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Résumé: Un modèle de firme dans un cadre principal-multiagent avec sélection adverse est analysé. L'efficacité de la firme dépend d'activités directement productives ainsi que de l'ajustement entre ces activités. Cet ajustement peut-être explicitement coûteux. La spécificité du modèle est que l'information privée d'un division ne peut pas être ordonné objectivement, comme il est possible dans les modèles standard utilisant la condition de Spence-Mirrlees. Cette spécificité induit un profil de rente non-monotone. Cependant, sous une certaine condition, l'optimum de premier rang peut être implémenté par le centre. Cette condition est reliée à la possibilité pour le centre de créer des incitations contraires "bayésiennes".

Abstract: We analyze a simple firm model in a principal multiagent framework under adverse selection. The firm's efficiency depends on the effort devoted to productive activities as well as on the fit between the divisions, for which costly coordination actions can be undertaken. The specificity of the model is that the hidden information may not be ranked objectively, as opposed to more standard models which assume the Spence-Mirrlees condition. This specificity ordinarily induces a non-monotous rent profile over the types and might lead to pooling. Nonetheless, a sufficient condition is given for the rents to be completely eliminated. It is related to the Principal's ability to create "bayesian" countervailing incentives.

Mots clés : multi-agent, sélection adverse, planification

Key Words : multi-agent, sélection adverse, planification

Classification JEL: L23, M11

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1 Introduction

This paper explores some aspects of decision making within a firm organized along functional lines (Chandler 1962). In such firms, planning may be associated to the elaboration of consistent material flow (purchasing, production, sales) of related and partially substituable products. During the year, the plan is regularly updated and the divisions are expected to adjust accordingly. To implement this process, the centre would like to enhance truth-telling from the divisions regarding the local uncertainties in order to elaborate and update the plan in the most profitable way. The divisions, on the other side, may prefer to report biased information so as not to support excessive adjustment costs during the year, and have no initiative by themselves if they are not compensated for. In this context, the role of the centre is to manage properly the interdependency of the divisions through a central plan. Following Simon (1973), Cremer (1980) studies the problem of "factoring the total system of decisions that need to be made into relatively independent subsystems, each one of which can be designed only with minimal concerns for its interaction with the others". Here we will focus on the efficiency of information gathering by the center, given the technological interdependency of the divisions and strategical behavior. Consider the example of an organization with a production division and a sales division. The production department should adapt products to the customers' demand, and alternatively the sales department can shift the customers' preferences towards the products of the firm through advertising and commercial behaviour. Each activity thus exert an externality on the marginal cost of the other.

The objective of this paper is to analyze this problem in a multiagent framework under adverse selection. The classical setting involves a productivity as private information of the agents, and under the Spence-Mirlees (or Single-crossing, or Sorting) assumption, the solution is well-known[1]. The most productive agents get the biggest rents, and less productive ones are asked a weaker effort than in the full-information. The optimal contract thus trades off the informational losses at the top of the types' distribution against effort inefficiencies at the bottom. For the multiagent setting, few papers in industrial economics consider the hidden information setting, none of them interdependency together with adverse selection. Demski and Sappington (1984) and Mookherjee (1984) give the main lessons for the multiagent case, building upon the standard single agent case. In short, the incentive scheme for one agent should depend

[1] The condition says that the marginal productivity varies the same way as productivity. A classical instance would be $\frac{\partial^2 c}{\partial e \partial \theta} > 0$ with c the cost of producing effort e and θ the efficiency parameter ($\frac{\partial c}{\partial \theta} > 0$). Screening by the principal is then possible through monetary transfers, because types differ in their trade-off between effort and allocation.
The seminal papers by Mirrlees (1971) and Spence (1973) illustrate well the economic content of this assumption, as well as the corresponding technical implications.

on the other's type if and only if it conveys information about the latter's. Efficiency results in case of correlation between types: the first-best outcome can be attained.

We show here that even in the absence of the classical screening tools (especially absent the Spence-Mirrlees condition), absent any correlation between types and absent risk-aversion, the principal (the centre) can still attain the first-best efficient outcome under some conditions. The key for the Principal is to create "bayesian countervailing incentives", or put differently, to build a contract that clouds the issue for each agent about the reorientation to be undertaken, by exploiting uncertainty on the other's type. In Cremer (1980), the optimal organization minimizes uncertainty at each division level, here it is important to preserve cross-divisional uncertainty for global efficiency. Another major difference with the model of Cremer (see also Aoki 1986) is the strategic aspect of information gathering. In these paper, the goals of the divisions are perfectly aligned with those of the whole firm.

To formalize the decision making under study, it is assumed that types have *relative* productive values relative to each other. In a sense, once the firm is on going, one should sell what has been produced and produce what has been sold. This setup may be formalized the following way: there exists an initial plan, that the centre would like to refine in response to shocks affecting the activity. When it takes place, the agent knows privately the realization of the exogenous shocks for their tasks, which induce deviations from the initial plan (they are the parallel of the costs shocks in Aoki (1986)). A new objective can be redefined to account for the private deviations. To adjust to the new plan, the agents incur a specific adjustment cost, in addition to the direct costs of operations. Thus the actions of the agents will be two dimensional efforts: the first dimension reflects the reorientation and the second reflects the productive effort (Ponssard and Tanguy 2001). While the second dimension is interpreted as the intensity of the effort in a strandard way, the first refers rather to the accuracy of the effort.

The paper is organized as follows: the next section describes the general model, the third presents the specificity of the effort in the single agent case, and the fourth part is dedicated to the coordination problem for two agents. The last part concludes and gives some perspectives for modelling an organizational structure subject to adjustment costs and incomplete information. All proofs are relegated to the appendix.

2 Description of the model

A Principal (she) owns a productive asset (a project) for which two agents s ($s = A$ or B) exert bidimensional efforts $\overrightarrow{e_s} = (e_s, \alpha_s)$. Each agent has type $\theta_s \in \Theta = \{\theta^i | i = 0..n\}$. Agent s incurs costs $C(\overrightarrow{e_s}, \theta_s) = A(e_s) + B(\alpha_s - \theta_s)$, and the corresponding total production is $F(\overrightarrow{e_A},\overrightarrow{e_B}) = \|\overrightarrow{e_A}+\overrightarrow{e_B}\|$.

