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Juin 2003

Cahier n° 2003-012

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# Political equilibrium with private or/and public campaign finance: A comparison of institutions

John E. Roemer<sup>1</sup>

Juin 2003

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**Résumé:** Nous proposons une théorie de la politique (deux partis, une seule question) dans laquelle les citoyens deviennent membre d'un parti en le finançant et dans laquelle l'influence d'un de ses membres sur la politique proposée par le parti est proportionnelle à sa contribution. L'électorat est constitué de votants informés et non-informés: Seuls les votants informés rejoignent les partis, et le budget de campagne d'un parti, la somme des contributions qu'il reçoit, est utilisé pour communiquer en direction des votants non-informés. Les partis sont en compétition stratégique par rapport à leur choix politique et leur communication. On propose une définition de l'équilibre politique dans laquelle l'appartenance partisane, les contributions et les politiques sont déterminées simultanément, pour quatre modes institutionnels de financement, allant d'un système non contraint purement privé à un système public dans lequel tous les citoyens ont le même impact financier. On compare les qualités des quatre systèmes en termes de représentation et de bien-être.

**Abstract:** We propose a theory of party competition (two parties, single-issue) where citizens acquire party membership by contributing money to a party, and where a member's influence on the policy taken by her party is proportional to her campaign contribution. The policy consists of informed and uninformed voters : only informed voters join parties, and the party campaign chest, the sum of its received contributions, is used to advertise and reach uninformed voters. Parties compete with each other strategically with respect to policy choice and advertising. We propose a definition of political equilibrium, in which party membership, citizen contributions, and parties' policies are simultaneously determined, for each of four financing institutions, running a gamut between a purely private, unconstrained system, to a public system in which all citizens have equal financial input. We compare the representation and welfare properties of these four institutions.

**Mots clés :** équilibre politique, représentation, financement des partis.

**Key Words :** political equilibrium, representation, campaign finance

**Classification JEL:** C72, D72

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## 1. Introduction

There have been, roughly speaking, two theories of party competition developed formally in the political economy literature. The first, due to Anthony Downs (1957), models parties as single candidates whose sole desire is to maximize the probability of winning office; alternatively, one might say there *are* no parties in Downs's model, only opportunistic politicians. The second, due first to Donald Wittman (1973), models parties as maximizing a preference order on the policy space. Although Wittman took the parties' preference orders to be exogenous, later writers modeled the *representation* aspect of parties – namely, that parties' preference orders should somehow reflect the preferences of their members. David Baron (1993), for example, and Ignacio Ortuno and J.E. Roemer (1998) explicitly model parties as having utility functions on the policy space that are averages of the utility functions of their (anticipated) supporters<sup>2</sup>.

These two theories are polar opposites: in the first, parties are completely opportunistic, in the sense of not acting in any explicit sense as the collective agent of a coalition of citizens, and in the second, they act as perfect representatives of coalitions of citizens. To be somewhat more precise, the parties in the Baron and Ortuno- Roemer models represent citizens in proportion to the votes they contribute to the party. But, at least in countries with private campaign financing, citizens make a second kind of contribution to parties -- their money. In this paper, I propose a theory of political competition in which parties do represent citizens, but according to their campaign contributions, not according to their (anticipated) votes.

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<sup>2</sup> To be precise, the Ortuno – Roemer paper represents the party's preferences as those of its average member.

Suppose we assume that parties are purely representative institutions. That is, at least for the present exercise, we ignore the opportunistic aspect, that parties are run by politicians who have career interests that deviate from the interests of their members. Clearly, both votes and money are important to parties, and one might ask, in a society where parties are privately financed, should we expect the preferences of these parties over policies to reflect their voters' preferences or their financial contributors' preferences? Which kind of contribution, the vote or the dollar, will purchase an *internal* party vote on what policy the party should propose? I claim that a promise to represent the interests of the citizens who vote for the party is incoherent (or, at least, not credible) in the sense that the citizen contributes her vote *after* the inter-party competition is completed. How, if we take the timing seriously, can the party represent a coalition of citizens that will not come into being until election day<sup>3</sup>? Financial contributions, on the other hand, take place before the election, and a given citizen can contribute money in a series of gifts, providing some accountability for the party's promise to represent his interests – assuming that the party knows who its contributors are<sup>4</sup>.

