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Horizontal differentiation and price competition with sequential entry

Pierre Fleckinger
Thierry Lafay

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Cahier n° 2003-023

LABORATOIRE D'ECONOMETRIE
1rue Descartes F-75005 Paris
(33) 1 55558215
http://ceco.polytechnique.fr/
mailto:labecox@poly.polytechnique.fr

Horizontal differentiation and price competition with sequential entry[1]

Pierre Fleckinger[2]
Thierry Lafay[3]

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Résumé: Nous étudions une classe de jeu à la Hotelling. Deux firmes choisissent une localisation et un prix dans l'espace des caractéristiques optimales des consommateurs. Un leader choisit d'abord ses deux variables, le suiveur observe ces choix avant de jouer. Nous montrons que dans une telle situation, tout équilibre a la propriété que le suiveur fait un profit supérieur à celui du leader. Un équilibre existe toujours pour des coûts linéaires and nous caractérisons les équilibres en fonction de la propension à payer des consommateurs pour la famille de produit. Quelques extension sont entreprises, en particulier sur les barrières à l'entrée.

Abstract: We study a class of differentiation games à la Hotelling. Two firms choose a price and a location in the consumers' space. The leader first chooses both variables, and the follower observes them before playing. It is shown that in such games, any equilibrium has the property that the follower always gets a greater profit (in the absence of entry barrier). An equilibrium is shown to always exist with linear costs and we qualify the equilibria with respect to the willingness to pay for the family of products. Some extensions are looked at. In particular, we investigate the effect of entry barriers on the strategies of the leader.

Mots clés : Différenciation horizontale, engagement en prix, compétition en catalogue, jeu séquentiel.

Key Words : horizontal differentiation, price commitment, catalog competition sequential moves.

Classification JEL: L11, D43, M3

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[2] Ecole Polytechnique, Laboratoire d'Econométrie
[3] Ecole Polytechnique, Laboratoire d'Econométrie

1 Introduction

In its pioneering paper 'Stability in competition', Hotelling (1929) modelled competition among firms with differentiated products. He departed from Cournot's and Bertrand's views of perfect substitutes, which implies pure quantity or pure price competition. Hotelling deals with a linear market (Main Street) and considers that all consumers, living along the street, buy one unit of good, choosing the one whose perceived price (mill price plus transportation costs) is smaller. The two firms compete in price, having different locations, or different characteristics in the product space. His main motivation was to show that competition has not always a "winner-takes-all" (namely, the leader in price) form, but that small changes in price only affects smoothly the quantity sold by a competitor. In equilibrium, when a firm lowers its price, it does not get the other's full market share, but only a fringe of them, varying continuously in price change. From this setting, another conclusion has be drawn by Hotelling: if firms choose first their locations, their will be a tendency towards homogeneity of the product: the firms will tend to choose almost the same location, at the center of the market. This is the so-called principle of minimum differenciation.

It is somehow paradoxical that the idea of 'stability' in competition put forth by Hotelling is actually invalid in his model, on account of indeterminacy of a stable configuration. Indeed, as pointed out by d'Aspremont & al. (1979), an equilibrium does not exist for all possible locations of the firms. If they are too close, a competitive price equilibrium is impossible. Thus the enunciated Principle of Minimum Differentiation does not hold even in Hotelling's model, because of a non-existence problem. d'Aspremont & al. continue the discussion with an adaptation of the standard model, changing the linear transportation costs into quadratic ones. As pointed out by MacLeod (1985) this implies that the mass of indifferent consumers is zero, because indifferent consumers are located at a single point. This restores equilibrium existence, as continuity of payoffs is restored. Moreover, in this setting, the reverse result holds about differentiation: the firms locate at the extreme points of the market. Many authors have subsequently found mitigated results about differentiation in equilibrium (Economides 1984, Hinloopen & van Marrewijk 1999 among others).

While differentiation models concern a huge literature, we focus in this paper on the sequentiality of location and price choices. Following Prescott and Visscher (1977), we use subgame perfection as equilibrium concept. In

most of the papers, it is assumed that firms choose location first (possibly sequentially) and once for all, then compete simultaneously in price. Lambertini (2002), for instance, studies sequential entry with discounting. In his setting, a leader chooses first a location for the whole game. He is first alone on the market and is thus free to set a monopoly price. After a certain while, a followers enters the market: he chooses a location and competes in price with the leader. This is consistent with the view that the location is a geographical parameter, or with the view that the design of a product is generically far less flexible than its price. It seems to fit relatively well products that have a long lifetime. Up to our knowledge, the only paper that modifies the assumption of simultaneous pice competition is Anderson (1987). He studies a price leadership configuration: the firms choose sequentially the locations, then a Stackelberg competition in price takes place. The subject of the paper is to determine which firm chooses to be the price leader; the answer is that the second entrant prefers to be price leader, and the first to be price follower. This is such the sequential setting that seems to be the most likely to appear.

One has now to remark that all these standard settings assume that price competition, whatever the form it takes, happens only *after* both location are fixed. However, situations exist where protagonists in a market commit to a price for a period as long as the product lifetime. Some firms, like chainstores, publish a catalogue (a product-price couple) and stick to it for a while. It could be, for example, that agency situations imply this inertia: an employee that work as a seller is not allowed to lower the price of the product it sells, he is forced to apply the announced price. Situations like these do not fit the classical assumption that price competition occurs last, after everyone has observed which products are in the market. It seems rather that some firms commit to a price, while others react to these leader firms. This is precisely the heart of the analysis we conduct here. In the present model, we study the case where one firm enters first the market and, in addition to the location of its product, commits to its price. Then a second firm chooses its location and price, knowing the other's catalogue. This is a Stackelberg setting for the couple of variables (location, price).

The first consequence of this setting is a second mover advantage, because only the second firm can use an undercutting strategy. The second is that an equilibrium always exists. We qualify them depending on the willingness to pay of the consumers for the family of products, as introduced by Lerner & Singer (1937) and studied specifically in Economides (1984). The equilibrium

is unique (up to the symmetry) as soon as this parameter is high enough so that the market is entirely served. On the other hand, when the willingness to pay is small, the firms can make two local monopolies, and many locations are feasible, leading to multiplicity of equilibria. Moreover, we find that no monotonicity appears in the shape of equilibria. For an intermediate range of the parameter, the leader may price above or beyond the follower, and the distance between locations varies from one half to one ninth. The differentiation is thus intermediate. Finally for very high values of the parameter, the strategies and payoffs do not depend on the parameter any more.

The first section of the paper introduces the model, the second solves the case where the willingness to pay is unbounded (as in Hotelling's original model). In the third section, the equilibrium is qualified when the reserve price is finite. The fourth section contains a discussion of the model about sequentiality assumptions, and examples are given where the results seem to have some predictive power. The last part concludes.

2 The model

We study a market where consumers belongs to a segment [0,1]. Consumers are spread over the market according to a distribution function $f(x)$. First firm A chooses a location a on the interval and a price p, then firm B chooses location b and price q after having observed A's moves[1]. A consumer x chooses to purchase a product according to its own utility maximization: Max $(v - d(x,a) - p, v - d(x,b) - q)$. v is the reserve price, and a consumer buys a product only if the perceived price is inferior to the reserve price. In Hotelling's original model, consumers always consume the good, they thus minimize their disutility Min $(d(x,a) + p, d(x,b) + q)$. This case is studied in the next section and it corresponds to the case where v is infinite. With or without a reservation price, consumer x prefers firm B whenever $d(x,a)+p \geq d(x,b) + q$.