The agents have private informations about their cost function: the orientation on which they start the game. This intends to capture the fact that the agents have a superior information about the realization of the hazards affecting their activity. They might be asked to change this orientation towards another one, at some cost.

Agent s has an initial type θ_s which belongs to the discrete set $\Theta = \{\theta^i | i = 0..n\}$. The types are uniformly distributed. So the principal has uniform beliefs on each agent's type, and each agent has uniform beliefs on the other's type. An agent always knows his type.

The set of actions is bidimensional and continuous: $\overrightarrow{e_s} = (e_s, \alpha_s)$. e is the norm and is called the "pure effort", α is the direction in which pure effort is exerted. α is of the same nature as the types: we assume $(e_s, \alpha_s) \in \mathbf{R}_+ \times [\theta^0, \theta^n]$. The monetary costs of efforts are additively separable in orientation and pure effort: $C(\overrightarrow{e}, \theta) = A(e) + B(\alpha - \theta)$, $A(e)$ for the realization of pure effort e and $B(\alpha-\theta)$ for the orientation effort where θ is the initial position of the agent (his type) and α the final position he achieves. Only the distance of the reoriention matters, not the path. We will mainly assume $A(e) = \frac{1}{2}e^2$ and $B(\alpha - \theta) = c|\alpha - \theta|^k$ with $k \in \{1, 2\}$. Note that with $k = 0$, the problem is a classical one, with a simple additive production function with respect to the pure effort, the costs being $1 + \frac{1}{2}e^2$.

The production function is the vector sum of efforts: $F(\overrightarrow{e_A},\overrightarrow{e_B}) = \|\overrightarrow{e_A}+\overrightarrow{e_B}\|$, so the best outcome is reached when $\alpha_A = \alpha_B$, that is when the agents exert aligned effort.

The problem for the principal is to define the better orientation for the project: which direction should she impose to the agents? We denote ϕ this strategical variable, we call the objective. A useful decomposition of the production function is the following, when ϕ is the ex-post realized orientation[2]:

$$F(\overrightarrow{e_A},\overrightarrow{e_B}) \equiv R(\overrightarrow{e_A},\overrightarrow{e_B},\phi) = e_A.\cos(\phi - \alpha_A) + e_B.\cos(\phi - \alpha_B)$$

In the following paragraphs, we examine the benchmark case (in complete information). We begin with the single agent case then study a two-agents setting. Then we come to the multiagent adverse selection problem.

3 Optimal production for a single agent

In this section, we look for the optimal behaviour of an agent, given ϕ. The Principal has an asset to which the agent can adjust, and she wants to induce the better combi-

[2] setting $\overrightarrow{u} = (1, \phi)$, $R(\overrightarrow{e_A},\overrightarrow{e_B},\phi) = \overrightarrow{e_A}.\overrightarrow{u} + \overrightarrow{e_B}.\overrightarrow{u}$ and if $\overrightarrow{u} = \frac{\overrightarrow{e_A}+\overrightarrow{e_B}}{\|\overrightarrow{e_A}+\overrightarrow{e_B}\|}$, the ex-post orientation, then $R(\overrightarrow{e_A},\overrightarrow{e_B},\phi) = \frac{\overrightarrow{e_A}+\overrightarrow{e_B}}{\|\overrightarrow{e_A}+\overrightarrow{e_B}\|}.(\overrightarrow{e_A}+\overrightarrow{e_B}) = \|\overrightarrow{e_A}+\overrightarrow{e_B}\|$.

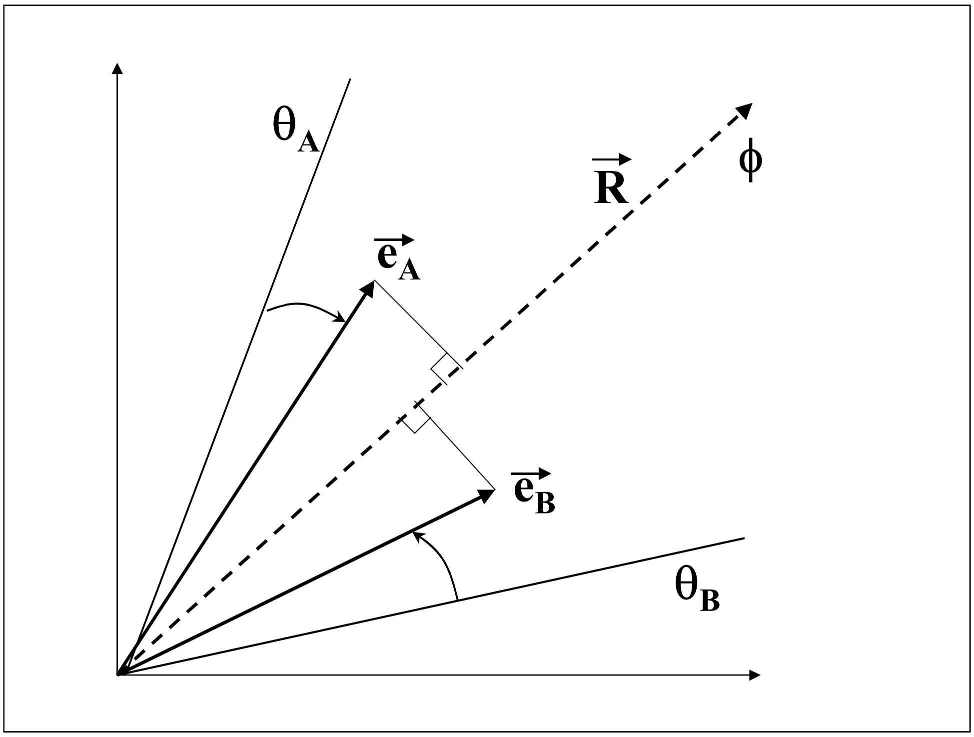

Figure 1: Graphical representation

nation of adjustment and quantitative effort. As the degree of adjustment modifies the productivity of the pure effort, the two efforts are partially substitutes and a trade-off appears between the two costs. The production function is the projection of the vector effort on the objective direction. In this case, the optimal efforts solve[3]:

$$\underset{e,\alpha}{Max} E_\theta[e.\cos(\alpha-\phi) - A(e) - B(\alpha-\theta)]$$

Note that the corresponding standard problem (without adjustment) would be: $\underset{e}{Max} E_\theta[e.\cos(\theta-\phi) - A(e)]$. The (interior) solution would be given by $A'(e^*) = \cos(\theta-\phi)$. α and θ together represent the classical productivity parameter.