I am here viewing parties as *empty vessels*, that is, organizations that will come to represent those who contribute to them. Not all parties, in reality, are of this type: sometimes a party is created by a coalition of citizens with common interests, and the party represents those interests because those citizens are its organizers. Labor parties or

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<sup>3</sup> Thus, the Baron and Roemer models referred to are really of the rational-expectations type: parties represent coalitions of citizens who, in equilibrium, turn out to be their supporters. Hence, my use of adjective 'anticipated.'

<sup>4</sup> The clause about the identification of contributors is important. Recently, Bruce Ackerman and Ian Ayers [2002] have proposed to amend political contribution law so that contributions remain private but

confessional parties, which were created by trade unions or churches, are examples. I do not think that all significant parties in democracies are of this type, however. And if private financial contributions are necessary for a party to succeed, there will arguably be pressure for the party to represent its contributors.

Under this logic, what might we expect to occur if a society finances its political parties publicly? In many (European) countries, parties receive public funds in proportion to their votes in the previous election. This system aligns the voters with the financial contributors to a party, and it suggests that representative parties would come to represent the interests of those who vote for them. (Here, we would naturally want to model the representation game as one taking place intertemporally.)

In a democracy in which political competition is organized through party competition, then, there are two loci at which the ‘one man one vote’ democratic desideratum may be applied: the first is in the *intra*-party preference formation process, and the second is in the *inter*-party election. There are many people – laymen as well as political theorists—who think that fair representation requires that the one-man-one-vote principle be applied at both loci. Thus, a political system in which parties can form freely and the franchise is universal, but parties represent their financial contributors rather than the coalition of citizens who vote for them, is (by many) considered to be imperfectly representative. It is, of course, this sentiment which has given rise to the legislation regulating campaign contributions in the United States, and the public financing of campaigns in Europe.

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donors are anonymous to parties. They believe this will reduce the pressure on parties to represent their contributors.

While my present purpose is to formulate a theory of political competition in which parties are empty vessels that come to represent their contributors, I reiterate that I do not claim that all parties are of this form: the two important exceptions are, first, parties in which the principal-collective agent problem is solved poorly, where the party organizers can use the party for opportunist purposes (of delivering the perks of office to themselves), and second, parties that are explicitly ideological, where their organizers are a priori committed to representing a particular viewpoint or coalition of citizens. The ‘empty vessel’ party is best seen as an intermediate form between the opportunist and ideological party.

In my view, a successful theory of political equilibrium, when parties are financed by private contributions from citizens, should contain the following elements, which I describe here informally, and model in the rest of the paper:

1. Parties compete with each other over policies that voters care about; that competition is strategic, in the sense that an equilibrium in the inter-party game should consist of a pair of policies that are mutual best responses.
2. Best responses according to what policy preferences? The preferences of its coalition of contributors, which is to say, preferences represented by some utility function defined on the policy space, which reflects the interests of contributors according to their financial contributions to the party. We identify the *members* of a party with its coalition of contributors.
3. But campaign contributions must play a double role. As just stated, they purchase influence in the intra-party struggle, or bargaining problem, over the party’s

line. Second, they comprise the party's campaign chest, which is used to advertise its position, and to win over voters who are not initially committed to the party's platform.

4. Parties compete with each other, then, not only with their policy proposals, but in their efforts to reach uncommitted voters, an enterprise in which the fuel is the campaign chest. Thus, parties' budgets (and hence the total of their members' campaign contributions) must also be in some way optimal for the members with regard to this effort. From the viewpoint of its members, the party's total campaign chest is a public good, whose value to members is its role in winning votes. As associations of very large coalitions of citizens, we suggest that parties have the ability to co-ordinate campaign contributions among their members so that this public good is optimally provided.

5. Party membership should be stable, in the sense that, at equilibrium, each contributor should prefer the policy of the party to which she contributes to the policy of the opposing party.

6. No individual citizen type should have noticeable influence on party policy or the campaign chest. (Thus, we wish to model a very large polity, where no individual citizen type has observable influence.)

From these desiderata, one can surmise that the theory will involve the simultaneous occurrence of three kinds of equilibrium:

- i) an equilibrium in campaign contributions, one for each party, among the set of contributors to the party (i.e., the members);
- ii) an equilibrium in policies, in a game played between the parties;
- iii) an equilibrium in the assignment of contributors to parties.