We will assume that when a consumer is indifferrent between firm A and B, it chooses the closest firm. If they have the same location or the distance is equal, we will only assume that some consumers choose A while others choose B. Moreover we will assume that a firm enters the market if and only if it makes a strictly positive profit.

[1] For convenience, we choose a and b as the distance of firms to 0. Note that in Hotelling 1929 b is the distance of product B to 1.

We look for the subgame perfect equilibrium of the game depicted in figure 1.

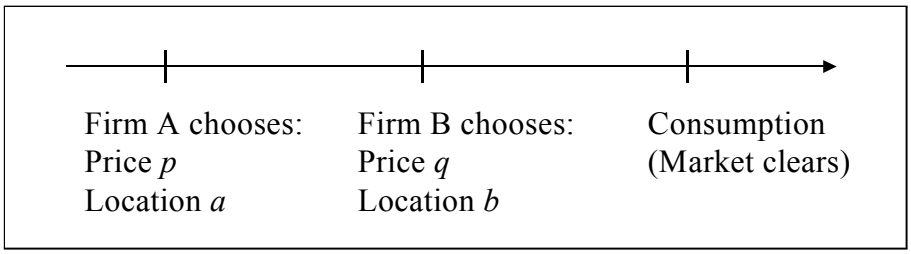

Figure 1: Game timing.

We may remark that if firm B chooses the same location as firm A, we must have $q \leq p$ otherwise firm B gets no client. By setting a slightly smaller price, firm B can get all the market. Thus if at equilibrium both firms choose the same location, both firms price at zero. This would correspond to a Bertrand competition, towards which Hotelling thought the firms would move. However this undercutting strategy should not be played at equilibrium. It is fundamental for the threat it represents, but if firms use it, they get no profits which suggests that at equilibrium firms choose different locations. As pointed out by Anderson (1987, p. 379), Hotelling seems not to have well taken these strategies into account, which lead him to a (partially) wrong conclusion. In the present setting, as the first firm commits to a price, it can not use an undercutting strategy. On the contrary, when firm B enters the market, it has a powerful threat with the undercutting strategy. As a result, we state the first proposition, which results does not depend on the distribution nor on the reservation price. The proof is also valid for any symmetric and single-peaked consumers' utility function.

Proposition 1 *If it exists a subgame perfect equilibrium, the follower makes a (weakly) greater profit than the leader.*

The proof is in the appendix.

We can derive from this first proposition that in our model the price followership effect is stronger than the location leadership effect. This statement is not general as Anderson (1987) in a different framework showed that a leader in location and price could earn slightly more than the follower in

location and price if firms first commit to a location before choosing their prices.

From now on we study the case where consumers are uniformly distributed along the segment $[0,1]$.

The follower has two different kind of strategies: either he decides to exclude the first player, this will be called the takeover strategy, or he decides to make a partial competition. The partial competition leaves firms with a monopoly on some of its demand. We could have alternatively called the takeover strategy the 'undercutting' strategy, but this would rather correspond to a setting where prices are chosen after both locations.

We will denote $\pi_i(a,p,b,q)$, for $i = A, B$ the profit of player i when strategies are a,p,b,q. Moreover we denote $\left(b^C, q^C\right) = \left(b^C(a,p), q^C(a,p)\right)$ the optimal strategy of firm B if he chooses to make a partial competition and similarly $\left(b^T, q^T\right) = \left(b^T(a,p), q^T(a,p)\right)$ the best strategy of firm B such that firm A is excluded.

Let's note that $\left(b^T, q^T\right)$ is a function of a,p which is not always well defined. Indeed while choosing the optimal takeover strategy, firm B acts such that consumers may be indifferent between the good offered by firm A and B. Thus everything depends on the convention on how the market is shared in this case. However this problem can be avoided because in the path of the subgame perfect equilibrium firm A cannot play a strategy such that this happens. We define $\Pi_B^T(a,p) = \pi_B\left(a,p,b^T,q^T\right)$ and $\left(b^T,q^T\right)$ such that firm B takes all the clients that are indifferent between firm A and B, we might note that whatever the way market is shared firm B always gets $\Pi_B^T(a,p)$ up to ϵ (by setting a price slightly under q^T, firm B gets the whole market). With this convention $\Pi_B^T(a,p)$ is a continuous function.

$\left(b^C, q^C\right)$ is a well defined function of (a,p) but the profits are not continuous. The first discountinuities appear when firm B plays a strategy such that the mass of indifferent consumers is not null. However if we make the classical assumption that indifferent consumers choose the closest firm there is only one discontinuity left, when $\left(b^C, q^C\right) = (a,p)$. However as we already stated, the first player cannot choose a situation where he would be excluded, thus in the path of the subgame perfect equilbrium we can arbitrarly define $\Pi_B^C(a,p)$ such that firm B takes only half of the remaining market when $\left(b^C, q^C\right) = (a,p)$. We may remark that in this case the threat of being excluded still exists with the takeover strategy. With this convention $\Pi_B^C(a,p)$ is a continuous function.

Without loss of generality we will assume that $a \in \left[0, \frac{1}{2}\right]$. We define $U_A(x) = v - d(x, a) - p$, moreover when there is no reserve price, we assume that $U_A(x) > 0$. The same notations are used for firm B.

Lemma 1 *In any subgame perfect equilibrium,* $\Pi_B^{C*} = \Pi_B^{T*}$.

This lemma will be useful for the characterization of the equilibria.

2.1 Without reserve price

In this subsection we assume that consumers always consume the good whatever the price. This assumption is the one used by Hotelling (1929).

In this case the optimal strategy of firm B if it decides to make a takeover on the leader's market is such that $\left(b^T, q^T\right) = (a, p)$ and thus by definition $\Pi_B^T(a, p) = p$.

Let us now compute firm B's best reply if it decides to make a partial competition. The optimal strategy is such that the consumer located at b earns the same from both firms, i.e. $q = p + (b - a)$. Indeed it is obvious that if it is not the case then an optimal deviation of firm B would be $(b - \epsilon, q)$ with ϵ sufficiently small (i.e. $0 < \epsilon < b - a + p - q$), this deviation obviously increases the size of firm B's market while the price remains constant. Thus firm B maximizes: $\Pi_B^C = (p + b - a).(1 - b)$ s.t. $0 \leq a \leq b \leq 1$ and $p > 0$, $q > 0$ (these constraints, that define the valid domain, will be implicit in the following). The profit is a concave function, first order condition implies: $\left(b^C, q^C\right) = \left(\frac{1+a-p}{2}, \frac{1+p-a}{2}\right)$. This solution is interior if and only if $a + p < 1$. Then $\Pi_B^C(a, p) = \frac{(1+p-a)^2}{4}$ and $\Pi_A^C(a, p) = p.\frac{1+a-p}{2}$. Moreover if $a + p \geq 1$ the second firm will prefer to take the whole market, thus the leader will choose $a < 1 - p$, or he gets zero profit. The program of firm A is then to maximize $\Pi_A^C(a, p)$ under the constraints

$$a < 1 - p \tag{1}$$

and

$$\Pi_B^T \leq \Pi_B^C \tag{2}$$

(the solution is interior *and* the follower prefers not to take the whole market). This last constraint yields $a \leq (1 - \sqrt{p})^2$ on the valid domain. As $\Pi_A^C(a, p)$ is increasing in a, one of these constraints will be binding at equilibrium.