Proposition 1 *The optimal production exhibits intermediate adjustment. Perfect alignment to the objective ($\alpha^* = \phi$) is optimal only if the agent is already perfectly aligned ($\theta = \phi$). With the specifications, the optimum is given by:*

$$e^* = \cos(\alpha^* - \phi) \tag{1}$$

$$B'(\alpha^* - \theta) = \frac{1}{2}\sin(2(\phi - \alpha^*) \qquad or \qquad \alpha^* = \theta \tag{2}$$

[3] The corresponding standard problem without adjustment would be: $\underset{e}{Max} E_\theta[e.\cos(\theta-\phi) - A(e)]$. The (interior) solution would be given by $A'(e^*) = \cos(\theta-\phi)$. α and θ together represent the classical productivity parameter.

The (simple) proof is omitted. The qualitative result is quite general: it holds for any functions A and B convex and any production exhibiting a C^1 maximum at ϕ ($\frac{\partial F}{\partial \alpha}(\phi) = 0$).

In the case of linear rotation costs, the optimal angle to reach take the following expression, where $\Delta(c) = \frac{1}{2}\arcsin(2c)$:

$$\begin{array}{cc} \text{If } |\theta - \phi| > \Delta(c) & |\alpha^* - \phi| = \Delta(c) \\ (C \text{ holds}) & e^* = \cos(\Delta(c)) \end{array}$$

$$\begin{array}{cc} \text{If } |\theta - \phi| \leqslant \Delta(c) & \alpha^* = \theta \\ (C \text{ does not hold}) & e^* = \cos(\theta - \phi) \end{array}$$

With linear adjustment costs, the optimal technology to use (α^*) does not depend on the type of the agent as soon as he is initially "far" from the objective. It might be that the agent is already sufficiently close to the objective to make adjustment worthless. The following picture illustrates these results.

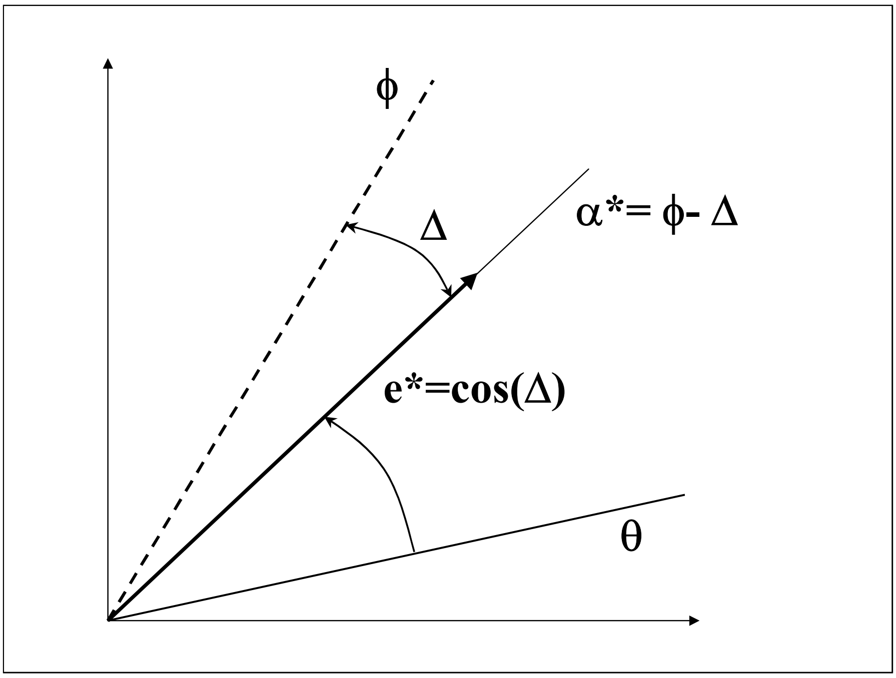

Figure 2: The single agent case

Given ϕ, we have seen how possible adjustment should be dealt with by the principal. We now come to the two agents setup. ϕ will no more be exogenous, but chosen by the principal in response to the deviations of each agent.

4 Team objective under adverse selection

4.1 The full information benchmark

The Problem of the Principal is now to define ϕ, which is now a strategical variable. There is no exogenous constraint on the objective that shall be used: only the agents' types condition the operations to be undertaken. The principal has to decide if adjustment will be incurred by the agents, and if so by which one(s), in order to have the team exploit the synergy, that is be aligned. Her program is:

$$\underset{\phi,e_A,e_B,\alpha_A,\alpha_B}{Max}[e_A\cos(\alpha_A-\phi)+e_B\cos(\alpha_B-\phi)]-\frac{1}{2}(e_A^2+e_B^2)-(B(\alpha_A-\theta_A)+B(\alpha_B-\theta_B))$$

It is clear that the objective should lie somewhere in the interval between the two types. This suggests that optimal adjustment implies a kind of compromise on the objective between the agents. The problem for the principal, learning from the single agent case, is to minimize the costs of adjusment, knowing that the adjustment conditions the productive effort. The first order approach is still valid, and we derive the following result from the first-order conditions:

$$e_A^*\sin(\alpha_A^*-\phi)-e_B^*\sin(\phi-\alpha_B^*)=0 \qquad (\text{FOC}_\phi)$$

We already know the four others from the preceding part, and substituting them in (FOC_ϕ), we get[4]:

$$B'(\alpha_A^*-\theta_A)=B'(\theta_B-\alpha_B^*) \qquad (3)$$

This relation has a strong implication: if the costs are convex, it implies that the optimal adjustment is symmetric, and thus $\phi^*=\frac{\theta_A+\theta_B}{2}$, the other variables being defined as in the single agent case. So the main point is that with convex adjustment costs, the optimum is to spread equally the adjustment costs between the agents. The Principal treat them exactly the same way, only the relative positions matter in the optimal contract. This is because adjustment is a matter of distance, not an absolute ranking problem. These results are synthetized in the following proposition:

Proposition 2 *If adjustment costs are strictly convex, the only optimal objective is the mean of types. If adjustment costs are linear, the principal can choose any objective in an interval (possibly reduced to one point) around the mean of types.*

This proposition is a straightforward consequence of the first one, and the proof is therefore omitted. We show in the following that this freedom of manoeuvre is crucial. It allows the principal to restore the first-best outcome under adverse selection.