In addition to these six desiderata, we may add, finally, a methodological desideratum:

7. Equilibria should be easy to compute, and locally unique, so that important questions of comparative statics may be studied.

One must note that the financing of parties is not the only locus at which money enters into politics, the other one being the lobbying process. Recently, Grossman and Helpman (2001) have studied the role of money in politics. Much of their book is concerned with modeling the lobbying process. They do, however, contribute, as well, to the theory of campaign finance: in chapter 10, they study political equilibrium in which campaign contributions of interest groups have an effect on the distribution of votes between parties. It will be more fruitful to compare their theory with the present one in the concluding section.

After having formulated the theory of political equilibrium with private campaign finance, we can easily amend it to model three other financing institutions:

- (1) privately financed parties with a legal cap on contributions;
- (2) publicly financed parties, where each party receives a public subsidy in proportion to its size (that is, each citizen brings an equal public subsidy to the party she joins);
- (3) publicly and privately financed parties, where each party receives public funds matching its private contributions.

(The American system is complex, but is approximated by a combination of institutions (1) and (3).)

We will make these amendments, and then compare the nature of the political equilibria that would obtain under the various financing institutions. Our central concern will be the degree to which these institutions produce results that conform to a common conception of good representation.

In section 2, I introduce a new concept of equilibrium in environments with cooperative ventures, which will later be a foundation of the concept of political equilibrium. In section 3, I describe the environment of the politico-economy. Section 4 presents the equilibrium concept for an environment under our first institution, unconstrained private campaign finance. Section 5 presents an interesting characterization of the equilibrium, and shows how to compute it. Section 6 applies the model to an example, computes equilibrium, and presents some comparative statics. Section 7 presents the equilibrium concept for an environment with constrained private campaign financing, where there is a legal cap on contributions, and computes the equilibria for our canonical example, for various values of the cap on contributions. Section 8 presents the definition of political equilibrium with public financing, for this environment, and computes the equilibria for the canonical example of section 6, under both public institutions described above. Section 9 concludes with a summary of the comparisons among the financing institutions, and some brief welfare analysis.

## 2. Kantian equilibrium

In this section, I introduce a general notion of equilibrium in a public-good setting. In later sections, it will be applied to the problem of campaign finance.

Consider an economy with  $N$  members. Each is endowed with a private good. The economy produces a public good according to the technology  $y = Q(x_1, \dots, x_N)$ , where  $x_i$  is the contribution to the public good by individual  $i$ , in units of the private good, and  $y$  is the level of the public good. The utility functions of the individuals are given by:

$$v^i(x_1, \dots, x_N) = c^i(x_i) + h^i(Q(x_1, \dots, x_N)), \quad (2.1)$$

which we express in terms of a personalized cost of giving and a personalized utility from the public good.

Denote by  $x$  the vector of contributions and by  $v$  the vector of utility functions.

We define:

**Definition 1.** A vector of contributions  $(x_1^*, \dots, x_N^*)$  is a *Kantian equilibrium* for the economy  $v$  if *no* individual would prefer that *all* individuals increase or decrease their contributions by *any* given factor  $r \geq 0$ .

Thus, at a Kantian equilibrium, there is *unanimous* agreement that, along the ray of possible contributions defined by the contribution vector  $x^*$ , the vector  $x^*$  itself is the best one.

In the differentiable case, if a contribution vector  $x^*$  is Kantian, then the functions of a real variable  $r$  defined by

$$u^i(r) = v^i(rx_1^*, \dots, rx_N^*) \quad (2.1b)$$

are maximized, for all  $i = 1, \dots, N$ , at  $r = 1$ .

In a Nash equilibrium, each agent assumes that if she alters her behavior, the behavior of all others remains fixed. In a Kantian equilibrium, she assumes that if she alters her behavior, all others will alter theirs in like manner (in the sense of equi-proportional changes). Our concept inherits its name from the Kantian imperative: Take

an action if and only if you would have all others do likewise. The idea was first introduced in Roemer (1994, Chapter 6), although it was not formalized there in the present manner.

A vector of contributions  $x$  is *Pareto efficient* if there is no vector of contributions at which all individuals are at least as well off, with at least one individual better off.