But $(1-\sqrt{p})^2 < (1-\sqrt{p})(1+\sqrt{p}) = 1-p$ for all $p > 0$. Thus constraint (2) is the binding one. Substituting in the objective function, A's program boils down to maximizing $p.(1-\sqrt{p})$ for $p > 0$. The value of the equilibrium parameters are in turn fully determined by using binding constraint (2) and the expression of the best-reply. This allows us to state:

Proposition 2 *The subgame perfect equilibrium of the game is such that:*

$$\begin{aligned}(a^*, p^*, b^*, q^*) &= \left(\frac{1}{9}, \frac{4}{9}, \frac{1}{3}, \frac{2}{3}\right) \\ \Pi_A^* &= \frac{4}{27} \text{ and } \Pi_B^* = \frac{4}{9}\end{aligned}$$

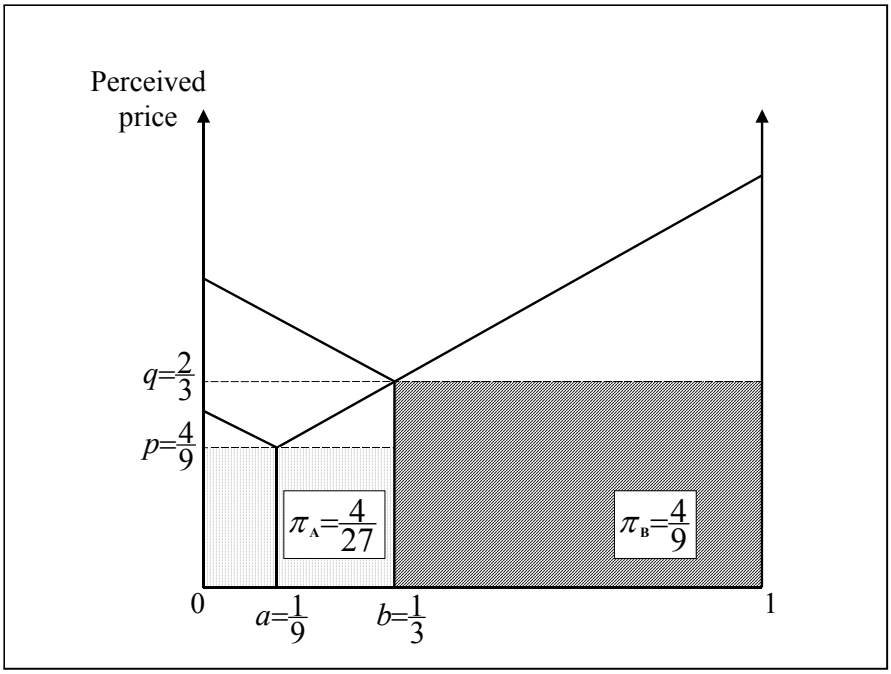

Figure 2: Equilibrium with infinite v.

The follower uses the umbrella price charged by firm A to fix its price, thus the follower prices higher than the leader.

We may also notice that the distance between the firms is such that no equilibrium with competitive price exists. It violates one of the existence conditions given by d'Aspremont, Gabszewicz and Thisse (1979). Thus if ex post

the leader was able to reset a new price after the entry of the follower then he would have an incentive to undercut the follower's price as in Hotelling (1929). However as there is no equilibrium in location which leads firms with positive profits if prices are chosen simultaneously, the leader wishes to irreversibly commit to a price in order to reach the equilibrium computed. The locations and prices at equilibrium are depicted in figure 2.

2.2 Extension when consumers have a reservation price

We now assume that each consumer has a reserve price at v. We first compute each best response of firm B according to the two strategies described above. The two following lemmas are quite general for any v, however some cases might not happen.

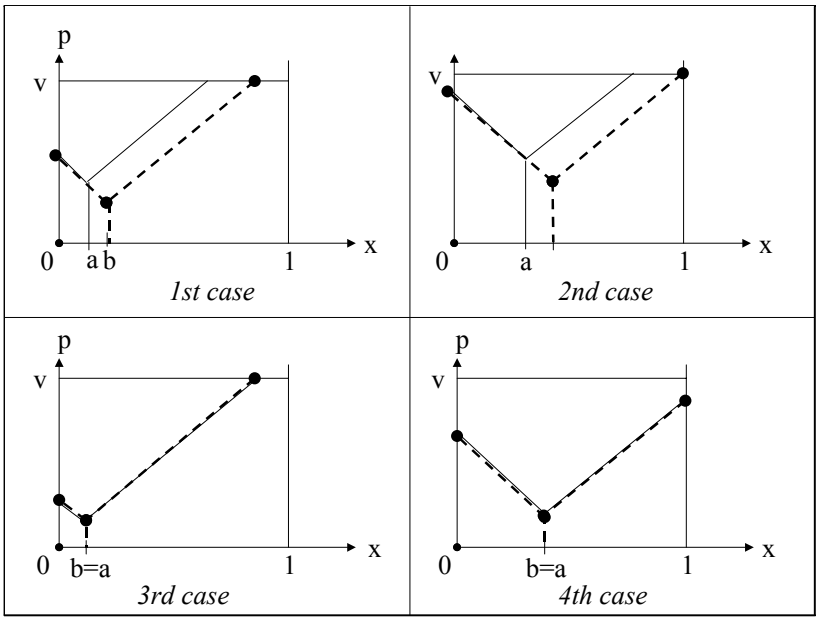

Figure 3: Takeover strategies.

Lemma 2 *(Takeover strategy)*

If $p - a > v - 1$ *and* $p + a < 2 - v$ *and* $p \geq \frac{v+a}{3}$ *then* $\left(b^T, q^T\right) = \left(\frac{3a+3p-v}{4}, \frac{v+a+p}{4}\right)$ *and* $\Pi_B^T = \frac{1}{8}(v + a + p)^2$.

If $p-a>v-1$ *and* $2-v\leq p+a$ *then* $\left(b^T,q^T\right)=\left(\frac{1-v+a+p}{2},\frac{v-1+a+p}{2}\right)$ *and* $\Pi_B^T=\frac{v+a+p-1}{2}$.

If $p-a>v-1$ *and* $p+a<2-v$ *and* $p\leq\frac{v+a}{3}$ *then* $\left(b^T,q^T\right)=(a,p)$ *and* $\Pi_B^T=p.(v+a-p)$

If $p-a\leq v-1$ *then* $\left(b^T,q^T\right)=(a,p)$ *and gets the profit* $\Pi_B^T=p$.

In the first case, the follower takes more market than what was initially served by the first firm, but the market is not entirely covered by firms. The second case corresponds to a case where the firm B takes all the market according to the strategy of the first firm. In the third case the follower imitates exactly the strategy of the first firm. Finally in the last case the firm A was initially covering the whole maket, the second firm can just do as well as the leader. The different cases are illustrated in figure 3.

