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From Natural Disasters to Terrorism**

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# Catastrophe Risk Sharing and Public-Private Partnerships: From Natural Disasters to Terrorism

Nathalie de Marcellis-Warin<sup>1</sup> and Erwann Michel-Kerjan<sup>2\*</sup>

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**Résumé:** Plusieurs catastrophes récentes appellent à reconsidérer le rôle des secteurs public et privé dans l'élaboration et le suivi de mécanismes de couverture des risques catastrophiques. Le développement de partenariats public-privé apparaît aujourd'hui l'une des voies prometteuses pour résoudre la problématique du financement des conséquences de tels événements à grande échelle. Néanmoins, à ce jour, peu de travaux analysent le rôle respectif des secteurs public et privé quant au développement de tels schémas de couverture adaptés à ces risques particuliers ainsi que l'efficacité des partages de risques public-privé qui en découlent.

Cet article propose des éléments de réponse à travers une approche de politiques économiques portant sur le partage de ces risques entre assureurs privés et un réassureur public disposant d'une garantie illimitée du Trésor. Ces deux acteurs participent à un partenariat national dédié spécifiquement à la couverture de risques extrêmes, dans lequel la couverture "catastrophes" est obligatoire et la politique de "surprime catastrophes", payée par les assurés, est décidée par la direction du Trésor.

Utilisant un modèle de jeu, nous montrons que dans un pays où les assureurs ont d'abord refusé de couvrir ces risques extrêmes par eux-mêmes, le gouvernement peut modifier sa politique de surprime pour les inciter à participer à ce partenariat. Suivant la politique de surprime retenue, deux types de comportements stratégiques seront adoptés par les assureurs : (1) se comporter en simples intermédiaires financiers entre les citoyens/entreprises assuré(e)s et le réassureur public. Dans ce cas, ce dernier supportera une part beaucoup plus importante des risques; (2) supporter une large part des risques pour bénéficier d'incitations financières qui découlent de la politique gouvernementale de surprime.

Le système de couverture contre les catastrophes naturelles établi en France en 1982, et les systèmes élaborés dans plusieurs pays après les attentats du 11 septembre 2001 pour couvrir contre le terrorisme de masse, illustrent l'analyse théorique.

**Abstract:** Recent extreme events showed how insurers, deprived of reinsurance capacity at an affordable price, could decide to stop covering for specific extreme events and rapidly let people and firms uncovered. Developing public-private partnerships could constitute one of the most appealing ways to solve the problem of financing the consequences of those extremes events by taking advantage of strengths of both sectors.

Catastrophic risks present, however, very specific characteristics which really challenge any traditional economic approach to analyse those issues. So as of today little has been done in the economic literature to reassess the role of public and private sectors with respect to making available protection to victims as well as better understanding how those risks are effectively shared between all partners in those partnerships.

This paper aims to provide a partial answer by analysing policy issues related to risk sharing between insurers and a dedicated state-backed governmental reinsurer, who are part of a national partnership of insurance against extreme events. The insurance is mandatory with premium policy that is decided by Treasury. We show that a government can modulate its premium policy levied against insured to make the private insurers in the country participating in the partnership instead of leaving the market and then to adopt two different strategies: (1) to behave as a simple financial intermediary between insured and the public reinsurer so as the former supports the largest portion of the risks or (2) to conserve the largest part of risks to benefit from government incentives.

National schemes for covering against natural hazards and those developed post 9/11 for emerging terrorism provide illustrations.

**Mots clés :** Partenariats public-privés; partage de risques; catastrophes naturelles; terrorisme

**Key Words :** Public-private partnership; Risk Sharing; Natural Disaster; Terrorism

**Classification JEL:** D80 ; G22 ; G38 ; H42

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## 1. Introduction

Facing unprecedented large-scale potential damage due to emerging extreme events, the private sector of insurance may be induced to severely restrict the insurance supply or even to refuse covering against those catastrophic risks. However, because those events are capable of inflicting a debilitating impact on the social and economic continuity of a country, covering them often constitutes a national issue. In that case, government has a social and economic responsibility to be assumed by providing alternative answers. In countries where there is no specific private insurance market offering coverage at an affordable price anymore, governments may look for dealing with the economic impact of those events by creating national systems of risk cover. This paper suggests a theoretical framework of public-private partnerships regarding catastrophic risk cover. This introduction presents a review of literature on the general context regarding those catastrophic risks in a perspective of risk cover and policy.

### *A new dimension of loss*

In a worldwide context, as far as the insurance against natural disasters is concerned, a dual trend has been observed since the beginning of the 90's: a growing number of important events<sup>1</sup> and the occurrence of catastrophic events that were just unthinkable a decade ago<sup>2</sup>. 1992 Hurricane Andrew inflicted \$20 billion of insured losses and the 1994 Northridge earthquake's costs were \$17 billion (in 2002 dollars) only for direct insured losses<sup>3</sup>. As results

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<sup>1</sup> According to Swiss Re, an "important event" is defined as an event that inflicts insured damage higher than \$70 million. Over the decade 1970-1980, on average, 40 important events per year occurred worldwide. Over the decade 1980-1990, the number increased to an average of 100 important events per year. In 1995, 130 important events occurred worldwide and no less than 160 in 1998 (Swiss Re, 2000). And the growing activities in prone natural hazard area cannot explain everything.

<sup>2</sup> In parallel to such an evolution, the cost of damage also increases. Between 1970 and 1999, 46 events worldwide inflicted insured damage higher than 1 billion dollars, 32 of which occurred between 1989 and 1999 (Swiss Re, 2000).

<sup>3</sup> When a major earthquake devastated Kobe in 1995, as the insurance coverage was pretty low in Japan, the insured damages were \$3 billion. But such a number shorts fall of nearly \$100 billion of total damages.

of that, lots of countries tried –and are still trying– to create national systems for bearing catastrophic losses that ensue from large-scale event<sup>4</sup>.

Terrorism is obviously an area we also consider in the paper. 9/11 terrorist attacks against New York City and Washington D.C. show how the nature of terrorism changed from small-size event (e.g. luggage bombing) to large-scale attacks. According to several authors, that changed would be directly caused by the change in the nature of terrorism itself (Hoffman 1998; Enders and Tandler, 2000). There is a rise of an increasing number of religious-based groups whose main objective would be to inflict mass casualties (from 2 of 64 groups in 1980 to 26 of 56 in 1995; Hoffman, 1997). After all losses are accounted for, 9/11 attacks will be the costliest insured property loss in history of world insurance, with current estimates ranging from \$40-70 billion (Swiss Re, 2000). Responding to a new situation, most of industrialized countries around the world have been trying to invent new solutions for covering against catastrophic risks due to terrorism. Most of those solutions are based on the creation of a partnership between government and the private sector of insurance and reinsurance (Kunreuther, Michel-Kerjan, and Porter, 2003).

Considering such a new scale of catastrophic risks, the issue of developing specific systems of risk cover and indemnification is obviously a key one to provide reimbursements, and therefore assure the continuity of activities of the devastated area. Indeed, private and public sector alone cannot provide definitive answers to the problem posed by extreme risks.

#### *Catastrophe risk sharing: the need to public-private partnerships*

With recent disasters and their resulting effect on the solvency of insurance/reinsurance companies, the question of liquidity and insurers' insolvency has become of great importance. Insurers may be reluctant to cover those risks alone. Traditionally, the insurance industry avoids those problems by transferring those risks to the reinsurance market (Borch, 1990).

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<sup>4</sup> Kunreuther and Roth (1999), Froot (1999), Godard et al. (2002).

However, the current reinsurance capacity for coverage against natural catastrophes and terrorism is limited and prices of catastrophe reinsurance are high<sup>5</sup>.

Another means of financing has been the development of “securitization” (property-catastrophe-risk financial instruments) since 1992 by looking at the \$19 trillion capital markets<sup>6</sup>. However, whereas the development of catastrophe modelling and simulation is effective owing to the advances in information technology, the development of financial instruments remains under potential. Still, we have not seen any definitive argument to show that the private insurance and reinsurance industry alone can handle the problem of insolvency in case of extreme financial consequences of natural disasters and large-scale terrorism.

Those low-probability/high-consequence events are difficult to be covered for other reasons. Dealing with a lack of historical data on those catastrophic events (as they are new in their nature or scale), the aversion for ambiguity leads insurers to set premiums high because of a lack of clarity on probability and events being insured (Kunreuther, Hogarth, and Maszeros, 1993). In some instances, that situation is counterbalanced by regulation of premium rates that obliges private insurers to sell insurance coverage at a lower price than necessary for business (Priest, 1996; Jaffee and Russell, 1997). However, the insurers will often prefer not covering against those risks or deeply limiting their involvements<sup>7</sup>. On the demand side, it is well known that potential purchasers may tend to underestimate the real level of risks and thus consider the insurance premiums as being too expensive (Camerer and Kunreuther, 1989)<sup>8</sup>. Moreover, the charity hazard, defined by Browne and Hoyt (2000) as

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<sup>5</sup> Very few reinsurers in place provide protection against industry-wide losses for catastrophic event greater than \$5 billion. Several arguments explain the reason why the prices of catastrophe reinsurance are high: insufficient reinsurance capital, reinsurers’ market power, inefficiency of the corporate form of reinsurance, high transaction costs, moral hazard and adverse selection at the insurer level (Froot, 1999, 2001).

<sup>6</sup> The idea was already suggested in Goshay and Sandor (1973). Among recent works regarding the development of derivatives and cat bonds, see D’Arcy and France, 1992; Niehaus and Mann, 1992; as well as Froot, 1999; Harrington and Niehaus, 1999; Cummins, Lewis, and Phillips, 1999; Cox, Fairchild and Pedersen, 2000; Cummins, Lalonde and Phillips, 2001. Since 9/11 attacks the possibility to transfer some terrorism risks to financial markets is also studied, see Woo (2002).

<sup>7</sup> Regarding the lack of terrorism coverage in the US one year after September 11, 2001, see Hale (2002).