[4] This is true at the optimum even if C_1 does not hold. If C_1 does not hold for both agents, then the first-order conditions for α_A and α_B implies that no adjustment is necesssary, then (3) is obvious. If C_1 is weakly satisfied for one agent at the optimum, it is for both.

4.2 General adverse selection setting

We now look for the optimal direct revelation mechanism. We assume perfect ex-post observability, so there is no moral hazard problem on the pure effort[5]. The problem for the principal is to have the agents announce their true deviations from the initial plan and to define the new objective to minimize adjustment losses. As the initial orientation is not known, adverse selection arise only because of the reorientation costs. The specificity is that the announce by an agent conditions not only his compensation, but also affects the objective chosen by the principal, therefore the surplus and the other's compensation. We denote $t_s(\overrightarrow{e_s}, \theta_s)$ $s \in \{A, B\}$ the wage of agent s when he announces that his type is θ_s and he exerts effort $\overrightarrow{e_s}$. To simplify notations, we denote $\overrightarrow{e_s} \equiv \overrightarrow{e_s}(\theta_s, \theta_{-s})$, but it is crucial to keep in mind that an agent's optimal effort is a function of the two types. The timing of the game studied from now on is detailled in the next figure.

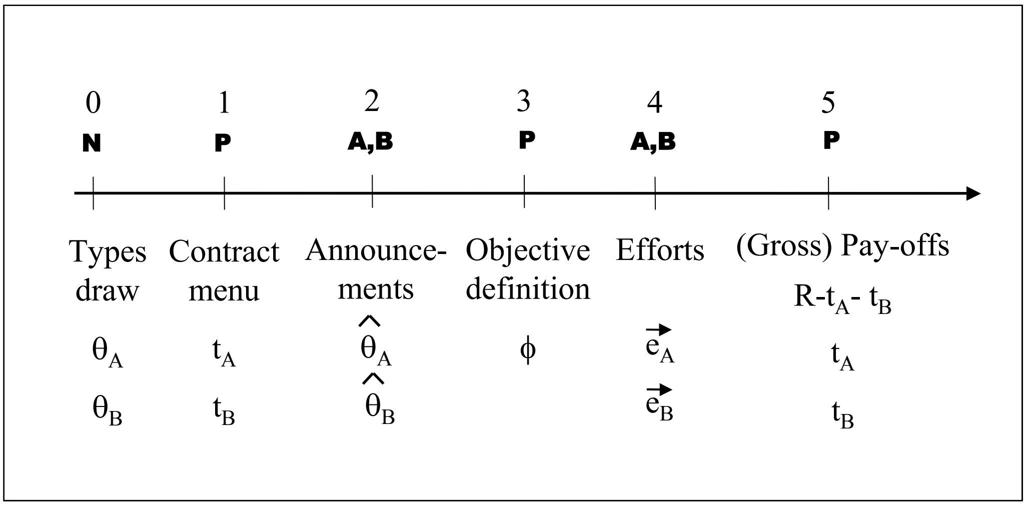

Figure 3: Contract timing

As each agent does not know the other's type, his compensation is defined in expectation over his teammate's type. P can build a contract in which the agents get a strictly positive wage if only if they behave according to the annoucement. This comes from the fact that we assumed complete ex-post observability. To summarize, the rationnality constraint is sufficient to impose the objective, and there is one for each type of agent. As the agents are identical ex-ante from the principal point of view, the contract menu is the same for both. Step 3 corresponds to the indication of which efforts the agents are intended to exert. As they do not know each other's type,

[5]In fact, the results hold even if only the personal contributions are observable. As the optimal behaviour to reach a given output is unique, the observation of $e.cos(\alpha - \phi)$ is a sufficient statistic for the costs of effort given his true type.

they need this coordinatioon signal by the principal. We use the convention: if $s = A$, $-s = B$ and vice versa.

The general Program for a revealing contract is:

$$\underset{t,\overrightarrow{e_A},\overrightarrow{e_B}}{Max} E_{\theta_A,\theta_B}[\|\overrightarrow{e_A} + \overrightarrow{e_B}\| - t(\overrightarrow{e_A}, \theta_A) - t(\overrightarrow{e_B}, \theta_B)]$$

$$s.t. \quad E_{\theta_{-s}}[t(\overrightarrow{e_s}, \theta_s) - A(e_s) - B(\alpha_s - \theta_s)] \geqslant E_{\theta_{-s}}[t(\overrightarrow{e_s}, \widehat{\theta_s}) - A(e_s) - B(\alpha_s - \widehat{\theta_s})] \quad \forall(\theta_s, \widehat{\theta_s}) \tag{IC}$$

$$t(\overrightarrow{e_s}, \theta_s) - A(e_s) - B(\alpha_s - \theta_s) \geqslant 0 \quad \forall \theta_s \tag{IR}$$

$$s = \{A, B\}$$

The first constraint states that agent s whose type is θ_s prefers (at least weakly) to report his true type, rather than any other $\widehat{\theta_s}$, in expectation over the other's type, θ_{-s}. The second constraint (IR), is the ex-post participation constraint, with reservation utility at zero. It states that whatever the other's type, an agent gets at least his reservation utility if he participates and behaves according to the contract. Recall again that the required effort is a function of both types. We do not make explicit the objective ϕ here, but instead use $(\alpha_{ij}, \alpha_{ji})$, the couple of positions that the agents must reach when the types are θ^i and θ^j. It is striclty equivalent for the principal to announce the two positions or to announce an objective. For the following analysis, it is however easier to use the individual directions. A star denotes the value of the variable in the First-Best solution.