We have the following result:

Proposition 1. *Let the functions  $c^i$  be convex and differentiable, and the functions  $h^i$  and  $Q$  be concave, differentiable, and strictly increasing. Let  $x^* > 0$  be a Kantian equilibrium. Then  $x^*$  is Pareto efficient.*

We prove the proposition after establishing two lemmas.

Lemma 1. *If there are positive numbers  $\{\alpha_i\}$  such that*

$$\sum \alpha_i \nabla v^i(x) = 0, \quad (2.2)$$

*then  $x$  is Pareto efficient.*

( $\nabla v^i$  is the gradient vector of  $v^i$ .)

Proof: By concavity of  $v$ 's, it suffices to show  $x$  is locally Pareto efficient. Suppose, to the contrary, there were a direction  $d \in R^N$  at  $x$  such that

$$\alpha_j \neq 0 \quad \nabla v^j(x) \cdot d \geq 0,$$

with at least one strict inequality. Then it follows from (2.2) that  $\sum \alpha_i \nabla v^i(x) \cdot d < 0$ , which shows that  $x$  is Pareto efficient.

Lemma 2. Let  $a_1, \dots, a_N$  be positive numbers; let  $(\alpha_1, \dots, \alpha_N)$  be a positive vector in the unit simplex  $S^N$ . Then there exist positive numbers  $\alpha_1, \dots, \alpha_N$  such that

$$\alpha_j \frac{\alpha_j a_j}{\sum_i \alpha_i a_i} = \alpha_j.$$

Proof:

$$\text{Let } \alpha_j = \frac{\alpha_j}{a_j}.$$

Proof of proposition 1:

Setting the derivative with respect to  $r$  of the functions  $u^i(r) = v^i(rx_1^*, \dots, rx_N^*)$  equal to zero at  $r=1$ , the condition for Kantian equilibrium gives:

$$\alpha_j h^j(x_j^*) \alpha Q(x^*) \cdot x^* = c^j(x_j^*) x_j^* \quad (2.3)$$

Compute that for any distinct pair  $(i, j)$ :

$$\frac{\partial v^i}{\partial x^i}(x^*) = \alpha c^i(x_i^*) + h^i(\alpha Q) \frac{\partial Q}{\partial x_i}(x^*),$$

$$\frac{\partial v^i}{\partial x^j}(x^*) = h^i(\alpha Q) \frac{\partial Q}{\partial x_j}(x^*).$$

It follows that (2.2) holds iff

$$\alpha_j \sum_i \alpha_i h^i(\alpha Q) \frac{\partial Q}{\partial x_j}(x^*) = \alpha_j c^j(x_j^*). \quad (2.4)$$

For each  $j$ , multiply both sides of equation (2.4) by  $x_j^*$  and substitute from (2.3), yielding:

$$\alpha_j x_j^* \frac{\partial Q}{\partial x_j} \sum_i \alpha_i h^i(\alpha Q) = \alpha_j h^j(\alpha Q) (\alpha Q \cdot x^*),$$

or:

$$\alpha_j, \frac{x_j^* \frac{\partial Q}{\partial x_j}}{\alpha Q(x^*) \cdot x^*} \sum_i \alpha_i h^i(\alpha Q) = \alpha_j h^j(\alpha Q). \quad (2.5)$$

Now let  $\alpha_j = \frac{x_j^* \frac{\partial Q}{\partial x_j}}{\alpha Q(x^*) \cdot x^*}$ ,  $a_j = h^j(\alpha Q)$ . The premises of Lemma 2 hold, and it follows

that there exists a vector of positive  $\alpha$ 's such that (2.5) holds. Hence, by Lemma 1,  $x^*$  is Pareto efficient. ■

There is a link between Kantian equilibrium and Lindahl equilibrium. Silvestre (1984) shows that, in the presence of convexity and differentiability assumptions (which we have in the premise of proposition 1), an allocation which is Pareto efficient and has the property that no citizen would like all citizens to *decrease* their contributions to the public good by the same amount is a Lindahl equilibrium. Our definition of Kantian equilibrium does not mention Pareto efficiency, but given proposition 1, a Kantian equilibrium in a convex, differentiable environment satisfies the premises of Silvestre's result. It is therefore a Lindahl equilibrium.