<sup>8</sup> The fact that risk-perception influences purchasing of insurance (Kunreuther, 1996) is shown by empirical studies (Browne and Hoyt, 2000; Ganderton and al., 2000).

“the tendency of an individual at risk not to purchase insurance or other risk financing as a result of a reliance on expected charity from (...) a government emergency program”, also induces potential purchasers of insurance to limit their coverage or even to prefer not be covered at all<sup>9</sup>.

As a result of those different effects, numerous citizens and firms could be left without coverage. However, under the public pressure in the aftermath of a catastrophe, the government would have to financially help citizens and firms, victims of the catastrophe (emergency measures, crisis management, and disaster relief to uninsured citizens...). The continuing increase in the costs of aid for governments<sup>10</sup> has caused policy-makers to look closely at possible national insurance programs to levy *ex ante* contributions and increases the general concerns on those issues (Kunreuther, 1997). The creation of specific public insurance may be one of the solutions (e.g. the US National Flood Insurance Program). However, on the one hand, that solution may be very costly for the government in implementation as well as day-to-day management (levying surcharges, estimating damage of a catastrophe, reimbursing insured [...]). On the other hand, insurers can easily add and manage an extra line of risk in their portfolio as well as taking advantage of their commercial network throughout a country to sell the associated policies. For that reasons, collaboration between the government and the insurers could, at least partially, be part of a global solution for catastrophic risk cover.

What would be strengths of the public sector in such a partnership? Government presents at least two key strengths. First, government constitutes a powerful source of wealth redistribution toward losses already occurred. What is concerned here is government ability to spread the costs of catastrophes over time and to obtain *ex post* contributions from the all taxpayers<sup>11</sup>. Time diversification constitutes a definitive strength for governments over

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<sup>9</sup> For instance, as written in Froot (2001), “since the late 70’s the Federal government has spent annually an average of \$8 billion (current) on disaster assistance. This is far greater than the average annual loss borne by reinsurers on US catastrophe coverage”.

<sup>10</sup> “Since the late 70’s the Federal government has spent annually an average of \$8 billion (current) on disaster assistance. This is far greater than the average annual loss borne by reinsurers on US catastrophe coverage” Froot (2001).

<sup>11</sup> The public sector is also able to achieve high diversification by pooling several sources of risks.

insurance companies (Gollier, 2002a-b)<sup>12</sup>. Government involvement into a public-private scheme of catastrophic risk cover could thus be a definitive advantage in a sense that may allow solving the problem of insurer's insolvency, for instance being a reinsurer of last resort for insurers. Second, government can constrain adverse selection phenomena by the enforcement of insurance purchase for a specific market segment exposed to catastrophic risks we are concerned about in this paper. Even if this governmental control of adverse selection does not reduce the risk itself, it also constitutes a strong vector of redistribution<sup>13</sup>.

For all those reasons, the idea of developing some kinds of public-private partnerships to deal with coverage against emerging catastrophic risk obviously emerges. As the private sector of insurance can simply refuse to cover those risks, a key question is to analyze how a government can take advantage of the strengths of national insurers to create *such public-private partnerships* based upon mutual interests<sup>14</sup>?

### *Information distribution*

Catastrophic risks such that large-scale terrorism and natural disasters may also present another particularity this paper takes into account. This paper focus on situations where the insurers are not the most informed stakeholders regarding that specific line of risks. As a result of that, the government (local to national, through its departments and specialized agencies) may be more informed on those catastrophic risks than the private sector of insurance. Although such a configuration may appear counter-intuitive for traditional lines of risks for which the know-how and experience of insurers is definitely better than any governmental agencies, it remains actually relevant here for at least two reasons. First, there could have been for a long time a lack of interest from insurers regarding those specific risks they did not really considered –and thus insured– before<sup>15</sup>. Second, regarding terrorism, risk knowledge may be kept secret for national security reasons. For instance, for obvious reasons, governmental intelligence services will keep information

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<sup>12</sup> Government are hence even to redistribute the cost of disasters among present and future generations of citizens and firms

<sup>13</sup>For a recent survey of adverse selection phenomena in insurance, see Dionne, Doherty and Fombaron, 2000.

<sup>14</sup> In using this expression we consider situations where the two sectors are capable of defining and mutually accepting specific configurations of risk-sharing that would respond to their own expectations.

<sup>15</sup> For instance because: (1) those risks represent only a small part of their portfolio; (2) they were not considered catastrophic until then and/or because insurers could have been largely reinsured by the reinsurance industry.



secret regarding the potentiality of terrorist attacks. They will not share such information with the private sector of insurance, which knowledge regarding terrorism risks remains poor.

Large-scale terrorism is hence a conclusive illustration of such an asymmetry of information in favor of the government sectors (Godard and al., 2002). To some instance, natural hazards can also be illustrative on that point in some countries where insurers did not pay attention to such risks<sup>16</sup>.

However, a key aspect is the following: that is not because insurers have only a priori information about the catastrophic risks that they decide to not cover those risks alone. But because they have decided at one time to stop covering them alone after a major event occurred (before that event that line of portfolio was not a big issue for them as being easily reinsured) that they are less informed than government agencies at the time of considering to accept the offer to be part of a partnership with government or not.

In that whole context, the paper presents a very simple model of public-private partnership. The government levies distinct extra-charges on specific basic insurance contracts that allow the insured being covered against a given catastrophic risk. If the insurers accept to participate in the system, they will receive those extra-charges (and thus bear the risks) as well as a participation commission from the government; a state-guaranteed public reinsurer can reinsure them. One question that the present paper address is what kind of national policies a government can develop regarding a partnership with the private sector of insurance. In other words, how do policymakers consider the system resulting from the partnership? Do they use the insurers as simple intermediaries between the government and citizens/firms it wants to be covered or do they really look for an effective risk sharing with the insurers? To our knowledge this problem of public-private partnership to develop national coverage against catastrophic risks has not been formalized in such a way in the literature.

This paper proceeds as follows. We start out in section 2 by presenting a basic model of public-private sector risk-sharing in the form of a game with Perfect Bayesian Equilibrium (PBE). The framework is presented in this section: the model's hypothesis, the players' action, the objectives of the government and the insurance industry. This section describes what we call the *governmental high-risk sharing (HRS) payment*: to induce the private insurers to bear more high risks, the government can decide to raise the level of premiums

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<sup>16</sup>For example, the French private insurers never had a deep interest in elaborating their own risk analysis for the natural catastrophes. The 13<sup>th</sup> July 1982 law provided the implementation of prevention measures. Municipalities should make risk analysis and implement "*Risk Prevention Plans*". In 2000, the French mutual and insurance companies created the French "Natural Hazards Mission". One of its main roles is to transfer the results of PPR into their database. That shows clearly that the information on risks described by the PPR have not been considered by the private sector for twenty years of the scheme's operation.

paid by insured citizens and firms. The government can vote different risk cover policies. We discuss two of them. Section 3 gives a characterization of the pooling or separating equilibria that correspond to those two cases (insurers as indemnification intermediaries; government looking for the autonomy of the scheme). We discuss the results of the model and possible extensions in section 2 too. Section 4 illustrates the theoretical approach with examples: first, the insurance against natural disasters in France as well as an application of the model for the US market; second, the post 9/11 national insurance schemes developed recently for covering against terrorism. Section 5 concludes the paper. An appendix provides detailed proofs for the two propositions of the paper in section 6.

## **2. A model of public-private partnership**

### *2.1. Framework of the public-private partnership*

In order to investigate the public-private partnership for catastrophic risks, we first characterize the general framework. We consider a state or a group of states where there is no insurance market for the class of risks we study here. To fill the gap, a partnership is created between the interested parties, the government and the private insurers. The government votes the application of an extra-charge on specific basic lines of insurance (e.g. property insurance); the extra-charge equals a certain percentage of that basic insurance premium. If the insurers accept to participate into the system, they levy those extra-charges and receive a participation commission from the government in return. For that line of risks, insurers can be reinsured by an unlimited state-guaranteed governmental (public) reinsurer, GRe. The different assumptions of the game are presented in figure 1.

[Insert Figure 1]

### *2.2. The game*

Our theoretical study confronts the public sector with private insurers. We consider a simple game of incomplete information. In order to simplify the model, assumptions are made on both the set of catastrophe risks and the possible actions. We are only interested in the events covered by the catastrophe risk guaranty during a given period of time.  $D$  denotes the total insured damage due to some catastrophes covered by the public-private partnership (e.g., terrorism, earthquakes or floods). The game has three players: the *Nature*, the *Public sector* and the *Private Insurers*.

### *Players' action*

The player 1, the *Nature*, can choose between two probability levels  $p_L$  and  $p_H$ <sup>17</sup>. We consider that there exist only two possible probabilities of occurrence  $p_L$  and  $p_H$  with  $p_L < p_H$ .

The player 2, the *Public sector*<sup>18</sup>, fixes the level of the extra-charge rate denoted  $\beta$ , taking into account its knowledge on the potential occurrence of the catastrophe damage.  $\beta$  is here a parameter.

Referring to the game theory literature, we call the strategy (or action) played by the government the “extra-charge policy”.  $\Pi$  is the total premiums of the basic contracts (e.g. Property and Casualty) on which the extra-charge is applied. Therefore, the total extra-charges collected for the line catastrophic risk are  $\beta \cdot \Pi$ . The government can choose between a low extra-charge policy, denoted by  $\underline{\beta}$ , and a high one, denoted by  $\bar{\beta}$ .

The player 3, the private insurance industry (or the insurers for brief), receives the total extra-charges collected for that line *catastrophe risk*,  $\beta \cdot \Pi$ . The insurance industry's behavior is assumed to be summarized by a single action: the reinsurance cession rate to the

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<sup>17</sup> The notion of Nature as a player is relative to the game theoretical literature.