Lemma 1 *With r^i the informational rent of type i agent, the constraints are equivalent to :*

$$r^i - r^j \geqslant E_{\theta^k}[B(\alpha^*_{jk} - \theta^j) - B(\alpha^*_{jk} - \theta^i)] \quad \forall(i, j) \tag{IC$_r$}$$

$$r^j \geqslant 0 \quad \forall j \tag{IR$_r$}$$

This formulation of the problem is till now general for a revealing contract. The principal's set of strategies consists of two elements: function ϕ (or indifferently functions α) that defines an objective for each possible agent couple and the rent profile over the types. The objective determines uniquely each optimal effort in cascade, and the rent profile gives the premium conceded to each agent to induce truthful announcement. These two levers together describe the contract and subsequent games entirely. We now make a crucial assumption: the principal can not commit not to take the optimal decision for him at $t = 3$, and we look for perfectly revealing contracts. From a purely technical point of view, it is a restriction on the set of admissible contracts.

However, it seems more realistic to assume that the managing board of a firm will not commit to an objective that is not ex-post optimal. Moreover, if we find one such objective that allows to attain the first-best, it is necessarily also optimal ex-ante. If it is not the case, this does not mean that the contract we found is the optimal one: we could have ex-ante better solutions with (partial) pooling, but not really credible from a managing perspective.

4.3 The rent profile

To compute the rent profile, we specify the adjustment cost function: $B(\beta) = \frac{c}{2}\beta^2$. Note that $B'(0) = 0$, so the condition (C) holds for any type and any objective: there will be interior solutions for α. The case where $B'(0) > 0$ only adds a little complexity (a limit angle comparable to $\Delta(c)$ in the linear case has to be considered), but it does not changes qualitatively the rent profile. As already seen, the Principal will always choose the mean of type as the objective, because B is strictly convex (From equation (3)).

Let us turn back to the incentive constraints. With these adjustment costs and types' set, they become:

$$r^i - r^j \geqslant \frac{c\pi}{80}(i-j).[2.(\alpha^*_{j0} + \alpha^*_{j1} + \alpha^*_{j2} + \alpha^*_{j3} + \alpha^*_{j4}) - \frac{5\pi}{8}.(i+j)] \qquad (\mathrm{IC}^{ij})$$

It is obvious to see that no agent will announce a deviation in opposite direction of his true one or a weaker one. We shall focus on the set $\{IC^{10}, IC^{01}, IC^{21}, IC^{12}, IC^{20}, IC^{02}\}$; by symmetry of the deviations, we can rule out the similar constraints for types 3 and 4.

Constraints (IC^{01}) and (IC^{10}) as well as (IC^{21}) and (IC^{12}) are compatible:

$$\frac{c\pi}{80}.[2.(\alpha^*_{00} + \alpha^*_{01} + \alpha^*_{02} + \alpha^*_{03} + \alpha^*_{04}) - \frac{5\pi}{8}] \leq r^1 - r^0 \leq \frac{c\pi}{80}.[2.(\alpha^*_{10} + \alpha^*_{11} + \alpha^*_{12} + \alpha^*_{13} + \alpha^*_{14}) - \frac{5\pi}{8}] \qquad (\mathrm{IC}_{10}+\mathrm{IC}_{01})$$

Similar brackets hold for $r^2 - r^1$ and $r^2 - r^0$. We show that the only sides where they bind are:

$$r^1 - r^0 \geqslant \frac{c\pi}{80}.[2.(\alpha^*_{00} + \alpha^*_{01} + \alpha^*_{02} + \alpha^*_{03} + \alpha^*_{04}) - \frac{5\pi}{8}] \qquad (\mathrm{IC}^{10})$$

$$r^2 - r^1 \geqslant \frac{c\pi}{80}.[2.(\alpha^*_{10} + \alpha^*_{11} + \alpha^*_{12} + \alpha^*_{13} + \alpha^*_{14}) - \frac{15\pi}{8}] \qquad (\mathrm{IC}^{21})$$

The optimal revealing contract is then entirely defined. The following proposition summarizes this.

Proposition 3 *The Principal leaves rents to the agents if the rotation costs are convex.*

The best fully revealing contract provides greater informational rents to medium-deviating agents than to non-deviating agents. The most deviating agents receive no rents.

It is worth noticing that the rent profile is not monotic. Actually, the middle agent's rent is a consequence of the small deviating agents' rent: the only incentives that agent θ^2 have to lie is to capture r^1. But doing it is costly as this pays only half of the time, thus $r^1 < r^2$. He is not interested in pretending a great deviation per se: because of the symmetry, incentives towards each side compensate mutually.

Had the principal the choice of one and only one agent, he would choose an agent θ^2 because he is the more versatile (he will induce less adjustment costs in expectation), but this agent does not get the biggest rent. By contrast, in classical adverse selection models, the preferred agent gets the biggest rents.

The next section is dedicated to a case where the principal can overcome the loss due to incomplete information. Demski and Sappington (1984) show that a correlation between types allows this for risk-neutral agents: an informative signal is provided by one agent's type about the other, and the principal can restore full-information pay-off by enforcing only one agent to tell his true type, the other playing his best response to that. In the present setting, the mechasism is somewhat different but also uses a link between the agents through the mutual adjustment.

4.4 Exploiting a freedom of manoeuvre

Let us now turn back to relation (3). We have seen that with strictly convex orientation costs, it implies a unique efficient objective, the symmetrical one. But with linear adjustment costs, there might be many optimal objectives. This comes from the fact that linear costs can be spread indifferently among the agents if reorientation has to be undertaken. This freedom in the distribution of adjustment is only constrained by the limit angle $\Delta(c)$, below which no reorientation is valuable.So if the agents are to close (including the alignment case), only the symmetrical objective is optimal, but when the agents are distant enough, there is a range in which any optimal objective is optimal. To clarify this assume w.l.o.g. that $\theta_A < \theta_B$. The formal expression of the preceding argument is: If $\theta_B - \theta_A < 2.\Delta(c)$, $\phi^* = \frac{\theta_A + \theta_B}{2}$, and if $\theta_B - \theta_A \geq 2.\Delta(c)$, $\{\phi^*\} = [\theta_A + \Delta(c); \theta_B - \Delta(c)]$.

The two extreme cases should be looked at briefly: If c is equal to zero, it is obvious that the agents can not gain anything from their private knowledges, the principal can ask perfect alignment at no cost. At the opposite, if c is very high[6], that is when $2.\Delta(c) > \underset{\theta_A, \theta_B \in \Theta^2}{Max} |\theta_A - \theta_B|$, reorientation should never occur, it is too costly. Thus no agent will be compensated for any adjustment, and they will have no incentive

[6] Δ is an increasing function of c.

to overstate their deviations: the full-information efficient case can be restored[7]. But between this two extreme cases, lies the problematic one, where adjustment is desirable. In this case, the principal can design a contract using the freedom of manoeuvre that has just been described to eliminate informational rents.