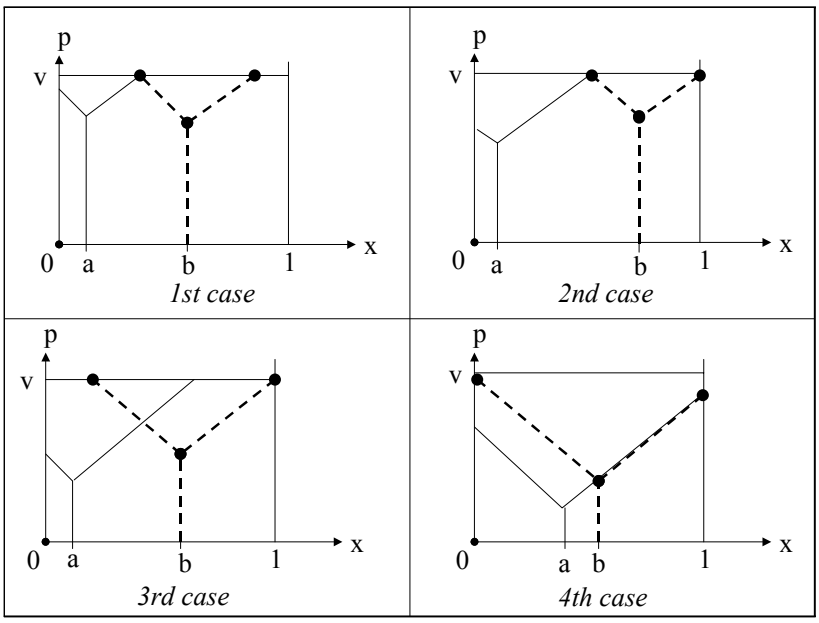

Figure 4: Partial competition strategies.

Lemma 3 *(Partial competition strategy)*

If $p-a\geq 2v-1$ *then* $\left(b^C,q^C\right)=\left(\frac{3}{2}v+a-p,\frac{v}{2}\right)$ *and* $\Pi_B^C=\frac{v^2}{2}$, $\Pi_A^C=p.(v+a-p)$

If $\frac{5}{3}v - 1 < p - a < 2v - 1$ *then* $\left(b^C, q^C\right) = \left(\frac{1+v+a-p}{2}, \frac{3v-1+a-p}{2}\right)$ *and* $\Pi_B^C = \frac{1}{2}.\left(3v - 1 + a - p\right).\left(1 - v + p - a\right)$, $\Pi_A^C = p.\left(v + a - p\right)$

If $v - 1 < p - a < \frac{5}{3}v - 1$ *and* $p + 3a < 3 - v$ *then* $\left(b^C, q^C\right) = \left(\frac{5-3v+p-a}{4}, \frac{1+v+p-a}{4}\right)$ *and* $\Pi_B^C = \frac{1}{16}\left(1 + v + p - a\right)^2$, $\Pi_A^C = p.\frac{3-v+a-p}{4}$.

If $p - a \leq v - 1$ *and* $p + a \leq 1$ *then* $\left(b^C, q^C\right) = \left(\frac{1-p+a}{2}, \frac{1+p-a}{2}\right)$ *and* $\Pi_B^C = \frac{1}{4}.\left(1 + p - a\right)^2$, $\Pi_A^C = p.\frac{1+a-p}{2}$.

In the competition case, it is not useful to define the best response in all points of the space as when this best reply is such that firm B enters at a then we know that it cannot be the subgame perfect equilibrium.

The first case is such that the leader left enough room for the follower to have a monopoly on its side. This case is the only one where the market is not entirely covered. In the second case the follower only takes the remaining market that the leader was not serving (this is the 'touching' equilibrium in Economides (1984)). In the third case, there is a real competition between firms and the utility of the indifferent consumer is stricly positive. In the last case the leader could alone serve the whole the market, and the competition is such that the follower chooses a different location but adjusts its price on the leader's one. The different cases are illustrated in figure 4.

Proposition 3 *For* $0 \leq v \leq \frac{1}{2}$ *both firms have a monopoly on their sides thus* $\Pi_A^* = \Pi_B^* = \frac{v^2}{2}$.

For $\frac{1}{2} \leq v \leq \frac{3}{2}$ *many configurations arise, as it is depicted on figures 5 and 6.*

For $v \geq \frac{3}{2}$ *the subgame perfect equilibrium does not depend on* v *any more, and firms have constant profits regarding* v*:* $(a^*, p^*, b^*, q^*) = \left(\frac{1}{9}, \frac{4}{9}, \frac{1}{3}, \frac{2}{3}\right)$ *and* $\Pi_A^* = \frac{4}{27}$ *and* $\Pi_B^* = \frac{4}{9}$.

We did not derive the explicit optimal strategy for firm A, as cases are too numerous especially for $\frac{1}{2} \leq v \leq 1$. We numerically calculate them on the basis of the best replies computed in lemmas 2 and 3. It is however straightforward to show that for $v \geq 2$ the optimal strategy is the same as the one defined in the case where consumers have no reservation price, because the only possible case of partial competition strategy is the fourth one[2] and thus we always have $U_A(1) > 0$. Further computation shows that

[2]Indeed the first case is only possible when $v \leq 1$, the second one when $v \leq \frac{3}{2}$ and the third one when $v \leq 2$.

this still holds when $v \geq \frac{3}{2}$. Indeed only the third and the fourth cases are then possible as best reply for firm B. The third case implies that the takeover strategy is the first one, thus the optimum for firm A can formally be computed. It is worse than if firm A chooses its optimal strategy with $U_A(1) > 0$.

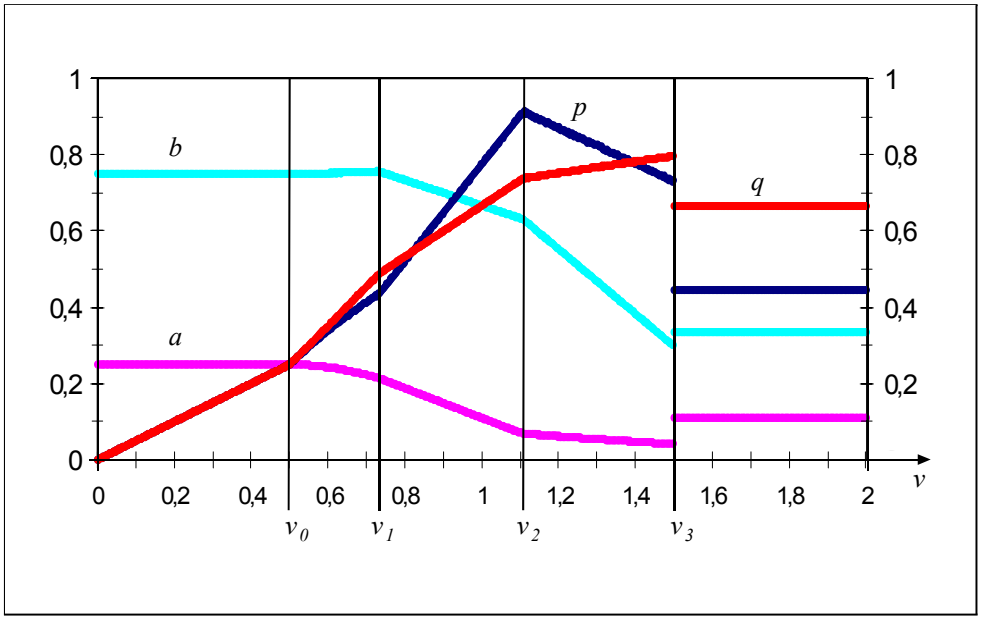

Figure 5: Equilibrium strategies as functions of v.

We may notice from figure 5 that there is a discountinuity for v close to $\frac{3}{2}$. This is due to the fact that while v is inferior to 2 there might be two local maxima for the profit of the first player (there are two when $v \leq 1.6$). The global maximum then switches from one to the other at $v = \frac{3}{2}$.