<sup>18</sup> The government actually, as part of the public sector.

governmental reinsurer (GRe)<sup>19</sup>, denoted by  $\alpha$ . For obvious matter of simplification, the private insurers are assumed to be only reinsured by the public reinsurer, GRe. They choose a cession rate to GRe  $\alpha$  in  $[\underline{\alpha}; \bar{\alpha}] \subset [0,1]$ . They can keep the largest part of the premiums and risks by playing a low cession rate  $\underline{\alpha}$  or choose to transfer the major part of them to GRe and then take the action “a high cession rate”. The high limit of reinsurance quota-share allows the system to share a minimum percentage of the risks with the insurers (they keep a minimum percentage  $(1-\alpha)$ ) whatever their decision of cession. The low rate limits the amount of losses the insurers would bear in case of catastrophic damage if they had decided to keep the largest possible part of risks. Moreover, the government offers the insurers a commission  $m$  in order to induce them to participate in the system ( $m$  is the private insurers’ *participation commission*).  $m$  is a poll commission, which does not depend on the level of risk kept by insurers.

#### *Players’ preferences*

In the standard theoretic framework, the two main stakeholders are (i) the public sector and (ii) the private insurers who decide according to their own preferences. The criteria for action are the following.

The government wishes all catastrophe victims to be compensated. However, the establishment of such an insurance scheme based on a national redistribution induces a social cost: the total amount of extra-charges paid by the insured,  $\beta\Pi$ . The payment of the participation commission  $m$  to the private insurers by the government has also to be taken into account. Moreover, as GRe is a public-owned company, the Treasury searches to balance the public reinsurer. Finally, as the government accepts a possible *ex post* public intervention (risk financing) by offering an unlimited guarantee to GRe, we have to consider the effect of

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<sup>19</sup> The GRe’s equalization reserves are assumed to be zero in the model. The introduction of such a parameter makes the model more realistic but leads to results difficult to interpret economically.

such an intervention by introducing a parameter  $\lambda$ , which represents the shadow price of public funds. The greater  $\lambda$ , the more reluctant to risk financing the government.

Adding to its own objective, we can consider that the government (here the Treasury) internalizes all the objectives of the public sector as a whole: the insured citizens and firms, the GRe, and itself. However, contrary to some traditional approaches à la Laffont-Tirole in which the Treasury, as a planner, looks for to maximize the total social surplus, it is not this goal that it wants to reach here. This is the case because the Treasury would like private insurers keep the largest portion of risks in order to avoid having to pay when the reserves of GRe are not high enough. Indeed, in that case – and only in that case – we have to take into account the shadow cost of public funds. In all other cases, the shadow cost does not appear. Neither the benefits or losses obtained by the insurers nor the benefits of GRe (because they are not transferred to the Treasury) are affected by his parameter in our model. The shadow cost of public funds is only considered when the Treasury has to operate its guarantee to the GRe. Taking into account such an argument in the definition of utility  $U(.)$  of the Treasury, it can be written as follows:

$$\begin{aligned} U_p(\beta; \alpha) &= -m - \beta\Pi + pD + \alpha(\beta\Pi - pD) \text{ if } \beta\Pi \geq D \\ U_p(\beta; \alpha) &= -m - \beta\Pi + pD + (1-p)\alpha\beta\Pi + \lambda\alpha p(\beta\Pi - D) \text{ otherwise} \end{aligned} \quad (1)$$

The first equation corresponds to the case in which the total amount of extra-charges is high enough to compensate all the victims of the catastrophe events (with total damage  $D$ ) that occur with a probability  $p$  (L or H) during the studied period of time. The *ex post* governmental intervention is not required because  $\beta\Pi > D$ . The benefit of the public reinsurer, which positively affects the expected social welfare, is the difference between the total premiums received from the private insurers and the total amount of reimbursement paid by the GRe in case of covered catastrophes.

The second equation corresponds to the case in which the extra-charge policy is not high enough. Indeed, the portion of premium received is not enough to compensate the part of

damage the public reinsurer would have to pay. GRe is virtually bankrupted. The state guarantee is triggered and the government pays the excess of insured losses that GRe is not able to pay, that is  $\alpha.(D - \beta\Pi)$  timed by the factor  $\lambda$ .

For the private insurers, we use a mean-variance utility function assuming they are risk averse for catastrophic risks<sup>20</sup>. When the insurers believe the probability of damage is  $p$  and with an extra-charge policy  $\beta$ , their utility  $V(.)$  is:

$$V(\alpha, \beta, pD) = (1-\alpha).(\beta\Pi - pD) - \frac{k}{2}(1-\alpha)^2 p.(1-p).D^2 + m \quad (2)$$

As the reinsurance treaty to the governmental reinsurer is assumed to be only on quota-share, whereas the portion of risk premium kept by the insurers and the participation commission offered by the government positively affect their utility, the expected payments and the variance in expected reimbursable damage affect it negatively.  $k/2$  is assumed to be positive (catastrophe risk aversion). We finally assume in this model that the commission  $m$ , which the government offers the insurers to cover their management costs, is always high enough to incite them to participate (i.e.  $V > 0$ ).

### *Players' information*

The public sector is assumed to be more informed on these risks than the private insurers. As explained in the previous section (??), in the model the government is then assumed to be the informed party whereas the private insurers have only prior beliefs on the distribution of types: there is a proportion  $\mu$  of low types of expected damages and a proportion  $(1 - \mu)$  of high types. The game is played within imperfect information of player 3, which is represented by two information sets in the extensive form of the game.

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<sup>20</sup> In the literature, insurers are often considered to be risk neutral. For catastrophic risks, the reinsurance demand shows explicitly their risk aversion. We use a mean-variance utility function because it is easily manipulated and it provides good intuition. But we know that this type of function has an undesirable property: it does not respect the first-order stochastic dominance (Borch, 1968). However, with a more correct version of the utility function, calculus are much more complicated and results are similar.

### *Sequence of decisions*

The game takes place over three “periods” (See Fig. 2). In period one, player 1 (Nature) chooses between two probability levels,  $p_L$  or  $p_H$ . In the second period, player 2 (the government) receives that information on the probability of catastrophes and gets to choose between two policies (nodes G1 and G2 in the extensive form of the game), a low extra-charge policy or a high level:  $\beta \in \{\underline{\beta}; \bar{\beta}\}$ . For choosing one of those two policies, the government takes account of its knowledge on risk and the reaction from the insurers it anticipates. Finally, in the third period, player 3 (private insurance industry), who has no other information on risk than its prior belief, receives either the signal of a high extra-charge policy (nodes I1 and I2) or the signal of a low extra-charge policy (nodes I3 and I4) and chooses a reinsurance cession rate  $\alpha \in [\underline{\alpha}; \bar{\alpha}]$ .

The insurers only receive indirect information on the nature of the risks they are going to insure when the government chooses the extra-charge policy. Observing that policy, they revise their prior beliefs using the Bayes’s rule. The government knows the prior beliefs of insurers. And both the government and the insurers obtain a certain level of payment according to their action and criteria. We can now draw the complete extensive form of the game as in figure 2.

[Insert Figure 2]

### *2.3. Main assumption*

As explained above, the government fixes the extra-charge. We make the following assumption concerning the level of this extra-charge. First, the low extra-charge equals the actuarial rate for the low risk ( $\underline{\beta}.\Pi = p_L.D$ ). Such a low governmental extra-charge policy, whose advantage is to limit the payment of the citizens and make the system popular, is not sufficient to enforce the balance of the indemnification process without any *ex post*

governmental intervention in the event of a natural catastrophe. However, whereas the low governmental policy is not sufficient in case of damage, one may consider that they levy a minimum level of extra-charges that allow reducing the total *ex post* governmental payment (if compared with a situation in which the system does not exist).

Moreover, the main difficulty for the government in creating this scheme may be to find a balance between the expected damages and the extra-charge policy in order to assure the autonomy of the scheme, that is to limit the *ex post* public intervention. This autonomy ends as soon as the total extra-charges levied during the period of time are not high enough to balance the public reinsurer's results. In this case, the state guarantee must be triggered in order to assure victim indemnification (equation (1)). To enforce the autonomy of the scheme, the government can define a high extra-charge policy that is sufficiently high so that the state guarantee is not required. Hence, the second part of this assumption is that  $\bar{\beta}.\Pi \geq D$ <sup>21</sup>. By so doing the government defines a higher extra-charge policy than the corresponding actuarial premium for high-expected losses ( $\bar{\beta}.\Pi > p_H D$ ). The profit may be higher for the insurers who keep the risks, as they may be also incited to be less reinsured in order to keep a higher part of the premiums.

We call such a policy the governmental “high-risk sharing payment” (HRS payment for brief) of the scheme. The HRS payment is a key part for a clear understanding of the scheme. Traditionally, the insurers accept only to insure “good risks” and refuse to insure “bad risks”. However, with the HRS payment, the opposite situation may occur. Since the government is likely to offer a higher payment than the actuarial premium, the insurers may make more profit than only the participation commission  $m$ . Thus, the insurers' retention rate, which represents their reaction function to the governmental extra-charge policy, could be non-decreasing with their prior beliefs on the proportion of high risks  $(1 - \mu)$ .

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<sup>21</sup> By taking into account of the CCR's equalization reserve  $R$ , the inequality becomes  $\alpha \bar{\beta}.\Pi + R - \alpha D \geq 0$ . So the government has to define a high extra-charge policy higher than  $D - \frac{R}{\alpha}$ , with  $\alpha$  the action decided by the insurers.



### 3. Public catastrophic risk management: Solutions of the model

In this section we make a focus on two policies that may be chosen by the government and we determine the corresponding equilibria. When the government announces its  $\beta$ , this does not necessarily correspond to its knowledge of expected damage. Insurers who observe the government's decision should update their beliefs and base their choice on the posterior distribution:  $\Phi(p.D \setminus \beta)$ , which depends on the signal  $\beta$  received and compatible with Bayes' rule. Observing  $\beta \in \{\underline{\beta}; \bar{\beta}\}$ , the insurers can use Bayes' rule to update  $\mu(\cdot)$  to  $\Phi(\cdot \setminus \beta)$ .