The virtue of the Kantian conception, in contrast to the Lindahl conception, is that there are no personalized prices for the public good. The interesting fact is that unanimous optimality along a ray (which is the definition of K-equilibrium) implies efficiency.

### 3. The political environment and the probability-of-victory function

#### A. The environment

There is a sample space of citizen types  $H$ , with generic type  $h$ , distributed according to a probability measure denoted  $\mathbf{F}$  on  $H$ . In the case that  $H$  is a real interval, we denote the distribution function of  $\mathbf{F}$  by  $F$ . There is a policy space  $T$  which we take to be an interval on the real line. Voters are endowed with money, and the amount of money they have will be an aspect of their type. A voter may make a contribution to a political party. A voter of type  $h$  who contributes  $m$  in campaign contributions to parties enjoys a utility of  $u^h(t, m)$  if  $t \in T$  is the realized policy. We assume that  $u^h(\cdot, \cdot)$  is a von Neumann-Morgenstern utility function for all  $h$ .

Political parties will eventually form and propose policies. We assume that, within each type, a fraction of voters are 'informed' and the remainder are 'uninformed.' An informed voter can observe the policy announcements of parties, and compute her utility. An uninformed voter cannot observe policies: he observes only campaign advertisements made by parties. We assume that uninformed voters tend to vote for the party whose ads they see more often, an assumption that will be formalized presently.

The key here is that policies influence the votes only of informed voters, and advertising influences the votes of only the uninformed.

For simplicity, we assume throughout that the fraction of informed voters is the same number  $\alpha$  in all types.

In the applications that we study, the probability distribution  $\mathbf{F}$  is assumed to be continuous. There is a continuum of types, and no type has positive measure.

To avoid generality that would be gratuitous, we specialize to the quasi-linear case, where utility is given by:

$$u^h(t, m) = v^h(t) + m.$$

### B. Electoral uncertainty

Suppose there are two parties, which will announce policies  $t^1, t^2 \in T$ . Define the set of types whose members prefer  $t^1$  as:

$$\Omega(t^1, t^2) = \{h \mid v^h(t^1) > v^h(t^2)\}.$$

By the quasi-linear assumption, this set of types is invariant over the vectors of campaign contributions that individuals have made. Facing a choice between  $t^1$  and  $t^2$ , all informed voters in  $\Omega(t^1, t^2)$  will vote for  $t^1$ , and so party 1 would immediately win a fraction  $\alpha \mathbf{F}(\Omega(t^1, t^2))$  of votes.

We now assume that the fraction of the uninformed vote going to the two parties depends upon their campaign budgets. Let  $m^J$  be the campaign chest of party  $J$ , measured in the dollars per capita that it can spend, which is assumed to be the contributions per capita it has received, where ‘per capita’ means per population member, not per party member. Call, now, the parties  $L$  and  $R$ . We assume that, if the campaign chests are  $(m^L, m^R)$  then the fraction of the uninformed voters who vote for parties  $L$  and  $R$  are given by:

$$\begin{aligned} \alpha^L &= \frac{\alpha \sqrt{m^L}}{1 + \alpha(\sqrt{m^L} + \sqrt{m^R})}, \\ \alpha^R &= \frac{\alpha \sqrt{m^R}}{1 + \alpha(\sqrt{m^L} + \sqrt{m^R})} \end{aligned} \quad , \quad (3.1)$$

where  $\alpha$  is a positive parameter. Note that  $\alpha^L + \alpha^R$  approaches one from below as  $m^L$  and/or  $m^R$  approach(es) infinity. At any finite levels of campaign finance, there will be a positive fraction of uninformed voters who are not convinced, by campaign ads, to vote for either party. Note also that

$$\frac{\alpha^L}{\alpha^R} = \sqrt{\frac{m^L}{m^R}},$$

a concavity which reflects the supposition that it becomes increasingly hard to locate new voters for the party as the population becomes saturated with ads.

(It may be worth noting that one could assume a model in which parties have a production function relating campaign finance to the number of ads broadcast, and that an uninformed voter casts her vote for the party whose ads she says more often. This leads in a natural way to a binomial distribution for votes cast, which is approximated by a Poisson distribution, giving formulae much more complicated than (3.1). I do not believe that that extra complexity is justified by the added realism of the Poisson model. The Poisson model has the qualitative features of the model (3.1).)