From figure 5 we can state that the leader chooses a more and more specific product while v increases. Indeed for given prices, firms can serve a bigger market and competition becomes then tougher. Thus firm A tries to specialize in order to avoid an undercutting strategy of firm B. While the market is well segmented when v is smaller than $v_0 = \frac{1}{2}$, the distance between the products of firm A and B decrease continuously with v (excepted for $v = \frac{3}{2}$). This observation is in fact in line with the principle of minimum differentiation: when v increases, the relative weight of the transport cost regarding the reserve price decreases thus products become more similar from

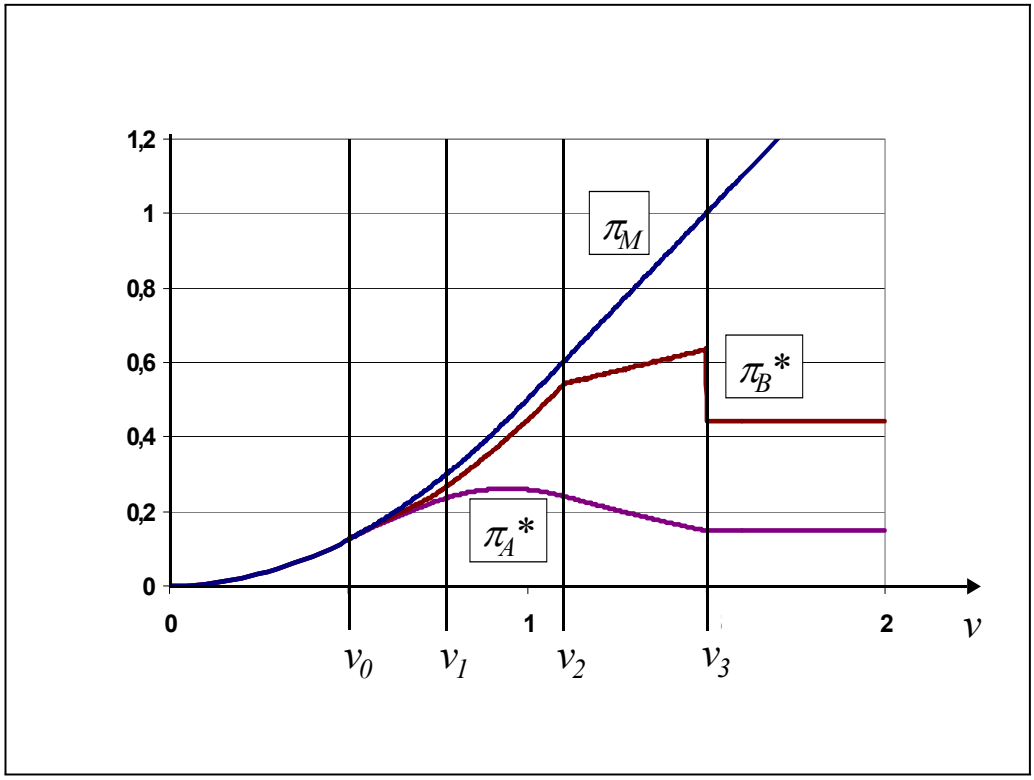

Figure 6: Equilibrium payoffs as functions of v.

the consumer's point of view and then firms apparently prefers to make closest products. The level of prices are almost increasing for both firms when v increase but it drastically decrease for $v = \frac{3}{2}$. Moreover price of firm A can be higher or lower than the one of firm B. Indeed firms have two levers regarding competition: price and location, and depending on the areas firms use either one or the other in order to capture more consumers.

Figure 5 also helps understanding how strategies change with v. When v is lower than v_1, the market is perfectly shared between firms: the indifferent consumer gets no surplus and he is located at $x = \frac{1}{2}$. However, the price of firm A is lower than the one of firm B in order to avoid a takeover strategy which allows firm B to almost get the monopoly profits. When $v_1 < v < v_2$ firm A still plays a strategy such that the firm B is not going to go for competition against the leader. The product chosen by firm A is more and

more specific but the price set by firm A increases and can become higher than the one of firm B. Indeed the opportunity of takeover strategies for firm B forces firm A to choose a specific product. When $v_2 < v < v_3$ the leader cannot avoid the competition with firm B, the indifferent consumer then gets a positive utility. The 'partial competition' strategy of firm B is then the third one. We can see from figure 5 that firm A then decrease its price slowly in order to be more competitive but keeps almost the same product. On the other hand, firm B prefers to choose a closer product to the one of firm A while increasing its price. For $v > v_3$ the strategy of firm A suddenly changes, firm A decreases hugely its price and chooses a product a little more generic, this is in fact its only way to avoid a takeover strategy of firm B. In this case the competition is tough as strategies and profits of both firms remain constant despite an increase in the reserve price of the consumers. This is the reason why B's profit is discontinuous at $v = \frac{3}{2}$. The strategy of firm A changes and in turn changes the form of *all* types of best reply, in particular the competition one, which is used in equilibrium, as well as the take over one.

v	a	b	p	q	π_A	π_B	A's MS*
0.5	0.250	0.250	0.250	0.250	0.125	0.125	0.500
0.75	0.207	0.750	0.458	0.500	0.229	0.250	0.500
1	0.109	0.776	0.667	0.667	0.259	0.444	0.333
1.25	0.058	0.845	0.509	0.759	0.203	0.576	0.241
1.5	0.111	0.444	0.333	0.666	0.148	0.444	0.333

* MS: market shares

Figure 7: Numerical examples

3 Discussion

3.1 Price commitment

In the litterature on horizontal differentiation, many different sequential settings have been studied. Hotelling (1929) who of course was not aware of Nash's work, studied a simultaneous price game given locations. As d'Aspremont, Gabszewicz and Thisse (1979) showed, there does not exist a Nash equilibrium when locations are too close. It turns out that there is a problem if firms can choose their locations first, because for any given location of firm A, firm B has an incentive to get the closest it can. Following Hotelling's reasoning, profits should in turn decrease to zero because of price war. Thus, whatever the setting (either sequential or simultaneous) of product choice, if prices are chosen simultaneously there is no subgame perfect equilibrium with linear transportation costs. In our model, we showed that a price commitment of one firm before the entry of the second firm restores the existence of a subgame perfect equilibrium. In a way there is some value to commit in a price for the first firm, because for some location and price, it discourages the other to make use of undercuting. However, in a sense, this price commitment is not credible as firm A has an incentive to change its price after firm B's moves. This is actually a standard problem with sequential price competition. This commitment has to be irreversible to be credible.

Another feature of the model that calls for a careful interpretation concerns relative flexibility of location and price. While in the geographical interpretation of the model the location choice is a long-term decision relatively to the price decision, one can think of markets where both variable are equally flexible. The catering industry seems to provide a good example of this. The product lifetime is very short, and the choice of the product can be a daily decision for independent restaurants, making product choice and price choice equally flexible. In contrast, for chains, uniform pricing and centralized decisions exhibits more inertia. In other words, chains make their price public by advertisement and national catalogues, while independent stores react to these leaders. The converse is obviously not true: local competition by a small competitor do not lead chains to revise their catalogue. Going deeper into this example, one can remark that the model also fits the observation that chain restaurants sell more specialized products (like pizzas or Tex-mex food), with relatively lower prices than small competitors.

This conclusion reached for v high enough corresponds to the consumers' behaviour when they leave office at noon to eat.