**Definition.** *A perfect Bayesian equilibrium with pure strategies is an action profile  $(\beta^*; \alpha^*)$  and posterior beliefs  $\Phi(p.D \setminus \beta)$  such that<sup>22</sup>:*

- (i)  $\forall pD \in \{p_L D; p_H D\}, \beta^*(pD) = \arg \max_{\beta} U_p(\beta; \alpha^*(\beta));$
- (ii)  $\forall \beta \in \{\underline{\beta}; \bar{\beta}\}, \alpha^*(\beta) = \arg \max_{\alpha} \sum_{pD} \Phi(pD \setminus \beta) V(\alpha; \beta; pD)$
- (iii)  $\forall \beta \in \text{Supp} \beta^*, \Phi(pD \setminus \beta)$  can be obtained from the *a priori* distribution  $\mu(\cdot)$  (discrete in the model) by using the Bayes' rule, *whenever feasible*.

(i) and (ii) are the perfection conditions: (i) says that the government takes into account the effect of  $\beta$  on insurers' decisions and determines its best response for each level  $pD \in \{p_L D; p_H D\}$ ; (ii) states that the insurers react optimally to governmental decisions given their posterior beliefs about  $pD$ . They choose the cession rate to GRe maximizing their utility. (iii) corresponds to the application of Bayes' rule by the insurers. It should be noted that if  $\beta$  is not part of government optimal action for some type, observing  $\beta$  is a probability-0 event, and Bayes' rule does not pin down posterior beliefs. Every posterior belief  $\Phi(\cdot \setminus \beta)$  is then admissible and every decision  $\alpha$ , which is the best response for certain beliefs, can thus be put into play.

Let us explain taxonomy of potential perfect Bayesian equilibria.

A *pooling equilibrium* is an equilibrium in which the government chooses the same action whatever the type  $p.D$ . The insurers do not update their beliefs when they observe the equilibrium action:

$$\Phi(p_L.D \setminus \underline{\beta}) = \mu = \Phi(p_L.D \setminus \bar{\beta}) \text{ and } \Phi(p_H.D \setminus \bar{\beta}) = 1 - \mu = \Phi(p_H.D \setminus \underline{\beta}).$$

A *separating equilibrium* is an equilibrium in which the government chooses two different actions depending on the type  $p.D$ . Observing the nature of the extra-charge played by the government, the insurers know the governmental type.

Moreover, when the observed level of extra-charge is inconsistent with the given equilibrium strategy, it's not possible to use Bayes's rule. We deal with this well-known problem by assuming that the private insurers will then set their beliefs via *forward induction* (see Fudenberg and Tirole (1991) for a rigorous and complete discussion of perfect equilibrium and its refinements). Under the latter the private insurers view any "surprising" (i.e out-of-equilibrium) action by the government as truly intentional (as opposed to being the result of some mistake). They first rule out the level of expected damage at which a rational government would not depart from the proposed equilibrium. Once their beliefs are updates accordingly, the private insurers' utility maximizing reaction must deter the level extra charge at any other expected damage level. An equilibrium obtain in this manner turns out to be unique modulo the private insurers' current state of mind.

### 3.1. *Pooling equilibrium: the indemnification intermediary behavior*

First, the government can decide to levy a low amount of extra-charges whatever the level of risk. That case appears precisely at the beginning of a scheme's operation when the government needs to obtain a consensus with citizens and firms to be covered in order that the

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<sup>22</sup> For a rigorous definition and complete discussion of perfect Bayesian equilibrium, see Fudenberg and Tirole

system is accepted. To encourage that acceptance, the government may choose a low extra-charge policy.

The question is whether, according to the model, equilibrium can be achieved with the government always choosing such a low policy. The insurers, receiving no information from the governmental signal  $\underline{\beta}$ , thus maximize their utility considering their prior beliefs.

**Lemma.** *The best response of the private insurance industry to a low extra-charge policy  $\underline{\beta}$  decided by the government whatever the type  $p.D$  is to transfer the largest portion of the risks to the public reinsurer.*

*Proof*<sup>23</sup>: If the government chooses the low level of extra-charge rate  $\underline{\beta}$ , insurers will choose the level of cession, which maximizes their utility according to this policy. Because the government plays the same action whatever the type  $p.D$ , the private insurance industry will choose its optimal level of cession, denoted  $\alpha(\mu)$ , by using their prior beliefs  $\mu$  on  $p.D$ .

Considering:

$$V(\alpha; \underline{\beta}) = \mu \left[ (1-\alpha)(\underline{\beta}.\Pi - p_L.D) - \frac{k}{2} \cdot (1-\alpha)^2 \cdot p_L \cdot (1-p_L) \cdot D^2 \right] \\ + (1-\mu) \left[ (1-\alpha)(\underline{\beta}.\Pi - p_H.D) - \frac{k}{2} \cdot (1-\alpha)^2 \cdot p_H \cdot (1-p_H) \cdot D^2 \right] + m$$

The first derivative can be written as follows:

$$V'_\alpha(\alpha; \underline{\beta})_{\alpha=\alpha(\mu)} = 0 = (1-\alpha(\mu)) \left[ \mu \cdot k \cdot p_L \cdot (1-p_L) \cdot D^2 + (1-\alpha(\mu)) \cdot k \cdot p_H \cdot (1-p_H) \cdot D^2 \right] \\ - \mu \cdot (\underline{\beta}.\Pi - p_L.D) - (1-\mu) \cdot (\underline{\beta}.\Pi - p_H.D)$$

Therefore, the sign of the first derivative (taken at  $\alpha = \alpha(\mu)$ ) is positive if and only if the inequality holds:

$$\alpha(\mu) < \frac{\mu \cdot (\underline{\beta}.\Pi - p_L.D) + (1-\mu) \cdot (\underline{\beta}.\Pi - p_H.D)}{\mu \cdot k \cdot p_L \cdot (1-p_L) \cdot D^2 + (1-\mu) \cdot k \cdot p_H \cdot (1-p_H) \cdot D^2}$$

As  $\underline{\beta}.\Pi = p_L.D$ , the inequality becomes:

$$\alpha(\mu) < 1 - \frac{(1-\mu) \cdot (\underline{\beta}.\Pi - p_H.D)}{\mu \cdot k \cdot p_L \cdot (1-p_L) \cdot D^2 + (1-\mu) \cdot k \cdot p_H \cdot (1-p_H) \cdot D^2}$$

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(1991).

<sup>23</sup> All proofs are put in appendix. This proof is in the main text to help the reader understanding the notation.

As  $\underline{\beta} \Pi < p_H D$ , the fraction is strictly negative. The inequality is equivalent to  $\alpha(\mu) < 1 + a$ , with  $a > 0$ . As  $\alpha \in [0,1]$ , the inequality always holds true whatever the beliefs out of the equilibrium and the first differential is always positive. Therefore, private insurers choose to transfer the largest portion of the risks to the public reinsurer when they receive no information from the governmental action.  $\square$

The lemma is very intuitive. As the insurers know the government has more information on the catastrophic risks, they know that the government could be induced to levy less extra-charge than the situation requires. However, to determine whether equilibrium can be reached, we have to show that the government has an interest in playing that action. Using the lemma, we have to show under which conditions, if any, the government has no interest in deviating from that action. Under the main assumption of the model, we find that whereas the government decides the same extra-charge policy for the catastrophe guaranty whatever the level of risk, the private insurers will basically behave as indemnification intermediaries of the scheme. This is equivalent to the following formal proposition. The demonstration is offered in appendix.

**Proposition 1. Insurers as intermediaries.**

*Consider the following actions and beliefs: the government chooses a low extra-charge policy  $\underline{\beta}$  whatever the probability of extreme risks, the private insurers choose the largest possible cession rate  $\bar{\alpha}$  to the public reinsurer and the insurers' beliefs are  $\Phi(p_L.D \setminus \underline{\beta}) = \mu$  and  $\Phi(p_H.D \setminus \underline{\beta}) = 1 - \mu$ .*

*There exists  $r_L$  and  $r_H$  such that: this pooling candidate is always an equilibrium iff*

$$\bar{\alpha} < \frac{1}{1 + (\lambda - 1)p_H} ; \text{ it can be an equilibrium depending on a special specification of the}$$

*insurers' beliefs out of the equilibrium as soon as  $\underline{\alpha} \leq \min\{r_L; r_H\}$ ; finally, it will never be an equilibrium iff  $\min\{r_L; r_H\} < \underline{\alpha}$ .*

**Proof.** See Appendix A.  $\square$

The rate bounds<sup>24</sup> are  $r_L = 1 - (\lambda - 1) \cdot \bar{\alpha} \cdot p_L$  and  $r_H = \bar{\alpha} \cdot p_L + \frac{(1 - p_L) \cdot (1 - \lambda \bar{\alpha} \cdot p_H)}{(1 - p_H)}$ .

Proposition 1 analyses a first configuration of public-private partnership. In that situation, the government asks private insurers to levy *ex ante* small extra-charges on the basic policies of their insured. The insurers, who have no other information on the nature of the risk than their prior beliefs, levy those “catastrophe extra-charge premiums” and choose to transfer the largest possible part of them (as authorized through the reinsurance treaty) to the public reinsurer. In that case, such a scheme uses the network and management of private insurers as what we could call a “financial indemnification intermediary” *per contra* the payment of a pool participation commission  $m$  for managing that catastrophic risk line in their portfolio. Government may have to pay *ex post* a portion of the damage corresponding to the state guarantee if triggered.

This pooling equilibrium is not sustainable if  $\underline{\alpha}$  is higher than either  $r_L$  or  $r_H$ . Indeed, in that case, taking into account the insurers' beliefs out of the equilibrium, the government would have an interest in deviating<sup>25</sup>.

It can also be noticed that the pooling equilibrium is only sustainable for “low” values of  $\lambda$ . When  $\lambda$  becomes too high, the government may estimate that the cost of financing the indemnification through the unlimited state guarantee becomes too expensive. Indeed, as the value of the bound rates  $r_L$  and  $r_H$  decreases with the exogenous parameter  $\lambda$ , above a particular level of  $\lambda^*$ , the government is incited to deviate from this equilibrium, which falls (see the proof of the proposition in the Appendix).