It follows that, if the policy-campaign finance vector is given by  $(t^L, t^R, m^L, m^R)$ , then the fraction of the population who are sure to vote for party  $L$  is

$$\alpha \mathbf{F}(\alpha(t^L, t^R) + (1 - \alpha)\alpha^L,$$

and the fraction of the population whose vote, thus far, is undetermined is

$$(1 - \alpha)(1 - \alpha^L - \alpha^R).$$

We now suppose that this undetermined vote will be determined by issues of candidate personality, scandals which may be revealed during the campaign, and other stochastic elements. In particular, the effect of these elements is likely to be correlated across the population of undecided voters, not independently distributed. To model this correlation in a simple way, we assume that the fraction of the undetermined vote which eventually goes to party  $L$  is given by a uniformly distributed random variable, denoted  $X$ , on the support

$$[0, (1 - \alpha)(1 - \alpha^L - \alpha^R)].$$



Consequently, the fraction of the population who votes for  $L$  is

$$\Phi(F(\pi(t^L, t^R)) + (1 - \pi)\pi^L + X,$$

and it follows that the probability that  $L$  wins the election is given by:

$$\begin{aligned} \Phi(t^L, t^R, m^L, m^R) &= \text{prob}[\Phi(F(\pi(t^L, t^R)) + (1 - \pi)\pi^L + X > \frac{1}{2}] \\ &= \text{prob}[X > \frac{1}{2} - \Phi(F(\pi(t^L, t^R)) + (1 - \pi)\pi^L)] \\ &= I[1 + \frac{\Phi(F(\pi(t^L, t^R)) + (1 - \pi)\pi^L - \frac{1}{2})}{(1 - \pi)(1 - \pi^L - \pi^R)}], \end{aligned} \quad (3.2a)$$

where  $I$  is the ‘truncated identity function,’ defined by:

$$I(x) = \begin{cases} 0, & \text{if } x \leq 0 \\ x, & \text{if } 0 < x < 1 \\ 1, & \text{if } x \geq 1. \end{cases}$$

(The last line in expression (3.2a) is computed using the knowledge that  $X$  is uniformly distributed on its support.)

We have thus defined the probability- of- victory function.

By substituting in for the expressions  $\pi^j$  in (3.2) and simplifying, we have:

$$\Phi(t^L, t^R, m^L, m^R) = I[1 + \sqrt{m^L} + \frac{(\Phi(F(\pi(t^L, t^R)) - \frac{1}{2}))(1 + (\sqrt{m^L} + \sqrt{m^R}))}{1 - \pi}], \quad (3.2b)$$

a nice, simple concave function of the campaign budgets.

The assumption that some voters are influenced only by policy and some voters only by campaign ads is clearly extreme; it is a stylized assumption that leads to fairly simple formulae in the analysis below.

#### 4. Political equilibrium with private finance

### A. Determination of policy

We propose how policy is determined, if the membership of parties is given and the contributions of members to their parties are given. Thus, let

$$H = L \sqcup R, \quad L \cap R = \emptyset,$$

be a partition of the space of types, where the informed voters of type  $h \in L$  form party  $L$  and the informed voters of type  $h \in R$  form party  $R$ . We assume that there is perfect coordination among informed citizens of the same type, so that every informed citizen of given type makes exactly the same campaign contribution. Let

$\{m^h \mid h \in L\}$  and  $\{m^h \mid h \in R\}$  be the campaign contributions of the parties' members to their parties. Thus the (population) per capita contributions are given by:

$$m^L = \int \int_{h \in L} m^h d\mathbf{F}(h), \quad m^R = \int \int_{h \in R} m^h d\mathbf{F}(h).$$

Each informed citizen is interested in his party's proposing a policy that maximizes his expected utility, given the policy that the opposition party is proposing. For instance, given that party  $R$  proposes policy  $t^R$ , a member of type  $h$  of party  $L$  would like her party to propose the policy  $t$  that maximizes

$$\int (t, t^R, m^L, m^R) v^h(t) + (1 - \int (t, t^R, m^L, m^R)) v^h(t^R) \cdot m^h. \quad (3.3)$$