As already explained, she has to deal with the incentives of the "mid-deviating" agents (θ^1 and θ^3) to overstate their deviations. This comes from the fact that an extreme agent will have a priori the greatest expected adjustment costs to incur. But the freedom of manoeuvre allows the principal to weaken these expected costs so as to make the "mid-deviating" agents (at least) indifferent between announcing their true type and exagerating. The matter is that this should not perturbate too much the other's incentives, for example that of a non-deviating agent (θ^2), as it is the case with convex costs. The following proposition tells that this can be done in the five-types setting. (We restrict to this case, which already counts 5^2 objectives to set...)

Proposition 4 *If the orientation costs are linear, there exists a revealing contract that restores the first-best pay-offs.*

The proof is constructive. The idea behind the contract is to always have the agent that has deviated the less bearing the adjustment costs; this minimizes the more-deviating agents expected wage, and in turn discourages the mid-deviating agents to overstate their deviations. As recognized by Lewis and Sappington (1989), creating countervailing incentives can lower the cost of information revelation. In the present case, the principal uses the flexibility of the objective to create informational countervailing incentives. Relying on the uncertainty about the other agent's type, she mitigates the incentives of overstating deviations. The technological interdependency is the source of potential inefficiencies, but it also plays the role of correlation in Demski and Sappington (1984)

5 Conclusion

The main purpose of this paper was to point out the role of adjustment costs and incomplete information in a functional hierarchy. As many papers regard the technology used as independent for each division, or focus on the allocation of tasks that are initially unrelated, they emphasize ex-post links between the efforts through the compensation scheme. By contrast, this paper intends to study the case where the agents are strongly interdependent, in the production itself. This typically applies to the case of sales and production or of supplies and production, or project management, where information on the solutions to adopt is revealed over time. The planification

[7] In fact, these two extreme cases arise with any function B with $B\prime(0) > 0$.

process should integrate this interdependency, which is not often explicitly modelled. The associated "qualitative" coordination problem can not be captured by a standard quantitative productivity parameter (cf next paragraph). It has been shown that in such a framework the means for having the agents contributions converge depends strongly on the nature of the adaptation costs. If they are linear, heterogeneity can not be a source of rents for the agents, thanks to the freedom of manoeuvre in the sharing out of adjustment costs. Having the more deviating agents bear the minimal costs of adaptation, in order to discourage others to lie in the planning stage. Thus this paper gives a formal rationale to the idea that a more flexible firm will be more efficient for reacting to local uncertainties, even under strategic behaviour of the different entities. The flexibility here lies in the freedom in allocating reorientation costs.

Moreover, one point has to be emphasized: relative productivity means that the Spence-Mirrlees condition does not hold, at least in this model. The Spence-Mirrlees condition states that the marginal utility (or equivalently here the costs of orientation) can be sorted with respect to the hidden parameter. Considering the two divisions as a single player, it would hold, as the private information would consist of the distance between the two types, a parameter affecting monotonically the surplus. Thus under collusion of the units, the Centre would face the classical adverse selection problem. The closer the divisions, the greater rents they would earn. That is, two "aligned" divisions ($\theta_A = \theta_B$) have indeed an incentive to pretend they are not in phase to be compensated for adjustment they will not actually incur. This issue of collusion (not necessarily detrimental) within a firm has been dealt with in a somewhat different setting by Itoh (1992).

Finally, since the seminal work of Chandler, much attention has been devoted to the internal organization of the firm. Information economics has allowed to handle many issues in this context, and given some answers to the questions raised by Simon among others. Early formalized works have focused mainly on operations research aspects of information processing, with leading material being the theory of team by Marschak and Radner (1972). Authors like Cremer (1980), Arrow (1985) and Aoki (1986) have subsequently derived theories of organization relying on informational efficiency. The theory of incentives has then enriched this literature by taking into account the differing goals pursued by members within an organization (for example Demski and Sappington (1983), Mookherjee (1984)). Due mainly to technical tractability, very few papers deals also with technological issues. Itoh (1991, 1992) and Choi (1993) are among the few who have considered more sophisticated models of productions, allowing for some kind of interdependencies. Another important branch of the literature deals with complementarities among different activities within the firm, see Milgrom and Roberts (1995) for an overview of this issue. However, complementarities are present by assumption. One can see the adjustment effort proposed in this paper as explicit, costly activities which goal is to improve the fit between divisions, i.e. make their efforts more complementary. Much work remains for endogenizing compementarities, especially work combining informational and technological links between strategic units of a firm. Indeed no definitive model for studying realistic interactions between divisions has yet emerged.

6 Appendix

Proof of Proposition 1:

Proof. As θ is known by the principal, she maximizes the expression for each θ. The problem is convex and thus admits a unique solution (e^*, α^*) for each (θ, ϕ). With $A(e) = \frac{1}{2}e^2$, the first order condition for e (FOC$_e$) gives:

$$e^* = \cos(\alpha^* - \phi) \tag{1}$$

And the condition for an interior solution for α is:

$$B'(0) < |\frac{1}{2}\sin(2(\phi - \theta))| \tag{C}$$

Note that this condition is (weakly) satisfied for any (ϕ, θ) if $B'(0) = 0$ (which is the case for quadratic costs). If (C) holds, the (unique) optimal α^* is defined by:

$$B'(\alpha^* - \theta) = \frac{1}{2}\sin(2(\phi - \alpha^*) \tag{2}$$

else we have

$$\alpha^* = \theta \text{ (no adjustment undertaken)} \tag{2'}$$

■

Proof of lemma 1:

Proof. The set of constraints is rewritten as follows (With this expression the index s is no more needed. i, j and k represents elements of the types' set. More precisely, the real type is θ^i, the announcement θ^j, the other agent's type is θ^k):

$$E_{\theta^k}[t(\overrightarrow{e_{ik}}, \theta^i) - A(e_{ik}) - B(\alpha_{ik} - \theta^i)] \geqslant E_{\theta^k}[t(\overrightarrow{e_{jk}}, \theta^j) - A(e_{jk}) - B(\alpha_{jk} - \theta^i)] \quad \forall (i,j) \tag{IC'}$$

$$t(\overrightarrow{e_{ik}}, \theta^i) - A(e_{ik}) - B(\alpha_{ik} - \theta^i) \geqslant 0 \quad \forall (i,k) \tag{IR'}$$