<sup>24</sup> If we consider the GRE's equalization reserves noted  $R$ , the bound are quite complicated. For instance,  $r_L = 1 - \frac{[\lambda \bar{\alpha} \cdot D(1 - p_L) + (1 - \lambda) \cdot R] p_L \cdot \bar{\alpha}}{\bar{\alpha} \cdot D(1 - p_L) - R} + \frac{\bar{\alpha}^{-2} \cdot D(1 - p_L) \cdot p_L}{\bar{\alpha} \cdot D(1 - p_L) - R}$  and we obtain

$$r_H = \frac{(\bar{\alpha} \cdot D(1 - p_L) - R) - [\lambda \bar{\alpha} \cdot D(1 - p_L) + (1 - \lambda) \cdot R] p_H \cdot \bar{\alpha} + \bar{\alpha}^{-2} \cdot D(1 - p_H) \cdot p_L}{\bar{\alpha} \cdot D(1 - p_H) - R}.$$

<sup>25</sup> When  $\underline{\alpha} \leq \min \{r_L; r_H\}$ , we find that conditions for no deviation depend on the specification of the insurers' beliefs out of the equilibrium. Taking  $v(p_L \cdot D \setminus \beta) = 1$  as the insurers' beliefs out of the equilibrium is sufficient to guarantee the sustainability of the equilibrium.

An alternative way for the government in that case – if it wants to continue playing that action whatever the type of risk – is to reduce the maximum possible cession rate to the governmental reinsurer (i.e. decreasing  $\bar{\alpha}$ ). By so doing, policy makers in government limit the amount of damage that the Treasury will pay in case of a disaster when the state guarantee is triggered.

A similar remark can be made regarding the levels of probability of an extreme event covered by the scheme: higher probability of damage ( $p_L$  and  $p_H$ ), less susceptible the government to play the same low extra-charge policy whatever the type of risk.

### *3.2. Separating equilibrium: the autonomy of the system*

This section focuses on another situation in which the government looks for the autonomy of the system, i.e. making the system financially self-sufficient so as the state guarantee is never triggered. This is specifically the case in a country where the probability of catastrophes is high (repeated events). Defining a high extra-charge policy will be necessary to guaranty that autonomy. By so doing, the *government may also create conditions that will induce private insurers to keep the largest part of the risks*; that is to play the action  $\underline{\alpha}$  in the model.

As proved in the preceding subsection, the pooling equilibrium can only be obtained with the government playing the action  $\underline{\beta}$ . But, in so doing, the government knows that the insurers will take an intermediary course, which is not the government's objective. In other words, the equilibrium candidates for the scheme's autonomy can only be separating equilibria.

In a separating equilibrium, the government's choice reveals its type and so gives a signal to the insurers.

Moreover, the members of the government ought never to decide a low extra-charge policy when they know that the probability of damage is high. Indeed, private insurers would decide

to transfer the highest part of risk to GRe. And, because the total premiums transferred to the public reinsurer would not be sufficient to counterbalance the cost of indemnities in case of an event, the state guarantee would always be triggered in that case. Conversely, when a high extra-charge policy is decided although the probability of damage is low, the insurer will keep the highest part of premiums. Whereas the government levies more charges than necessary, the insurers will choose to transfer a minimal portion of premiums. This policy could never be beneficial from the public sector's point of view.

The following proposition gives necessary and sufficient conditions to obtain an equilibrium in which the private insurance industry would accept to bear the largest part of the high risks. Under the assumptions of the model, the government, looking for the scheme's autonomy for high risks, will choose a specific extra-charge policy for each level of risks. Moreover, it offers a sufficient HRS payment, which induces the insurers to conserve the largest share of high risks. This is equivalent to the following formal proposition.

**Proposition 2. Looking for the scheme's autonomy**

*When the governmental HRS payment is sufficiently large, so that  $(\bar{\beta}.\Pi - p_H.D)$  is higher than  $(1-\underline{\alpha}).k.p_H.(1-p_H)D^2$ , there exists a separating PBE such that the action profile  $\{(\underline{\beta}(p_L), \bar{\alpha}(\underline{\beta})) ; (\bar{\beta}(p_H), \underline{\alpha}(\bar{\beta}))\}$  and posterior beliefs  $\Phi(p_L.D \setminus \underline{\beta}) = \Phi(p_H.D \setminus \bar{\beta}) = 1$  define it, if and only if the two following inequalities are true:*

$$r_H = \bar{\alpha}.p_L + \frac{(1-p_L)(1-\lambda.p_H.\bar{\alpha})}{1-p_H} \leq \underline{\alpha} \leq 1-p_L(\lambda-1)\bar{\alpha} = r_L$$

**Proof.** See Appendix A.  $\square$

*On the conditions of equilibrium*

When the probability of catastrophic damage is low, the extra-charge policy decided by the government equals the actuarial premium. The insurers agree to participate in because they receive a participation commission such that their utility  $V$  is positive. Nevertheless, the insurers have no real interest, without making any profit with the premiums, to bear a higher

portion of the risks than the minimum required to earn the commission. They prefer to act as an indemnification intermediary. That explains the action profile  $(\underline{\beta}(p_L), \bar{\alpha}(\underline{\beta}))$  in proposition 2. When the probability of damage is high, because the best strategy for the government is a high extra-charge policy, the insurers have to consider the trade-off between keeping the largest portion of premiums (with a profit  $(1 - \underline{\alpha}).(\bar{\beta}\Pi - p_H.D)$ ) or transferring a larger portion of the risk to GRe.

By assumptions, such a trade-off depends on all the parameters in the model.

At the light of the governmental HRS payment, the government, who would like in this section the insurers to conserve the greatest part of the high risks, has to offer them a HRS payment (the difference between  $\bar{\beta}\Pi$  and  $p_H.D$ ) sufficiently high (above a given threshold) so that insurers play such an action.

We find that the condition is  $(\bar{\beta}\Pi - p_H.D) \geq (1 - \underline{\alpha})k.p_H.(1 - p_H)D^2$ .

Regarding the insurers' catastrophic risk aversion ( $k/2$ ), this threshold can also be written in order to consider their aversion, which must not be too high in order to guarantee the equilibrium as described by proposition 2 to be sustainable ( $k \leq \frac{1}{(1 - \underline{\alpha})p_H.D}$ ). This condition

is intuitive: the more risk averse the insurers, the less susceptible to keep the high risks<sup>26</sup>.

Regarding potential insured damage, that threshold means that  $D$  has to be lower than the

expression  $\frac{1}{k.p_H(1 - \underline{\alpha})}$ <sup>27</sup>.

### 3.3. Discussion

<sup>26</sup> As soon as their risk aversion is too important for bearing such risks or that the potential damage would be really catastrophic for them, they will transfer the largest possible portion of those catastrophic risks to GRe. So they behave as financial intermediaries.

<sup>27</sup> In appendix, proof of Proposition 2 explains the other strategies of insurers when this condition is not sustainable. For intermediary levels of risk aversion or damage – that is  $\frac{1}{p_H.D(1 - \underline{\alpha})} < k < \frac{1}{p_H.D(1 - \bar{\alpha})}$  or

$\frac{1}{k.p_H(1 - \underline{\alpha})} < D < \frac{1}{k.p_H(1 - \bar{\alpha})}$  – the insurers will play a cession rate (best response) included in the possible cession range offered by GRe.



*Frontier between pooling and separating equilibrium areas:* Proposition 2 deals with that trade-off by taking into account the necessary and sufficient conditions for the government and the insurance industry to play a specific equilibrium in which the insurers conserve the largest portion of risks.

It can be noticed that, according to Proposition 1, as soon as the minimal possible cession rate offered by the public reinsurer is higher than a threshold, here

$\underline{\alpha} \geq r_H = \bar{\alpha} \cdot p_L + \frac{(1-p_L)(1-\lambda \cdot p_H \cdot \bar{\alpha})}{1-p_H}$ , the pooling equilibrium is not sustainable anymore.

According to Proposition 2, in that case we can balance to the separating equilibrium (depending on the level of HRS).

That threshold  $r_H$  is thus also the equation of the frontier between pooling and separating equilibrium areas for the two-specific sub-cases studied here.

*The cost of risk financing:* The parameter  $\lambda$  may also be interpreted as the weight attributed by the Treasury to an *ex post* governmental payment (risk financing, the state guarantee being triggered) in order to balance the system<sup>28</sup>. How does such a frontier move with the cost of risk financing  $\lambda$ ? When the cost of *ex post* financing damage through governmental expenditure increases and is relatively high, that is when  $\lambda$  is relatively high, that threshold  $r_H$  decreases. So does pooling equilibria's area. In particular, the Treasury has a clear interest in that case to induce insurers keeping the largest possible portion of the risks (for instance by levying *ex ante* higher amounts of extra-charges on the insured) when  $\lambda$  is large, i.e. playing the separating strategy.

Interestingly enough, it can also be shown that as soon as the parameter  $\lambda$  is higher than 1 (which is always the case as soon as this parameter is the shadow cost of public funds),

the social welfare –defined as the sum of  $U$  and  $V$ – at the separating equilibrium is always higher (or at least equal) than the social welfare associated with the pooling equilibrium of the model. In a more practical sense, that means that the government should be induce to look for a effective risk-sharing with the private insurers instead of just using them as simple intermediaries. It may do that by sharing more systematically information on catastrophic risks with them.

*The basic contract premiums:* Further extensions can be considered. From a catastrophic risk cover point of view, we may want to create partnerships as those proposed by this paper (or to imagine related systems) to specific countries. In that spirit, another parameter is essential: the level of basic contract premiums, which the governmental extra-charge policy is applied on, i.e.  $\Pi$  in the model.

For obvious reasons, government has interest in applying the extra-charge policy on the largest possible segment of insurance policies (we illustrate that point with the US and French markets in section 4). Intuitively, when the level of total premiums on which an extra-charge policy is applied is large, the model may more often lead to a pooling equilibrium: for a given governmental weight attributed to finance *ex post* the cost of a catastrophe, the government could be more susceptible to choose a low extra-charge rate whatever the type given by the nature. So they could simply use insurers as indemnification intermediaries. Conversely, when the financial basis  $\Pi$  for the application of the catastrophe extra-charge policy is low; government may tend to inform the private insurers of the true type more systematically, in order to develop and sustain a real risk-sharing partnership with them<sup>29</sup>.