We now assume that the members of each party bargain with each other over policy, where the bargaining power of a particular type is proportional to its contributions to the party. In this bargaining game, the *threat point* for members of party  $L$  is the utility realized if their party fails to agree on a policy, and hence the opposition party wins the election by default, in which case all members of party  $L$  sustain the utility

$$v^h(t^R) \square m^h. \quad (3.4)$$

The *utility gain* of member  $h$  of  $L$  at a policy bargain  $t^L$  reached in  $L$  from the threat point, is hence the difference between the expressions (3.3) and (3.4), which we write as:

$$\square(t^L, t^R, m^L, m^R) \square v^h(t^L, t^R), \quad (3.5a)$$

where  $\square v$  is the difference operator:

$$\square v(x, y) = v(x) \square v(y).$$

In like manner, the utility gain to member  $h$  of party  $R$  from her threat point, when facing a policy  $t^L$  from the opposition, is:

$$(1 \square \square) \square v^h(t^R, t^L). \quad (3.5b)$$

Expressions (3.5 a and b) have the natural interpretation that the utility gain is the product of the probability of one's party's victory and the utility difference enjoyed from one's party's victory.

We now model the intra-party bargaining process by taking a cue from the Nash bargaining game: that is, we assume that the policy bargain reached maximizes the product of the bargainers utility gains from their threat points, raised to the powers of their bargaining powers. Expressing this in logarithmic form, we have that:

$$\begin{aligned} t^L &= \arg \max_{t \in T} \int_L m^h \log[\square(t, t^R, m^L, m^R) \square v^h(t, t^R)] d\mathbf{F}(h), \\ t^R &= \arg \max_{t \in T} \int_R m^h \log[(1 \square \square)(t^L, t, m^L, m^R)) \square v^h(t^L, t)] d\mathbf{F}(h). \end{aligned} \quad (3.6)$$

To summarize, we say that:

**Definition 3.** A policy pair  $(t^L, t^R)$  is a *policy equilibrium* for the parties  $L$  and  $R$  in a partition  $H = L \sqcup R$  at contribution levels  $\{m^h \mid h \in L\}$  and  $\{m^h \mid h \in R\}$  if equations (3.6) hold.

It goes almost without saying that each member of a party prefers its party's policy to the opposition's policy, because, were that false, then the logarithms in (3.6) would be undefined. So *if* a policy equilibrium exists, we are guaranteed that every party member prefers his party's policy to the opposition's.

We note that the present theory is incapable of explaining what sometimes occurs in reality, that some citizens contribute money to more than one party; see Steen and Shapiro (2002) for discussion. It would seem that the phenomenon of 'walking both sides of the street' is explained by the desire of contributors to have 'access' to office holders after the election; in contrast, in our environment, all issues are decided prior to the election. Access after the election would be important if the platform is an 'incomplete contract,' so to speak, and so many issues will be settled as they come up over time, after the election. In the complete contract setting of the present theory, access to the winner after the election would be of no value.

#### B. Determination of campaign contributions

We next propose how campaign contributions are determined, at a given pair of policies  $(t^L, t^R)$ . Here, we invoke the notion of Kantian equilibrium. Denote an (infinite vector) of campaign contributions to party  $L$  by  $M^L$ , with the analogous meaning for  $M^R$ . At a particular vector of policies, and given the campaign contributions of the opposite party, we can write the expected utility of an informed voter of type  $h$  of party  $L$  as a function of the (infinite) vector of campaign contributions made by his party's members as:

$$U^h(M^L, m^h; m^R) = \alpha(t^L, t^R, m^L, m^R)v^h(t^L) + (1 - \alpha)v^h(t^R)\alpha m^h, \quad (3.7a)$$

where  $m^L$  is derived from  $M^L$  according to the formulae provided at the beginning of part A of this section. The analogous representation for the utilities of party  $R$ 's members is given by the same formula, that is:

$$U^h(M^R, m^h; m^L) = \alpha(t^L, t^R, m^L, m^R)v^h(t^L) + (1 - \alpha)v^h(t^R)\alpha m^h, \quad (3.7b)$$

We are now in the environment of a Kantian equilibrium, where the relevant utility functions of the party members are the functions  $U^h$ . Clearly, if every party member were to increase (decrease) his contribution by a factor  $r$ , then the budget of the party would increase (decrease) by the same factor.