3.2 Sequential setting

When one introduces sequentiality in differentiation models, six possible sequential settings are to be considered. We already know that the (two) cases where firms choose prices simultaneously have no equilibrium. Up to our knowledge, Anderson (1987) is the closest paper about sequential prices and differentiation. He studies the two cases (2 and 3 in figure 8) where firms first sequentially choose their location and then sequentially choose their prices. He proves then that the most likely equilibrium to appear is the one with the location leader being subsequently the price follower (case 3). The explana-

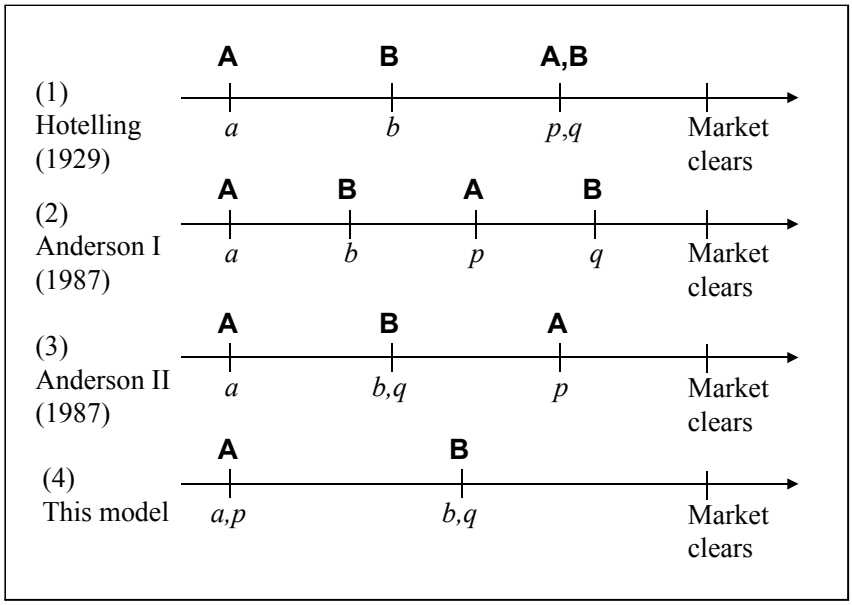

Figure 8: Timing comparison.

tion is that the location leader can choose a product such that the location follower prefers to be the price leader: the location leader then locates at the center of the market. The follower's profit in this case is 0.428.

A discussion arises about the credibility of the equilibrium he favours, using our timing structure. If the location follower decides to wait for the

location leader to post its price before choosing its product (which is our timing structure), it makes a greater profit 0.444. Thus if ex ante of the game the follower can credibly commit to make an attrition game until the leader posts a price then the most credible timing sequence of a game where each firm can choose its location and decide to post prices whenever it wants is that there will be a location leader, which will at the same time post its price and be a leader on price before the follower chose its location and price. However this commitment is not credible per se in our timing sequence: if the leader choose its optimal strategy as in Anderson's model ($a = 0.5$) then the follower has a huge incentive to choose its product before the leader posts its price. Indeed if the leader posts its price before the entry of the second firm he has to fix a really small price in order to prevent an undercutting strategy of the follower and thus he spoils the market for both firms.

Thus we can distinguish two cases. If the choice of product can become irreversible whithout fixing a price then Anderson's model is more credible: the leader chooses a generic product ($a = .5$) then the follower choose a specific product ($b = 0.869$) and its price while the leader is the Bertrand follower; prices are relatively high ($p_A = 1.277$, $p_B = 1.185$) as well as profits ($\Pi_A = 0.815, \Pi_B = 0.428$). We can note that in this case the leader is better off than the follower. On the contrary if the commitment of firms can only be a product-price couple thus if firm A cannot commit to a product without giving its price or if firm B can credibly commit to enter the market only after firm A chose its price, then our model is the most credible . The second firm has a second mover advantage. The leader chooses a specific product (a=1/9) with a low price ($p_A = 0.444$) in order to secure its market while the follower thrives under the umbrella price of the leader by choosing a more generic product ($b = .333$,$p_B = 0.667$). The profits are then low ($\Pi_A = 0.148, \Pi_B = 0.444$). We can note that this catalogue competition is fiercer than the Anderson's one. Moreover there is a value to commit to a product independently of a price only for the leader.

Our model is thus more consistent with markets where information that is usually available consists of catalogue, a product-price couple. The conclusion from this comparison is that a 'catalogue competition' is at least as credible as Anderson's alternate leadership equilibrium on markets where product choice is flexible.

4 Conclusion

Since Hotelling's 1929 work, many economists have investigated the so called principle of minimum differentiation. Whereas d'Aspremont Gabszewicz and Thisse (1979) show that there is in fact no equilibrium when prices are chosen simultaneously, we show that a sequential timing in both locations and prices is sufficient to restore the existence of an equilibrium. The equilibrium is such that: 1) the follower earns always more than the leader and this result holds whatever the distribution of consumers along the space and the (symmetric) costs structure; 2) when consumers are uniformly distributed, the leader makes the follower indifferent between excluding him from the market and choosing a location such that each firm has a fraction of the market. When the willingness to pay of the consumers is small, competition is soft between firms and it becomes tougher when the reserve price increases. Actually when consumers are not too picky on the product ($v \geq \frac{3}{2}$), the strategies of firms become constant regarding v, the competition is then fierce and firms choose close locations. As Economides (1984), we find that the value of the reservation price changes the structure of equilibrium, especially regarding the location of the products. The differentiation is indeed never minimal at equilibrium, and is even very high for low reservation values. One actually does not need to take quadratic distance costs à la d'Aspremont & al (1979) to obtain differentiated products in equilibrium.

The second mover advantage present in this model calls for a comparison with Anderson's (1987) model. While he privileged the case where the location follower is (endogenously) price leader, we show here that ex ante the follower will prefer to wait until the location leader announces its price before entering the market. However ex post of a leader's decision of location the follower will agree on being a price leader. Thus we might have opposite conclusion depending on the credibility of firms commitment in price as well as in location. In addition, when there is a sunk cost to bear to enter the market, there is a threshold entry cost such that the leader will deter entry.

This type of model can be interpreted as a model of competition through catalogue. There exist many markets where consumers get information only through advertisement and catalogue, and as such activities are costly, firms do not go continuously into them. In these situations, price commitment and price inertia are equivalent in terms of consequences on the market. The dynamic of prices should also be analyzed in terms of sequentiality of announces and heterogeneous reactive abilities in horizontal differentiation

models. It seems that the conclusions of the present model are consistent with the fact that big firms (firm A) still publish catalogues, even if they anticipate that smaller, more reactive firms will compete with them and earn bigger margin. The first entrant's catalog exhibits relatively low prices and specialized products, as can be seen from restaurants chains. The first catalog serves as reference for the followers, for prices as well as for the type of products. The credibility of such price-location commitment stems from the fact that in absence of commitment, profits are driven to zero by Hotelling's like location dynamics in which firms tend to get very close in locations.

5 Appendix

5.1 Entry barriers

In this paragraph, we investigate the role of entry barriers in the model. First, if we assume that each firm has the same entry cost, the first proposition implies that there are no entry deterring strategies played in equilibrium in the game with n possible entrants. Indeed, as a follower can always make as well as the last firm who entered the market, if the last firm was covering its entry cost, the follower is also able to do it. Thus as the leader can always earn a stricly positive profit, if the entry cost tends to zero, there is no upper bound on the number of firms to enter: the economy is fragmented and not segmented. This result is in line with the one of Shaked and Sutton (1987) on horizontal differentiation even though here firms choose sequentially both prices and locations.