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<sup>28</sup> For this, the literature on risk-managers' choices between mitigation and reparation can help us. Mitigation measures, which permit the reduction of loss probability *ex ante*, are often less costly than a reparation *ex post*. The government may prefer spend public expenditures to improve mitigation programs.

<sup>29</sup> The mathematical proofs of these arguments are not presented here in order to facilitate reading.

All things being equal, if we consider that the couple  $(\Pi, \lambda)$  characterizes a given country (or state), it might be interesting to get some data and test empirically the results of the model by applying to it different values of the national parameters  $\lambda$  and  $\Pi$ .

*The role of the asymmetry of information:* Finally, it might be useful to consider proposition 2 on the light of the recent economic literature on asymmetry of information in insurance markets. In a traditional approach a la Rothchild-Stiglitz (R-S)<sup>30</sup> (1976) –i.e. an insurer/insured relationship where the most informed part regarding the risk is the insured– high risks are entirely covered when low risks are only partially covered at the equilibrium. Proposition 2 tells, however, that low risks are mostly covered by GRe when high risks are only partially covered (transferred) by the public reinsurer. Are those results inconsistent with more traditional R-S results? This is actually not the case. Our model is not similar with R-S models in its formalisation. And –and perhaps more important– there is a crucial difference regarding the nature of the asymmetry of information. In our model, because of the specific risks the paper is focused on, the insured part is not the most informed one. The public reinsurer (the insuring part), which covers the insurers (insured part), is the most informed part. Under such an assumption, our results are consistent with the recent literature that analyses the implications of such a reversed asymmetry of information and shows that, in such a context, the low risks are fully insured at the equilibrium when high risks are only partially insured (Villeneuve, 2000; Henriët and Michel-Kerjan, 2003).

#### **4. Illustrations**

This section considers the extent to which the risk-sharing aspects analysed by the model can fit with more practical situations at a national level. We consider two illustrations: (1) the insurance scheme established 20 years ago in France to cover against natural catastrophes and

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<sup>30</sup> Or Stiglitz (1977) for a monopolistic insurer.

well as the results of its application to the US insurance market of insurance; (2) those issues in the context of emerging large-scale terrorism risks.

#### *4.1. Risk-sharing in the French insurance “Cat. Nat. System” and beyond*

Until 1982, French insurers had refused to cover against natural hazards. The discussions between government and insurers that took place after the 1981 flooding episodes led the creation of a specific partnership between the insurance industry and the French government, the “Cat.Nat. System”. Insurers receive a governmental commission to levy a “natural catastrophe” surcharge against all their property and casualty policyholders and to manage the indemnification process in case of a catastrophe. Insurers have the possibility to be reinsured for that special line of their portfolio by the Caisse Centrale de Reassurance (CCR), an unlimited state-guarantee reinsurance company. The French Treasury decides the level of surcharge. Interestingly enough the evolution of that scheme follows the theoretical results obtained through the two propositions (de Marcellis and Michel-Kerjan, 2001). At the infancy of the scheme, a period of low damages, government used to levy a low surcharge (5.5% of the insurance premiums) and the insurers’ cession rate to the CCR was high<sup>31</sup>. With time, the cost due to natural disasters increased, as did the surcharge set by government<sup>32</sup>. Insurers, who received a substantial governmental commission, kept more premiums instead of being largely reinsured by the CCR<sup>33</sup>. With those specificities, the scheme remains unique worldwide<sup>34</sup>.

A system similar to that one could also be established with some benefit in the US. As some natural hazard mapping has been developed there, the surcharge could depend on the level of risk. For instance, the introduction of an extra-charge on every US property insurance

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<sup>31</sup> The maximum cession rate to the CCR was 90%; on average, the insurers used to transfer 85% of the risks.

<sup>32</sup> The surcharge rate increased from 5.5% of the basic premium to 9% in 1988 and to 12% in 2000.

<sup>33</sup> Between 1988 and 1999, with a minimum cession rate required of 40%, the average cession rate to the public reinsurer has remained stable around 43% (CCR, 2000).

<sup>34</sup> Magnan (1995) gives a description of the French “Cat.Nat. System”; for analyses of its strengths and limitations, see de Marcellis (1997) ; Michel-Kerjan (2001).

policies, with a rate varying between 5 and 20% and a two-thirds of the basic extra-charge on base premiums for motor vehicles has been studied. The results of that estimation are illustrative. “If this schedule of surcharges had been applied between 1977 and 1993 to the relevant lines of property insurance, annual receipts would have been enough to cover all private insurance payments for natural catastrophes *as well as* all federal disaster payments and still leave a cumulative surplus of nearly \$2 billion at the end of the period” (Moss, 1999, p 346). After some devastating natural disasters in the US –hurricane Andrew in 1992, the Mississippi floods in 1993 and the Northridge earthquake in 1994– the question of the limitation of the insurability of such events by the private sector alone as well as the need to the development of public-private partnerships have been raised. It must be stressed that if such an extra charge policy had been applied to *all* property and casualty lines, as done in the French scheme, the surplus at the end of 1993 would have totaled no less than \$148 billion (Moss, 1999); i.e. enough liquidity to cover damages due to three events without requiring any federal payment or extra reimbursement from insurers.

#### 4.2. *The case of terrorism risk*

Prior to September 11, 2001 terrorism coverage in most European countries as well as in the US came essentially free of charge by virtue of its being included in most standard commercial policy packages with no premium to speak of associated with these events<sup>35</sup>. Insurers and reinsurers had simply not had to pay close attention to their potential losses from terrorism. The 9/11 terrorists attacks killed more than 3000 citizens from over 80 countries, illustrating the *transnational* character of terrorism (Sandler and Enders, 2004). From an economic perspective, they inflicted insured damage estimated at nearly \$40 billion; i.e. the most costly event in the world history of insurance (Swiss Re, 2002-b). Hence 9/11

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<sup>35</sup> In the UK where, in the wake of terrorist attacks in the City of London in April 1992 and an announcement seven months later by British insurers that they would exclude terrorism coverage from their commercial policies, the UK established a mutual insurance organization (*Pool Re*) in 1993. Pool Re charges a separate,

confronted governments and the insurance industry with an entirely new loss dimension. The larger question being debated was whether terrorism is an insurable risk and what is the role of governments in developing effective policies that aim at curbing such kind of religious-based fundamentalism terrorism (Kunreuther, Michel-Kerjan, and Porter, 2003; Sandler and Enders, 2004).

Although both terrorist activities and natural disasters have now a recognized potential to create catastrophic losses, there is a significant difference between those two risks that raise some challenges for the private sector to provide insurance protection without some type of partnership with the public sector.

The sharing of information on terrorism risk between all the stakeholders is clearly different than with respect to natural hazards. In the latter case, the asymmetry of information regarding the risk, when it exists, is mostly due to some historical reasons of a lack of interest of the insurers for that types of markets. But to some instances, with the creation and the development of public-private partnerships, new scientific information on the risk could be shared among the stakeholders.

The situation is different with respect to terrorism, as information on possible attacks or current threats is kept secret by government agencies for national security reasons. Traditionally, government intervention into markets is justified by market failure due to problem associated with asymmetric information between buyers and sellers. The problem posed by terrorism appears to be, however, one where there is *symmetry of non-information* on the risk between insured and insurers for covering terrorist attacks. Terrorism is not only a new catastrophic risk with limited available historical data; it is also a risk where the government is the most informed party, which raises some fundamental question for political economy (Michel-Kerjan, forthcoming)<sup>36</sup>. Because of this feature, the question with respect to

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optional premium for terrorism coverage that can be calculated as a percentage of the total sum insured under a fire and accident policy.

<sup>36</sup> Similarly to natural hazard loss estimation models developed since the end of the 80's, the development in 2002 of a first generation models for terrorism risk (by firms like RMS, AIR and EQECAT) provide better

the terrorist risks is not whether the government has to intervene in insurance markets because of market failure, but rather how government and the insurance industry can work together in providing coverage for citizens and firms.

The need for public-private partnerships has been recognized in the aftermath of 9/11 in a lot of countries. In the UK, the scale of 9/11 attacks led to changes –with effect on January 1, 2003– in operation of *Pool Re* to clarify the insurers' involvement. In France discussions between insurance industry and the government led to the creation of a special reinsurance pool for terrorism, the *GAREAT*, which has been operating since January 1, 2002. In Germany discussions between German Federal Government and the insurance industry led to the creation of a special insurer for terrorism risks, *Extremus AG*. The company has been operating since November 2002<sup>37</sup>. In the US, that need was recognized too on November 26, 2002 when the Terrorism Risk Insurance Act of 2002 (*TRIA*) was passed and signed into law by the President the next month. TRIA established a temporary partnership between insurance industry and the US federal government for covering against terrorist attacks up to \$100 Billion (Kunreuther, Michel-Kerjan, and Porter, 2003).

## **5. Concluding remarks**

Although the theoretical approach simplifies the complexity of these emerging issues, the underlying insights in the model presented here reasonably capture some important elements of any public-private partnership of risk cover. Hence it may also open the window for future analysis. At the end, a key question for every policy implementing national schemes for covering against catastrophic risks is who should pay for the costs due to those extreme events? If they are viewed as a national problem with the costs borne by all taxpayers rather

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estimation of potential losses associated with specific scenario of attacks; the asymmetry of information between governments and the insurers is relative to  $p$  and not to  $D$  (Kunreuther, Michel-Kerjan and Porter, 2003).

than just those who suffer losses, then some type of tax on all citizens might be appropriate. If on the other hand, Governments feel that those costs should be borne by those who are at risk, then the owners of the property at risk will be required to pay the cost.

In developing public-private partnerships for catastrophe coverage, whether natural disaster or large-scale terrorism, there is a set of similar questions to be addressed: What type of risks would be covered by this insurance? How will premiums be determined for different insured given that the risk of future losses is highly uncertain? What coverage limits will be placed on different types of property in different parts of the country? What portion of the losses will the private sector cover and what portion will the government cover?