We denote by e^*_{ik} the optimal effort (from the first-best) for an agent θ^i when the other is θ^k, and similarly for α^*_{ik}. The objective ϕ_{ik} is implicit in α^*_{ik} and α^*_{ki}. As argued before, the principal can enforce these optimal behaviours because information is ex-post public. That is, given the annoucements, say $\widehat{\theta_A} = \theta^i$ and $\widehat{\theta_B} = \theta^k$, agent A will get a positive wage if and only if $\overrightarrow{e_A} = (e^*_{ik}, \alpha^*_{ik})$. The rent profile will be such that:

$$t(\overrightarrow{e_{ik}}, \theta^i) = \left\{ \begin{array}{cc} r^i + A(e^*_{ik}) + B(\alpha^*_{ik} - \theta^i) & \text{if } \overrightarrow{e_{ik}} = \overrightarrow{e^*_{ik}} \\ 0 & \text{if } \overrightarrow{e_{ik}} \neq \overrightarrow{e^*_{ik}} \end{array} \right|$$

Where r^i is the informational rent of agent i (because of the ex-post participation constraint, the informational rent can not depend on the teammate's type). With this wage profile, the incentive constraint (IC) is rewritten as follows :

$$r^i - r^j \geqslant E_{\theta^k}[B(\alpha^*_{jk} - \theta^j) - B(\alpha^*_{jk} - \theta^i)] \quad \forall(i,j) \qquad (\text{IC}_r)$$

The second hand represents the expected gain for an agent θ^i of pretending that his type is θ^j.

By symmetry of types' space, we will have $r^j = r^{|n-j|}$. The ex-post participation constraint boils down to:

$$r^j \geqslant 0 \quad \forall j \qquad (\text{IR}_r)$$

■

Proof of proposition 3:

Proof. The set of constraints that might be binding is $\{IC^{10}, IC^{01}, IC^{21}, IC^{12}, IC^{20}, IC^{02}\}$ (and those who are pairwise symmetrical).
We set $r^0 = 0$ (we will see that it always works), and find r^1 and r^2.
(IC^{10}) gives $r^1 \geq \frac{c\pi}{80}.[2.(\alpha^{01} + \alpha^{02} + \alpha^{03} + \alpha^{04}) - \frac{5\pi}{8}]$
from (IC^{01}), noticing that $\alpha^{1i} = \frac{\pi}{8} + \alpha^{0i-1}$ for $i = 1,2,3$ we get $r^1 \leq \frac{c\pi}{80}.[2.(\alpha^{02} + \alpha^{03}) + \frac{5\pi}{8}]$
We see that if (IC^{10}) is binding, (IC^{01}) is satisfied.
(IC^{20}) gives $r^2 \geq \frac{c\pi}{80}.[2.(\alpha^{01} + \alpha^{02} + \alpha^{03} + \alpha^{04}) - \frac{10\pi}{8}]$
which is always satisfied because $\alpha^{0i} \leq i.\frac{\pi}{16}\ \forall(i,c)$,
and summing: $(2.(\alpha^{01} + \alpha^{02} + \alpha^{03} + \alpha^{04}) - \frac{10\pi}{8}) \leq 0$
(IC^{02}) imposes $r^2 \leq -\frac{c\pi}{80}.[2.(\alpha^{20} + \alpha^{21} + \alpha^{22} + \alpha^{23} + \alpha^{24}) - \frac{10\pi}{8}]$
now $\alpha^{20} + \alpha^{24} = \frac{\pi}{2}$, and $\alpha^{21} + \alpha^{23} = \frac{\pi}{2}$, so we rewrite it: (IC^{02}): $r^2 \leq \frac{c\pi}{80}.\frac{10\pi}{8}$
(IC^{21}) gives $r^2 \geq r^1 - \frac{c\pi}{80}.[\frac{15\pi}{8} - 2.(\alpha^{10} + \alpha^{11} + \alpha^{12} + \alpha^{13} + \alpha^{14})]$
As $\alpha^{1i} \leq \frac{(i+1)\pi}{16}\ \forall c, \forall i \geq 1$,and $\alpha^{10} < \frac{\pi}{8}$, $\frac{c\pi}{80}.[\frac{15\pi}{8} - 2.(\alpha^{10} + \alpha^{11} + \alpha^{12} + \alpha^{13} + \alpha^{14})] > 0$.
(IC^{12}) gives $r^2 \leq r^1 + \frac{c\pi}{80}.\frac{5\pi}{8}$.
It is now clear that we can saturate constraints (IC^{10}) and (IC^{21}), which ensures the smallest informational rents. This leads to:
$r^1 = Max(\frac{c\pi}{80}.[2.(\alpha^{01} + \alpha^{02} + \alpha^{03} + \alpha^{04}) - \frac{5\pi}{8}], 0)$
$r^2 = Max(r^1 - \frac{c\pi}{80}.[\frac{15\pi}{8} - 2.(\alpha^{10} + \alpha^{11} + \alpha^{12} + \alpha^{13} + \alpha^{14})], 0) \leq r^1$. ■

Proof of proposition 4:

Proof. We only have to define to define the different objectives, through function α^*.

