forward recurrence time of the orbit retrial time after the end of the nth service time. Then the process Q_n has now the following representation as a SRS:

$$Q_{n+1} = (Q_n + \xi_n)^+, \tag{15}$$

where

$$\xi_n = N_\lambda(\sigma_n) - \mathbb{I}\{\pi(n) < \gamma_n\}. \tag{16}$$

Theorem 2 *If R is nonlattice and*

$$\lambda \mathbb{E}\sigma_1 < \frac{[1 - r^*(\lambda)]}{\lambda \mathbb{E}r_1}, \tag{17}$$

then the process $\{Q_n\}$ is strong coupling convergent to a unique stationary ergodic regime.
If $\lambda\mathbb{E}\sigma_1 > (1 - r^*(\lambda))/(\lambda\mathbb{E}r_1)$. Then the process Q_n converges in distribution to an improper limiting sequence.

PROOF. We have

$$\mathbb{E}\xi_n = \lambda\mathbb{E}\sigma_1 - \mathbb{P}(\pi(n) < \gamma_n). \tag{18}$$

Since the interarrival times are exponentially distributed then so is the residual arrival time γ_n, hence

$$\mathbb{P}(\pi(n) < \gamma_n) = \int_0^{+\infty} \mathbb{P}(\pi(n) < t)\ \lambda\ e^{-\lambda t} dt. \tag{19}$$

Since we are interesting on steady state behaviour of the system and by assuming a nonlattice (also called non-arithmetic) distribution $R(t)$ for orbit retrial times, then from a well known result in renewal theory (see Cox, 1962) we have the following asymptotic distribution for the forward recurrence time

$$\mathbb{P}\left(\pi(n) < t\right) = \frac{1}{\mathbb{E}r_1} \int_0^t \left[1 - R(x)\right] \, dx. \tag{20}$$

The formula (19) becomes

$$\mathbb{P}\left(\pi(n) < \gamma_n\right) = \frac{1}{\mathbb{E}r_1} \int_0^{+\infty} \left[1 - R(x)\right] \int_x^{+\infty} \lambda e^{-\lambda t} dt \, dx \tag{21}$$

$$= \frac{1}{\mathbb{E}r_1} \int_0^{+\infty} \left[1 - R(x)\right] \, e^{-\lambda x} \, dx = \frac{1}{\mathbb{E}r_1} \left[\frac{1 - r^*(\lambda)}{\lambda}\right]. \tag{22}$$

Now if condition (17) is satisfied then $\mathbb{E}\xi_n < 0$. Since ξ_n is $\mathcal{F}^\sigma$-measurable (generated by σ_n) then it is a stationary ergodic sequence. From this and example 11.1 in Borovkov (1998), there exists a stationary sequence of renovation events with positive probability for $\{Q_n\}$. Hence, using Theorem 11.4 of Borovkov (1998), the sequence $\{Q_n\}$ is strong coupling convergent to a unique stationary sequence $\widetilde{Q}_n$ obeying the equation $\widetilde{Q}_{n+1} = (\widetilde{Q}_n + \xi_n)^+$, the ergodicity of $\widetilde{Q}_n$ follows from the fact that $\widetilde{Q}_n$ is an $U-$shifted sequence ($\widetilde{Q}_n = U^n \widetilde{Q}_0$, with $\widetilde{Q}_0$ $\mathcal{F}_0^\sigma$-measurable) generated by the stationary and ergodic sequence ξ_n. The instability condition $\lambda \mathbb{E}\sigma_1 > \left[1 - r^*(\lambda)\right] / \lambda \mathbb{E}r_1$ yields to $\mathbb{E}\xi_n > 0$, and it is well known that for SRS of the form $Q_{n+1} = (Q_n + \xi_n)^+$ this implies the convergence of the process Q_n to an improper limiting sequence (see Theorem 1.7 of Borovkov (1976)).

2.1 Exponential retrial times

By assuming an exponential distribution with parameter θ for retrial times, that is $R(x) = 1 - e^{-\theta x}$, it is well known that $r^*(s) = s/(s+\theta)$, and $\mathbb{E}r_1 = 1/\theta$. The condition (17) will read up, after some algebra, as follows

$$\lambda \mathbb{E}\sigma_1 < \frac{\theta}{\lambda + \theta}. \tag{23}$$

Which is the condition obtained in the paper of Kernane and Aïssani (2006), in exponential retrial context.

3 Retrial Control Policy

Consider a single server retrial queue with a control retrial policy. Primary customers enter from the outside according to a Poisson process with rate λ. If a primary customer finds the server busy upon arrival it joins the orbit

to connect later according to the control retrial policy, which is described as follows. Just after the end of a service time a generally distributed retrial time begins to find the server free. If the retrial time finishes before an external arrival, then one customer from the orbit (at the head of the queue or a randomly chosen one if any) receives its service and leaves the system. We assume that the sequence of retrial times $\{r_n\}$ is an i.i.d sequence having $r(\cdot)$ as pdf, $R(\cdot)$ as cdf and Laplace transform $r^*(\cdot)$, with finite mean $\mathbb{E}r_1$. The nth service duration of a call is σ_n and we assume that the sequence of service times $\{\sigma_n\}$ is stationary and ergodic with $0 < \mathbb{E}\sigma_1 < \infty$.