At a European level, the devastating floods that beset several countries in the summer 2002 and caused economic damage of \$15 billion (Swiss Re, 2003) also raise the question of developing appropriate solutions.

The European Union Solidarity Fund (EUSF) was created recently to assist people in affected regions of the member states and countries involved in accession negotiations in the event of a major disaster. It may constitute a first step in the recognition by the European Commission of the necessity to be involved in building up better-dimensioned answers. Hence EU may still be at the beginning of global process<sup>38</sup> to create a scheme for covering against catastrophes that would be really operating. Managing catastrophic risks in a EU of now 25 countries and 450 million people with several new entrants with various economic and political agendas poses, however, major problems to be solved (CEC, 2002); it may take time and create strong opposition because of historical reasons, cost concerns, legal impediments and national concerns as well (Roland and Verdier, 2003).

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<sup>37</sup> For a comparative analysis of the current terrorism insurance schemes developed after 9/11 in the US, France and Germany (relative to each country's market), see Michel-Kerjan and Pedell (2003).

<sup>38</sup> For instance the generalization of risk mapping to all European countries, the development of common standards for risk mitigation or the enforcement law.



The emergence of a new dimension of losses is today recognized; that urges the development of efficient European policies based upon partnerships between governments, insurers and reinsurers as well as future research.

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## Appendix A

**Proof of Proposition 1.** To prove that  $(\underline{\beta}; \bar{\alpha})$  constitutes the action profile at the pooling equilibrium, we have to show that considering some private insurers' beliefs out of equilibrium, say  $(\nu; 1 - \nu)$ , the government has no interest in deviation.

We note  $U_i$ ,  $i \in \{L; H\}$ , the expression of the public-sector utility when knowing the probability of a catastrophe is  $i$ .

**When Nature chooses  $p = p_L$ :** According to the lemma, the members of the government know that if they choose the low extra-charge policy  $\underline{\beta}$ , the insurers will play the largest cession rate action. In such a case, because  $\underline{\beta} \cdot \Pi - D < 0$ , the Treasury will have to support GRe.

It will never be in the government's interest to deviate considering the insurers' beliefs  $\nu$  and action out of equilibrium  $\alpha(\nu)$  iff  $U_L(\underline{\beta}; \bar{\alpha}) \geq U_L(\bar{\beta}; \alpha(\nu))$ .

As, by playing the low extra-charge policy, the state guarantee will be triggered, we have:

$$-\underline{\beta} \cdot \Pi + (1 - p_L) \bar{\alpha} \underline{\beta} \cdot \Pi + \lambda \bar{\alpha} \cdot p_L \cdot (\underline{\beta} \cdot \Pi - D) \geq -\bar{\beta} \cdot \Pi + \alpha(\nu) \cdot (\bar{\beta} \cdot \Pi - p_L \cdot D).$$

With the expression of the extra-charge policies  $\underline{\beta} \cdot \Pi$  and  $\bar{\beta} \cdot \Pi$ , it becomes:

$$\alpha(\nu) \leq 1 + \frac{(1 - p_L) \bar{\alpha} p_L D + \lambda \bar{\alpha} \cdot p_L \cdot D \cdot (p_L - 1)}{D - p_L \cdot D}.$$

$$\text{That is } \alpha(\nu) \leq 1 - \frac{(\lambda - 1)(1 - p_L) \cdot \bar{\alpha} \cdot p_L}{(1 - p_L)} = 1 - (\lambda - 1) \cdot \bar{\alpha} \cdot p_L \equiv r_L.$$

Therefore,  $U_L(\underline{\beta}; \bar{\alpha}) \geq U_L(\bar{\beta}; \alpha(\nu))$  iff  $\alpha(\nu) \leq r_L$ .

As  $\alpha \in [\underline{\alpha}; \bar{\alpha}] \subset [0, 1]$ , we have:

- As soon as  $\bar{\alpha} \leq r_L$ , the action profile  $(\underline{\beta}; \bar{\alpha})$  is **always** sustainable for  $p = p_L$  and that whatever the insurers' beliefs out of the equilibrium ;
- If  $r_L < \underline{\alpha}$ , the action profile  $(\underline{\beta}; \bar{\alpha})$  is **never** sustainable for  $p = p_L$  and that whatever the insurers' beliefs out of the equilibrium: the government will always be incited to deviate from the candidate;
- When  $\underline{\alpha} \leq r_L \leq \bar{\alpha}$ , the action profile  $(\underline{\beta}; \bar{\alpha})$  may be sustainable for  $p = p_L$  depending on the insurers' beliefs out of the equilibrium.

As the threshold  $r_L = 1 - (\lambda - 1)\bar{\alpha}.p_L$  depends on the upper limit of the possible cession rate range,  $\bar{\alpha}$ , we can rewrite the conditions of existence of such a candidate:

- A soon as  $\bar{\alpha} < \frac{1}{1 + (\lambda - 1)p_L}$ , the action profile  $(\underline{\beta}; \bar{\alpha})$  is **always** sustainable for  $p = p_L$  and that whatever the insurers' beliefs out of the equilibrium;
- If  $1 - (\lambda - 1)\bar{\alpha}.p_L < \underline{\alpha}$ , the action profile  $(\underline{\beta}; \bar{\alpha})$  is **never** sustainable for  $p = p_L$  and that whatever the insurers' beliefs out of the equilibrium: the government will always be incited to deviate from the candidate;
- When  $\underline{\alpha} \leq 1 - (\lambda - 1)\bar{\alpha}.p_L$  and  $\frac{1}{1 + (\lambda - 1)p_L} \leq \bar{\alpha}$ , the action profile  $(\underline{\beta}; \bar{\alpha})$  may be sustainable for  $p = p_L$  depending on the insurers' beliefs out of the equilibrium. For instance, one takes  $\nu(p_L D) \setminus \beta = 1 = 1 - \nu(p_H D) \setminus \beta$  out of the equilibrium in order to lead insurers to play the « low cession rate » strategy.

**When Nature chooses  $p = p_H$ :** Using similar arguments, it becomes that the government has no interest in deviation from  $(\underline{\beta}; \bar{\alpha})$  if and only if  $U_H(\underline{\beta}; \bar{\alpha}) \geq U_H(\bar{\beta}; \alpha(\nu))$ , that is:

$$\alpha(\nu) \leq \frac{D.(1 - p_L) + (1 - p_H)\bar{\alpha}.p_L D - \lambda \bar{\alpha}.D.p_H.(1 - p_L)}{D.(1 - p_H)}$$

$$\text{i.e. } \alpha(\nu) \leq \bar{\alpha}.p_L + \frac{(1 - p_L).(1 - \lambda \bar{\alpha}.p_H)}{(1 - p_H)} = r_H$$

Thus,  $U_H(\underline{\beta}; \bar{\alpha}) \geq U_H(\bar{\beta}; \alpha(\nu))$  iff  $\alpha(\nu) \leq r_H$ .

Using similar arguments than with  $p = p_L$  and after simplifications, we have:

- A soon as  $\bar{\alpha} < \frac{1}{1 + (\lambda - 1)p_H}$ , the action profile  $(\underline{\beta}; \bar{\alpha})$  is **always** sustainable for  $p = p_H$  and that whatever the insurers' beliefs out of the equilibrium;
- If  $\underline{\alpha} > r_H = \bar{\alpha}.p_L + \frac{(1 - p_L).(1 - \lambda \bar{\alpha}.p_H)}{(1 - p_H)}$ , the action profile  $(\underline{\beta}; \bar{\alpha})$  is **never** sustainable for  $p = p_H$  and that whatever the insurers' beliefs out of the equilibrium: the government will always be incited to deviate from the candidate;
- When  $\underline{\alpha} \leq r_H = \bar{\alpha}.p_L + \frac{(1 - p_L).(1 - \lambda \bar{\alpha}.p_H)}{(1 - p_H)}$  and  $\frac{1}{1 + (\lambda - 1)p_H} \leq \bar{\alpha}$ , the action profile  $(\underline{\beta}; \bar{\alpha})$  may be sustainable for  $p = p_H$  depending on the insurers' beliefs out of the

equilibrium. For instance, take  $v(p_L D) \setminus \beta = 1 = 1 - v(p_H D) \setminus \beta$  out of the equilibrium in order to lead insurers to play the strategy low cession rate.

In summary, a *sufficient condition* for the profile  $(\underline{\beta}; \bar{\alpha})$  to be a pooling equilibrium is  $\bar{\alpha} \leq \min\{r_L; r_H\}$ , that is as soon as  $\bar{\alpha} < \frac{1}{1 + (\lambda - 1)p_H}$ . That result is true whatever the insurers' beliefs out off the equilibrium.

Conversely, when we take by chance two different beliefs out off the equilibrium, it is easy to obtain, with at least one of these beliefs, a contradiction with the fact that the action profile  $(\underline{\beta}; \bar{\alpha})$  may be chosen. The condition under which such a contradiction (interest to deviation) never appears is that the highest possible cession rate to GRe is lower than a given threshold, here  $\bar{\alpha} < \frac{1}{1 + (\lambda - 1)p_H}$ .

Moreover, a *necessary condition* for the profile  $(\underline{\beta}; \bar{\alpha})$  to be a pooling equilibrium, with specific insurers' beliefs out of the equilibrium, is  $\underline{\alpha} \leq \min\{r_L; r_H\}$ . Conversely, taken into account  $\underline{\alpha} \leq \min\{r_L; r_H\}$ , all beliefs out of the equilibrium will not lead the pooling candidate to be an equilibrium, but some of them work.

Finally, as soon as  $\min\{r_L; r_H\} < \underline{\alpha}$ , there is no sustainable pooling equilibrium with the government playing the low extra-charge action. Conversely, the condition under which the candidate will fail for every belief out off equilibrium is that  $\min\{r_L; r_H\} < \underline{\alpha}$ .  $\square$

**Proof of Proposition 2.** When do separating equilibria exist? In these equilibria, a low type public sector decide a low extra-charge policy,  $\underline{\beta}$ , thus revealing its type and its utility is  $U_L(\underline{\beta}; \bar{\alpha}_L)$ . Indeed, the best response of the insurers, who think bearing low risks in that case, is the highest possible cession rate to the CCR (cf. Lemma).