The following contract gives the first-best pay-offs (Min and Max appear to make the case general whatever the value of $\Delta(c)$):

·Two identical agents should not move:

$\alpha^*_{ii} = \theta^i$

·Symmetrical objective when the agents are symmetrical:

$(\alpha_{i|4-i|}, \alpha_{i|4-i|}) = (Max(\theta^2 - \Delta, \theta^i), Min(\theta^2 + \Delta; \theta^{|4-i|}))\forall i < 2$

·The most-deviating agents bear the maximal adjustment:

$(\alpha^*_{01}, \alpha^*_{10}) = (\theta^0, Min(\theta^0 + 2\Delta; \theta^1))$

$(\alpha^*_{02}, \alpha^*_{20}) = (\theta^0, Min(\theta^0 + 2\Delta; \theta^2))$ and symmetrically $(\alpha^*_{24}, \alpha^*_{42}) = (Max(\theta^4 - 2\Delta; \theta^2), \theta^4)$

$(\alpha^*_{03}, \alpha^*_{30}) = (\theta^0, Min(\theta^0 + 2\Delta; \theta^3))$ and symmetrically $(\alpha^*_{14}, \alpha^*_{41}) = (Max(\theta^4 - 2\Delta; \theta^1), \theta^4)$

$(\alpha^*_{12}, \alpha^*_{21}) = (\theta^1, Min(\theta^1 + 2\Delta; \theta^2))$ and symmetrically $(\alpha^*_{23}, \alpha^*_{32}) = (Max(\theta^3 - 2\Delta, \theta^2), \theta^3)$

The other variables come directly from α^*:

$e^*_{ik} = \cos(\frac{\alpha^*_{ik} + \alpha^*_{ki}}{2})$ if $i \neq k$

$e^*_{ik} = 1$ if $i = k$

$t(\overrightarrow{e_{ik}}, \theta^i) = \frac{1}{2}e^{*2}_{ik} + b(\alpha^*_{ik} - \theta^i)$ if $\overrightarrow{e_{ik}} = \overrightarrow{e^*_{ik}}$

$t(\overrightarrow{e_{ik}}, \theta^i) = 0$ if $\overrightarrow{e_{ik}} \neq \overrightarrow{e^*_{ik}}$

It is now sufficient to check that an agent has no incentives to deviate. (The rest is only calculation).

First, (IR) is binding if the agent takes the specified actions, and he will because if not, he gets nothing.

We now verify that the incentive constraint induces truthfull announcement. It is obvious that they will always exert e^*. We must only verify that $E_{\theta^k}[b(\alpha^*_{jk} - \theta^j) - b(\alpha^*_{jk} - \theta^i)] \leq 0\ \forall (i,j)$ which is equivalent in this linear case to $B_j \equiv \sum_k |\alpha^*_{jk} - \theta^j| \leq \sum_k |\alpha^*_{jk} - \theta^i|$ $\forall (i,j)$ (IC^{ij})

The agents θ^0 (and symmetrically θ^4) have obviously no incentive to deviate. For example, concerning agent 0: $\sum_k |\alpha^*_{jk} - \theta^j| \leq \sum_k |\alpha^*_{jk}|\ \forall j$

It is moreover obvious that an agent will have no incentive to announce a type that is on the other side of θ^2.

There remains only the constraints $IC^{10}, IC^{12}, IC^{21}, IC^{20}$ (the symmetrical case is identical).

·For constraint IC^{10} we have:

$\sum_k |\alpha^*_{0k} - \theta^1| = 4.|\theta^0 - \theta^1| + |\theta^2 - \Delta - \theta^1| = 4.\theta^1 + |\theta^1 - \Delta| > B_0 = \theta^2 - \Delta$

·For constraint IC^{20} we have:

$\sum_k |\alpha^*_{0k} - \theta^2| = |\theta^0 - \theta^2| + |\theta^0 - \theta^2| + |\theta^0 - \theta^2| + |\theta^0 - \theta^2| + |\theta^2 - \Delta - \theta^2| = 4\theta^2 + \Delta > B_0 = \theta^2 - \Delta$

$\cdot$For constraint IC^{12} we have:
if $\Delta < \frac{\theta^1}{2}$
$\sum_k |\alpha^*_{2k} - \theta^1| = |2\Delta - \theta^1| + |\theta^1 + 2\Delta - \theta^1| + |\theta^2 - \theta^1| + |\theta^3 - 2\Delta - \theta^1| + |\theta^4 - 2\Delta - \theta^1|$
$= \theta^1 + \theta^1 + \theta^2 - 2\Delta + \theta^3 - 2\Delta = 7\theta^1 - 4\Delta$
$> B_2 = 2\theta^1 - 2\Delta + \theta^1 - 2\Delta + \theta^1 - 2\Delta + 2\theta^1 - 2\Delta = 6\theta^1 - 8\Delta$
if $\Delta \geq \frac{\theta^1}{2}$
$\sum_k |\alpha^*_{2k} - \theta^1| = |2\Delta - \theta^1| + |\theta^2 - \theta^1| + |\theta^2 - \theta^1| + |\theta^2 - \theta^1| + |Max(\theta^4 - 2\Delta; \theta^2) - \theta^1|$
$= 2\Delta + 3\theta^1 + 2.Max(\theta^1 - \Delta, 0)$
$> B_2 = |Min(\theta^0 + 2\Delta; \theta^2) - \theta^2| + 0 + 0 + 0 + |Max(\theta^4 - 2\Delta; \theta^2) - \theta^2| = 4.Max(\theta^1 - \Delta, 0)$

As $2\Delta + 3\theta^1 > 2.Max(\theta^1 - \Delta, 0)$, IC_{12} is satisfied.

$\cdot$For constraint IC^{21} we have:
if $\Delta < \frac{\theta^1}{2}$
$\sum_k |\alpha^*_{1k} - \theta^2| = |2\Delta - \theta^2| + |\theta^1 - \theta^2| + |\theta^2 - \Delta - \theta^2| + |\theta^4 - 2\Delta - \theta^2|$
$= 5\theta^1 - 3\Delta > B_1 = 2\Delta + 0 + 0 + (\theta^1 - \Delta) + (3\theta^1 - 2\Delta) = 4\theta^1 - \Delta$
if $\Delta \geq \frac{\theta^1}{2}$
$\sum_k |\alpha^*_{1k} - \theta^2| = |\theta^1 - \theta^2| + |\theta^1 - \theta^2| + |Max(\theta^2 - \Delta; \theta^1) - \theta^2| + |Max(\theta^4 - 2\Delta; \theta^1) - \theta^2|$
$= 2\theta^1 + Min(\theta^1, \Delta) + |Max(2(\theta^1 - \Delta); -\theta^1)| > \frac{5\theta^1}{2}$
And $B_1 = 0 + 0 + 0 + |Max(\theta^2 - \Delta, \theta^1) - \theta^1| + |Max(\theta^4 - 2\Delta; \theta^1) - \theta^1|$
$= Max(\theta^1 - \Delta, 0) + Max(\theta^3 - 2\Delta, 0) < \frac{5\theta^1}{2}$
Thus the constraint system is satisfied: the agents are induced to tell the truth and to exert the first-best efforts. ∎

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