If we now assume that the follower has to bear an entry cost f while the leader is an incumbent who has no set up cost, then there might exist strategies deterring the entry of a new firm. Its obvious strategy is to set a price $p = f$ and to choose the midpoint location $a = \frac{1}{2}$. The takeover strategy of the follower allows him to have a profit just less than f, and it is clearly its optimal strategy as a follower (competition strategies of the follower yields $\Pi_2^C = \frac{1+2f}{4}.\frac{3-2f}{4} < f$, because the solution is interior only if $f < \frac{1}{2}$). This strategy of firm A is optimal if $f > \frac{4}{27}$, as it allows the leader to get profit f, while when $f \leq \frac{4}{27}$ the leader has no interest in deterring entry of firm B. There is thus a threshold value of f over which firm A prefers to remain alone on the market, and is able to do so.

5.2 Proof of proposition 1:

Proof.

We prove that we cannot have $\Pi_B < \Pi_A$.

We assume that firm A chose a^*, p^* and firm B chose b^*, q^* we can prove that firm B has a better reply at $a^*, p^* - \epsilon$.

Let $\Delta = \Pi_A - \Pi_B > 0$.

It is obvious that if firm B leaves b then the support of captive consumers of firm A is larger. By pricing at $p^* - \epsilon$ more consumers will buy the product of firm B than what they would do if only firm A was there.

Thus the support $S'' = \{x | v - d(x, a^*) - p^* + \epsilon \geq 0\}$ is larger than $S' = \{x | v - d(x, a^*) - p^* \geq 0\}$.

Moreover S' is larger than $S = \{x | v - d(x, a^*) - p^* \geq 0 \text{ and } d(x, a^*) + p^* \geq d(x, b^*) + q^*\}$.

Let $\mu = \int_S f(x) .dx$

Thus the profit of firm B is: $\Pi_B(a^*, p^* - \epsilon) = (p^* - \epsilon.) . \int_{S''} f(x) .dx \geq (p^* - \epsilon.) . \int_S f(x) .dx \geq \Pi_1 - \epsilon.\mu$

Thus if $0 < \epsilon < \frac{\Delta}{\mu}$ then $-\epsilon.\mu > \Pi_B - \Pi_A$ thus we proved that $\Pi_B(a^*, p^* - \epsilon) > \Pi_B(b^*, q^*)$ and the deviation is profitable. By contradiction we can conclude that $\Pi_B \geq \Pi_A$.

■

5.3 Proof of lemma 1:

Lemma 4 *1 If we assume that indifferent consumers always choose the nearest firm and whatever the convention on how the market is shared (when both firms have the same location)*[3] *then when the set such that* $\Pi_B^C(a^*, p^*) = \Pi_B^T(a^*, p^*)$ *is non empty, there is a subgame perfect equilibrium and firm A's optimal strategy is* (a^*, p^*).

Proof. Clearly, if an equilibrium exists, firm A makes a positive profit; thus it must be the case that firm B prefers not to exclude firm A: $\Pi_B^C(a^*, p^*) \geq \Pi_B^T(a^*, p^*)$.

[3] In fact we only need to assume that either the leader keeps some consumers when both firms have the same location and price, or that the second firm prefers a partial competition when it is indifferent between making a takeover or a partial competition. Otherwise there is still a discountinuity problem and there might be no subgame perfect equilibrium.

Let us assume that $\Pi_B^T(a^*, p^*) < \Pi_B^C(a^*, p^*)$: we will show that it is impossible in equilibium. First, in this case, the optimal response of firm B is necessarily such that $b^* > a^*$. We separate in three cases:

· If $U_A(0) < 0$, firm A would have a better response, shifting position to the left while keeping the same price. This softens competition for firm A, without increasing the takeover desirability for firm B, thus enlarges (at least weakly) the support of consumers of firm A.

· If $U_A(0) > 0$, we show that firm A has a profitable deviation: $(a^* + \epsilon, p^* + \epsilon)$, for ϵ small enough. The optimal competition response of firm B does not change, as U_A remains the same on $[a^* + \epsilon, 1]$. However $\Pi_B^T(a^* + \epsilon, p^* + \epsilon) > \Pi_B^T(a^*, p^*)$: the desirability of takeover has increased. But as Π_B^T is continuous with respect to a and p, there exists ϵ small enough so that $\Pi_B^T(a^* + \epsilon, p^* + \epsilon) < \Pi_B^C(a^*, p^*)$, $U_A(0)$ is still positive and $a^* + \epsilon < b^*$. Firm B's optimal reaction is unchanged, the support of clients of firm A is the same and the price is higher, thus Π_A increases.

· If $U_A(0) = 0$ $(v - p^* - a^* = 0)$, first note that $p^* < \frac{v}{2}$ as otherwise $\Pi_B^T(a^*, p^*) = \Pi_M \geq \Pi_B^C(a^*, p^*)$. Moreover the support S^* served by firm A is stricly superior than $a^* = v - p^* > \frac{v}{2} > p^*$. We have thus $\Pi_A(a^* + \epsilon, p^* + \epsilon) = (p^* + \epsilon).(S^* - \epsilon) = \Pi_A^* + \epsilon.(S^* - p^* - \epsilon)$ thus as $S^* - p^* > 0$ we can take an $0 < \epsilon < S^* - p^*$ such that this deviation increases Π_A.

Finally, by contradiction we can conclude that we must have $\Pi_B^T = \Pi_B^C$ in any subgame perfect equilibrium.

On this set the second player always chooses a partial competition and $\left(b^C, q^C\right) \neq (a, p)$, thus $\Pi_A^C(a^*, p^*)$ is continuous. Moreover as both functions are continuous on this particular set, this set is closed. Thus as the first player maximizes a continuous function on a compact, there is at least one solution when this compact is non empty. Moreover every solution gives the same profit to the first player.

■

In the following, we will make extensive use of the notations:
$\overline{x_i}$: the greatest point at which utility of consumer buying product i is weakly positive.
$\underline{x_i}$ is defined similarly.
$\overline{y}$ is the indifferent consumer whenever it exists and is unique.

5.4 Proof of lemma 2:

Proof.

For this set of strategy, firm B wants to price as high as possible, under the constraint that it excludes firm A from the market. It is expressed, for instance, by $U_A(a) \leq U_B(a) \Leftrightarrow v - p \leq v - b + a - q$. And as firm B wants to maximize the price it sets, this constraint will be binding. We thus look for strategies having the property $b = p + a - q$.

1. Let's first assume that
$$v - 1 < p - a \qquad (\overline{S_1})$$
In this case, firm B does not necessarily serve consumers at 1. The profit is: $\Pi_B = q.Min\,(1, \overline{x_B}) = q.Min\,(1, v + a + p - 2q)$.
If an interior solution exists, it solves $Max\,[q.\,(v + a + p - 2q)]$ which solution is $q = \frac{v+a+p}{4}$, $b = \frac{3a+3p-v}{4}$ when interior and gives profit $\Pi_B = \frac{1}{8}\,(v + a + p)^2$.
This solution is valid if $a \leq b$, so if $a \leq \frac{3a+3p-v}{4} \Leftrightarrow$
$$3p - a \geq v \qquad (\mathrm{T}_1)$$
and if $\overline{x_B} \leq 1$, or
$$p + a \leq 2 - v \qquad (\mathrm{T}_2)$$

2. If $p+a > 2-v$, Firm B serves the whole market, being only constrained to let some positive utility to consumers at 0. The optimal strategy is $q = \frac{p+a+v-1}{2}$ and $b = \frac{p+a+1-v}{2}$ it gives a profit of $\Pi_2 = \frac{p+a+v-1}{2}$.