If the government, although knowing it was low risk, decided a high extra-charge policy,  $\bar{\beta}$ , it would convince the private insurers that it was high type and would obtain  $U_L(\bar{\beta}, \alpha_H^*)$ , where  $\alpha_H^*$  is the best response from insurers to a high extra-charge policy (thinking receiving high risks).

Thus, a first necessary condition for the existence of a separating equilibrium is:

$$U_L(\underline{\beta}; \bar{\alpha}_L) \geq U_L(\bar{\beta}, \alpha_H^*) \quad (S_L)$$

Using the same argument with high-risk public sector, it appears that a second necessary condition is:

$$U_H(\bar{\beta}, \alpha_H^*) \geq U_H(\underline{\beta}; \bar{\alpha}_L) \quad (S_H)$$

The *necessary conditions* for the existence of a separating equilibrium is thus the following system:

$$\begin{cases} U_L(\underline{\beta}; \bar{\alpha}_L) \geq U_L(\bar{\beta}, \alpha_H^*) \\ U_H(\bar{\beta}, \alpha_H^*) \geq U_H(\underline{\beta}; \bar{\alpha}_L) \end{cases} \quad (S)$$

Conversely, suppose that system (S) is satisfied. Let consider the following strategies and beliefs. The low type public sector chooses a low extra-charge policy and the insurers correctly infer that the risks are low when observing the low extra-charge policy and then play their best strategy, that is  $\bar{\alpha}$ . The high type public sector chooses a high policy and the insurers (correctly) infers that the public that the transferred risks are high, and therefore play their best strategy,  $\alpha_H^*$ , which depends on the level of governmental HRS.

Let now determine the expression and value of  $\alpha_H^*$ .

As  $\Phi(p_H D \setminus \bar{\beta}) = 1$ , the insurers choose the level of cession  $\alpha_H^* \in [\underline{\alpha}; \bar{\alpha}]$  which maximizes

$$V(\alpha; \bar{\beta}) = (1 - \alpha) \cdot (\bar{\beta} \cdot \Pi - p_H D) - \frac{k}{2} (1 - \alpha)^2 p_H \cdot (1 - p_H) \cdot D^2 + m.$$

Here, as the utility function of the insurers is concave with  $\alpha$  ( $V''_{\alpha} = -k \cdot p_H \cdot (1 - p_H) \cdot D^2 < 0$ ), the first order condition is sufficient for the maximization

and is given by  $(V'_{\alpha}(\alpha; \bar{\beta}))_{\alpha=\alpha_H^*} = 0$ , which can be written  $\alpha_H^* = 1 - \frac{(\bar{\beta} \cdot \Pi - p_H D)}{k \cdot p_H \cdot (1 - p_H) D^2}$ .

As  $\alpha \in [\underline{\alpha}; \bar{\alpha}]$ , there would have three cases to be discussed:

(a)  $\alpha_H^* \leq \underline{\alpha}$ , which appears as soon as the governmental HRS is sufficiently high so that:

$$(\bar{\beta} \cdot \Pi - p_H D) \geq k(1 - \underline{\alpha})p_H \cdot (1 - p_H) \cdot D^2.$$

In that case, the insurers play the lowest possible cession rate to GRe,  $\underline{\alpha}$ , keeping the largest portion of risks. The system (S) becomes the system (S-a) that follows:

$$\begin{cases} U_L(\underline{\beta}; \bar{\alpha}_L) \geq U_L(\bar{\beta}, \underline{\alpha}) & (S_L - a) \\ U_H(\bar{\beta}, \underline{\alpha}) \geq U_H(\underline{\beta}; \bar{\alpha}_L) & (S_H - a) \end{cases} \quad (S - a)$$

That case is precisely the one we would like to study. However, in order to offer a complete demonstration of possibility, we describe also the two other possible cases. We will come back on that first case after.

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(b)  $\alpha_H^* \in ]\underline{\alpha}; \bar{\alpha}[$ , which appears for intermediary level of governmental HRS, that is when:

$$k(1 - \bar{\alpha})p_H.(1 - p_H).D^2 < (\bar{\beta}.\Pi - p_H D) < k(1 - \underline{\alpha})p_H.(1 - p_H).D^2.$$

In that case, the insurers play their best cession rate response  $\alpha_H^* = 1 - \frac{(\bar{\beta}.\Pi - p_H D)}{k.p_H.(1 - p_H).D^2}$ .

The system (S) becomes the system (S-b) that follows:

$$\begin{cases} U_L(\underline{\beta}; \bar{\alpha}_L) \geq U_L(\bar{\beta}, 1 - \frac{(\bar{\beta}.\Pi - p_H D)}{k.p_H.(1 - p_H).D^2}) \\ U_H(\bar{\beta}, 1 - \frac{(\bar{\beta}.\Pi - p_H D)}{k.p_H.(1 - p_H).D^2}) \geq U_H(\underline{\beta}; \bar{\alpha}_L) \end{cases} \quad (S-b)$$


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(c)  $\bar{\alpha} \leq \alpha_H^*$ ; which appears when the HRS is too low for inducing the insurers not to transfer the largest possible portion of risks to GRe, that is:

$$(\bar{\beta}.\Pi - p_H D) \leq k(1 - \bar{\alpha})p_H.(1 - p_H).D^2$$

In that case, the insurers choose the highest possible cession rate to the GRe whatever the level of risk,  $\bar{\alpha}$ , transferring the largest portion of risks. The system (S) becomes the system (S-c) that follows:

$$\begin{cases} U_L(\underline{\beta}; \bar{\alpha}_L) \geq U_L(\bar{\beta}, \bar{\alpha}) \\ U_H(\bar{\beta}, \bar{\alpha}) \geq U_H(\underline{\beta}; \bar{\alpha}_L) \end{cases} \quad (S-c)$$


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Let now go back to (S-a). The condition  $(S_L - a)$  can be written:

$$-\underline{\beta}.\Pi + (1 - p_L)\bar{\alpha}\underline{\beta}.\Pi + \lambda.\bar{\alpha}.p_L.(\underline{\beta}.\Pi - D) \geq -\bar{\beta}.\Pi + \underline{\alpha}.(\bar{\beta}.\Pi - p_L.D)$$

That is for condition (S-a):

$$\underline{\alpha} \leq 1 - p_L(\lambda - 1)\bar{\alpha}$$

We find one of the same conditions that for the pooling case.

However, the second one allows differentiating the pooling area and the separating ones.

The condition  $(S_H - a)$  can be written:

$$-\bar{\beta}.\Pi + \underline{\alpha} .(\bar{\beta}.\Pi - p_H .D) \geq -\underline{\beta}.\Pi + (1 - p_H)\bar{\alpha} \underline{\beta}.\Pi + \lambda \bar{\alpha} .p_H .(\underline{\beta}.\Pi - D),$$

that is:

$$\underline{\alpha} \geq \frac{1 - p_L}{1 - p_H} + \bar{\alpha} .p_L + \frac{\lambda .p_H .(p_L - 1)\bar{\alpha}}{1 - p_H}$$

Condition  $(S_H - a)$  can be rewritten as follows:

$$\underline{\alpha} \geq \bar{\alpha} .p_L + \frac{(1 - p_L)(1 - \lambda .p_H .\bar{\alpha})}{1 - p_H}$$

Precisely, such a condition eliminates possible pooling equilibria (cf. proof of Proposition 1).  $\square$

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Fig. 1. The public-private partnership

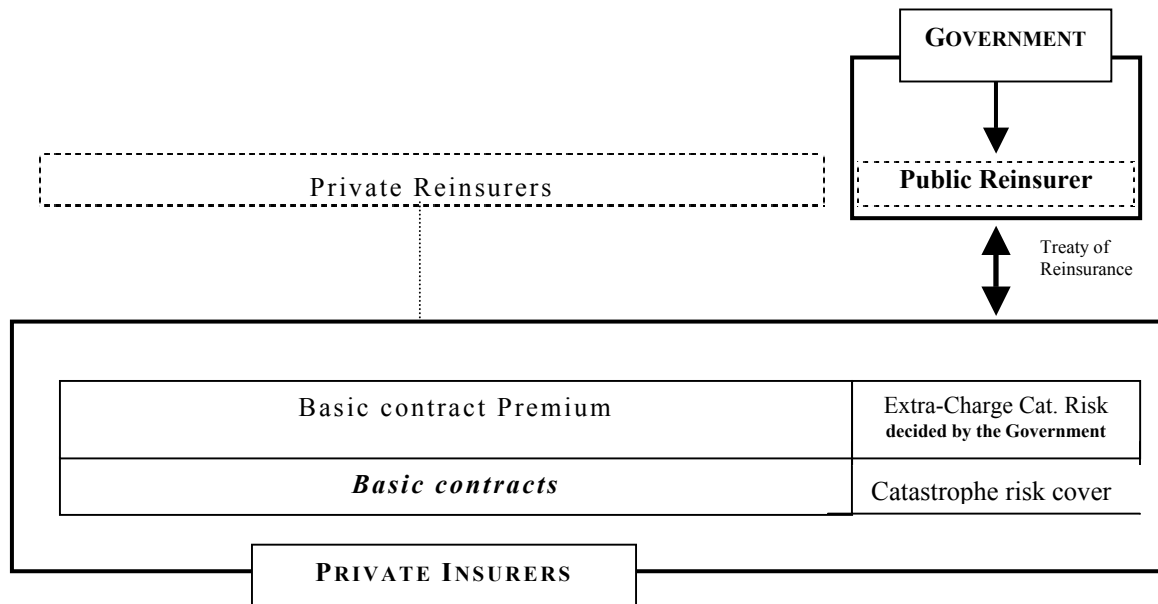


Fig. 2. The extensive form of the game.