The process $\{Q_n\}$ has the following representation as a stochastic recursive sequence SRS:

$$Q_{n+1} = (Q_n + \xi_n)^+, \tag{24}$$

where

$$\xi_n = N_\lambda(\sigma_n) - \mathbb{I}\{r_n < \gamma_n\}, \tag{25}$$

where γ_n is the residual arrival time of an external call at the end of the nth service period.

Theorem 3 *Assume that*

$$\lambda\mathbb{E}\sigma_1 < r^*(\lambda). \tag{26}$$

Then the process Q_n is strong coupling convergent to a unique stationary ergodic regime.
If $\lambda\mathbb{E}\sigma_1 > r^*(\lambda)$. Then the process Q_n converges in distribution to an improper limiting sequence.

PROOF. The proof is similar to that of Theorem 2, by noting that $\mathbb{E}\xi_n = \lambda\mathbb{E}\sigma_1 - r^*(\lambda)$.

3.1 Exponential retrial times

Assume that the retrial times are exponentially distributed with mean $1/\theta$, then $r^*(\lambda) = \theta/(\lambda + \theta)$ and the stability condition (26) becomes:

$$\lambda\mathbb{E}\sigma_1 < \frac{\theta}{\lambda + \theta}. \tag{27}$$

This condition is quite evident since it can be obtained from the constant policy from the memorylless property of the exponential distribution.

3.2 Hyperexponential distribution for retrial times

Assume now that the retrial times follow the hyperexponential distribution with density $r(x) = p\theta \exp(-\theta x) + (1-p)\,\theta^2 \exp(-\theta^2 x)$, $0 \leq p < 1$. Then $r^*(\lambda) = \theta\left[\lambda\left(p + (1-p)\,\theta\right) + \theta^2\right] / (\lambda+\theta)\left(\lambda+\theta^2\right)$, and the stability condition (26) in this case is

$$\lambda \mathbb{E}\sigma_1 < \frac{\theta\left[\lambda\left(p + (1-p)\,\theta\right) + \theta^2\right]}{(\lambda+\theta)\left(\lambda+\theta^2\right)}. \tag{28}$$

3.3 The Erlang distribution for retrial times

The Erlang distribution has been found useful for describing random variables in queueing applications. The density of an $Erlang\,(n,\mu)$ distribution is given by $r(x) = \mu^n \exp(-\mu x) x^{n-1} / (n-1)!$, $x > 0$ and $n \in \mathbb{N}^*$. Its Laplace transform is $r^*(s) = \mu^n / (s+\mu)^n$. Then the control policy model will be stable if

$$\lambda \mathbb{E}\sigma_1 < \left(\frac{\mu}{\lambda+\mu}\right)^n. \tag{29}$$

Remark 4 *It should be noted that the assumption $\mathbb{E}\xi_n = 0$, and weak dependence among the ξ_n, does not preclude the possibility that the process $\{Q_n\}$ converges to a proper stationary regime.*

Remark 5 *Conditions for the stability of modified models with general retrial times, such as allowing breakdowns of the server, two types of arrivals, negative arrivals and batch arrivals models may be obtained easily following the procedure used in Kernane and Aïssani (2006). The conditions of the stability will be written by replacing the left hand side of the classical linear policy by the left hand sides of the case of a linear versatile policy obtained in Kernane and Aïssani (2006). For the constant policy, we have to make the appropriate changes to the driving sequences in the SRS modeling the dynamics of the modified models in Kernane and Aïssani (2006) by considering the residual orbit time as shown here in Section 3, the conditions of stability will follow directly after some algebra.*

Remark 6 *We may also consider the versatile retrial policy by incorporating the residual orbit retrial time $\pi(n)$ in the equation 2 and considering the whole retrial times of the customers in orbit $r_1(n), ..., r_{Q_n}(n)$ as follows*

$$\xi_n = N_\lambda(\sigma_n) - \mathbb{I}\left\{\min(\pi(n) + r_1(n), ..., \pi(n) + r_{Q_n}(n)) < \gamma_n\right\}, \tag{30}$$

the condition of Theorem 1 still holds for this versatile retrial policy, by noting

that in the proof we have to consider

$$\mathbb{E}\xi_n^* = \lambda\mathbb{E}\sigma_1 - \left[1 - \int_0^\infty \int_0^t \lambda e^{-\lambda t}\ P\left(r_1(n) \geq t-s, ..., r_C(n) \geq t-s\right)\ dG(s)dt\right], \tag{31}$$

where $G(s)$ is the cdf of the residual orbit retrial time satisfying

$$G(s) = \frac{1}{\mathbb{E}\alpha_1} \int_0^s [1 - A(x)]\ dx, \tag{32}$$

with $\mathbb{E}\alpha_1$ the mean of the orbit retrial time and $A(x)$ its cdf.

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