3. If $p < \frac{v+a}{3}$, the solution is not interior because firm B's unconstrained optimum does not permit to take A's market. So firm B chooses price p and location a. The profit is $\Pi_B = p.\,(v + a - p)$.

4. We now assume that $U_A\,(1) \geq 0$:
$$p - a \leq v - 1 \qquad (\mathrm{S}_1)$$
then the least costly strategy to exclude A is obviously $\left(b^T, q^T\right) = (a, p)$, yielding profit $\Pi_B^T = p$. ■

5.5 Proof of lemma 3:

Proof.

1. If firm A's strategy verifies $1 - \overline{x_A} \geq v \Leftrightarrow$

$$2v - 1 \leq p - a \tag{S$_2$}$$

firm B can act as a (local) monopolist. There might be a multiplicity of optimal strategies for firm B if it has more room than necessary for a monopoly. The profit is $\Pi_B = \frac{v^2}{2}$ with price $q = \frac{v}{2}$ and any location satisfying $\overline{x_A} \leq \underline{x_B} \leq 1$ or $\frac{3}{2}v + a - p \leq b \leq 1 - \frac{v}{2}$.

Assume now

$$v - 1 \leq p - a < 2v - 1 \tag{$\overline{S_1}$ \& S$_2$}$$

which says two things: there is no room for B to make a monopoly and $U_A(1) \leq 0$. First we can state that as firm A does not serve consumers at point 1, if firm B serves them, their utility level is zero. Moreover, it is always advantageous to take the consumers on the free side of the market for firm B, rather than not serving them. Thus in equilibrium we could have only $U_B(1) = 0$, or $b = 1 - v + q$.

2. The first possible strategy for firm B is to serve only the remaining market. The best of these strategies is to locate in the middle of the free market: $b = \overline{x_A} + \frac{1-\overline{x_A}}{2} = \frac{1+v+a-p}{2}$, and to set the minimal price to cover the free market: a price such that $U_B(\overline{x_A}) = 0 \Leftrightarrow q = \frac{3v-1+a-p}{2}$. This yields the profit $\Pi_B = \frac{1}{2}(3v - 1 - p + a)(1 - v + p - a)$.

3. The second possible strategy is to compete for a part of A's market in addition to take the free market. The profit of firm B is in this case $\Pi_B = q.(1 - y)$. We maximize this profit along $b = 1-v+q$. Substituting $y = \frac{1}{2}(a+b-p+q)$ and b, it becomes $\Pi_B = \frac{q}{2}(1 + v - 2q + p - a)$. It is concave in q and the first order condition implies: $-4q+1+v-a+p = 0 \Leftrightarrow q = \frac{1+v+p-a}{4} < v$ and $b = \frac{5-3v+p-a}{4} < 1$. The profit of this strategy is $\Pi_B = \frac{1}{16}(1 + v + p - a)^2$. However, for the solution to be valid, it should not correspond to a takeover case: we must have $U_A(a) > U_B(a)$. This yields the constraint

$$p + 3a < 3 - v \tag{C$_1$}$$

And in addition, this solution is only defined if $U_B(y) \geq 0 \Leftrightarrow 2v + a - b - q - p \geq 0$. Substituting q and b, it yields: $2v + a - \frac{5-3v-a+p}{4} - \frac{1+v-a+p}{4} - p \geq 0 \Leftrightarrow$

$$p - a \leq \frac{5}{3}v - 1 \tag{C$_2$}$$

To summarize, if $\frac{5}{3}v - 1 \leq p - a \leq 2v - 1$, the first type of strategy is preferred by firm B.
If $v - 1 \leq p - a < \frac{5}{3}v - 1$ and $p + 3a < 3 - v$, firm B prefers the second type of strategy, and finally if the last constraint is not satisfied, firm B prefers to take the whole market.

4. Assume finally that

$$p - a < v - 1 \tag{S$_1$}$$

which means $U_A(1) > 0$. Firm B will capture consumers at 1. The best reply will be such that $U_A(b) = U_B(b)$, as we have seen in the first part. Thus we have $b = a + q - p$ and $\Pi_B = q.(1 - b) = q.(1 - a - q + p)$. The profit is concave in q, and has an interior maximum if (first-order condition): $2.q = 1 - a + p \Leftrightarrow q = \frac{1+p-a}{2}$. For this solution to be valid, we must have $0 < q < v \Leftrightarrow -1 < p - a < 2v - 1$. But by assumption, $p - a < v - 1 < 2v - 1$. Moreover $b = \frac{1-p+a}{2}$ and we must have: $a < b \Leftrightarrow$

$$p + a < 1 \tag{C$_3$}$$

One can easily check that $b < 1$. Finally, the solution is interior if $p + a < 1$, and firm B gets $\Pi_B = \frac{1}{4}.(1 + p - a)^2$. When $p + a \geq 1$, firm B always prefers to make a takeover on the leader's market ($q^T = p$, $b^T = a$).

Finally the choices of strategy when firm B had different best response (i.e. in the first case) allow us to state that the profit of the firm A is always $\Pi_A = p.y$ where $y = \frac{a+b-p+q}{2}$.

■

References

[1] Anderson, S., 1987, Spatial competition and price leadership, *International Journal of Industrial Organization*, 5, 369-398.

[2] Bester, H., de Palma, A., Leininger, W., Thomas, J. and E.L. von Thadden, 1996, A noncooperative analysis of Hotelling's location game, *Games and Economic Behavior*, 12, 165-186.

[3] Dasgupta, P. and E. Maskin, 1986, The existence of equilibrium in discontinuous economic games, I: theory; II: applications, *Review of Economic Studies*, 53, 1-41.

[4] D'Aspremont, C., J. Jaskold Gabszewicz and J.F. Thisse, 1979, On Hotelling's 'Stability in Competition', *Econometrica,* 47, 1145-1150.

[5] Eaton, B.C. and R. Lipsey, 1975, The Principle of minimum differentiation reconsidered: Some new developments in the theory of spatial competition, *Review of Economic Studies,* 42, 27-50.

[6] Economides, N., 1984, The Principle of minimum differentiation revisited, *European Economic Review,* 24, 345-368.

[7] Hinloopen, J., and C. van Marrevijk, 1999, On the limits and possibilities of the principle of minimum differentiation, *International Journal of Industrial Organization*, 17, 735-750.

[8] Hotelling, H., 1929, Stability in competition, *The Economic Journal*, 39, 41-57.

[9] Lambertini, L., 2002, Equilibrium locations in a spatial model with sequential entry in real time, *Regional Science and Urban Economics*, 32, 47-58.

[10] Lerner, A. P. and H. W. Singer, 1937, Some notes on duopoly and spatial competition, *Journal of Political Economy*, 45(2), 145-186.

[11] MacLeod, W. B., 1985, On the non-existence of equilibria in differentiated product models, *Regional Science and Urban Economics*, 15, 245-262.

[12] Prescott, E.C. and M. Visscher, 1977, Sequential location among firms with foresight, *The Bell Journal of Economics*, 8, 378-393.

[13] Shaked, A., and J. Sutton, 1987, Product differentiation and industrial structure, *The Journal of Industrial Economics,* XXXVI, 131-146.