**Definition 4.** Vectors of contributions  $M^L$  and  $M^R$  to two parties at a given policy vector  $(t^L, t^R)$  comprise a *contribution equilibrium at  $(t^L, t^R)$*  if  $M^L$  is a Kantian equilibrium for the members of  $L$ , with respect to the utility functions  $\{U^h \mid h \in L\}$ , given  $M^R$ , and  $M^R$  is a Kantian equilibrium for the members of party  $R$  with respect to the utility functions  $\{U^h \mid h \in R\}$ , given  $M^L$ .

In other words, it is assumed that parties co-ordinate members' contributions in order to realize a Kantian equilibrium in contributions. Note that, in the thought experiment that Kantian equilibrium proposes, all members' contributions would increase by the same factor, and hence the relative bargaining powers of the members would not change.

We cannot immediately apply Proposition 1 to this environment, because in the proposition, the environment assumed a finite number of types, and here we are working with a continuum. Nevertheless, if we here assume a finite number of types, so that

$m^L = \prod_{h \in L} f^h m^h$ , then we can immediately observe, from equation (3.2b), that the

functions  $U^h$  are concave in the contributors' contributions, and hence a Kantian equilibrium in contributions is Pareto efficient from the contributors' viewpoints.

Let us review the motivation for invoking Kantian equilibrium here. We wish to model the idea that parties are associations of citizens that provide public goods to their members – two, in fact -- their policy and their campaign ads, financed by the campaign chest. We model policy as being produced by *competition* among members. We model the campaign chest as produced by *coordination* among members and *competition* between parties. How could a party coordinate member contributions? We have proposed a very simple rule: it can appeal to all members to increase (or decrease!) their contributions by a given proportion. This rule, as well as being extremely simple, has the virtue of not interfering with the process by which policy is arrived at, because a call to change proportionally all contributions will not alter the nature of the bargaining problem among party members over policy. And finally we have shown that this rule successfully coordinates individual behavior, in the sense that, when no such proportionate changes are justified, the provision of the public good of campaign finance is Pareto efficient for the set of party members.

### C. Political equilibrium with contributions

We now define a full political equilibrium by combining the previous two concepts.

**Definition 5** A *political equilibrium with unconstrained contributions* consists of:

- (1) a partition  $H = L \sqcup R$ ,  $L \sqcup R = \emptyset$ ,

(2) vectors of contributions  $M^L = \{m^h \mid h \in L\}$ ,  $M^R = \{m^h \mid h \in R\}$  from the informed members of types to their parties,

(3) policies  $t^L$  and  $t^R$  of the two parties,

such that:

(4)  $(t^L, t^R)$  is a policy equilibrium at contribution vectors  $M^L, M^R$ , and

(5)  $M^L$  and  $M^R$  comprise a contribution equilibrium at  $(t^L, t^R)$ .

We should note that the equilibrium concept fulfills the six desiderata listed in the introduction. Each policy is a best response to the other party's policy, where the 'utility function' of a party is that function which is optimized at the solution of a Nash bargaining game among the party's members, a game in which the power of individual types is proportional to their campaign contributions to the party. Campaign contributions play a double role: they determine the strengths of citizen types in intra-party bargaining, and they also determine the party's ability to reach uninformed voters. Moreover, with respect to this second purpose, the contributions are optimal as far as the the members are concerned, because they are a Kantian equilibrium in that regard, given the other components of the equilibrium. Finally, party membership is stable in the sense that every type prefers its party's policy to the opposition's, and by the continuum assumption, all types have negligible influence on donations and policy.

Two remarks are in order.

Remark 1. Party members do not determine their contributions with an eye to optimizing with respect to their bargaining power in the intra-party bargaining game. Now in the continuum model, individual types have no incentive to alter their contributions, because, in an atomless economy, no type can alter the objective functions

in (3.6) by altering its contributions. In a finite- type economy, we would have to consider this kind of strategic behavior, and then the political equilibrium would be over-determined: in most cases, no equilibrium would exist.

In the continuum model, as we shall see, equilibria do exist and are well-defined, that is, locally unique.

Remark 2. The party is an association which organizes the campaign-contribution behavior of its members in a cooperative fashion; we must say that there is space for this kind of cooperation precisely because, with the continuum assumption, no type can have any strategic gain by altering its contribution. We have modeled that cooperative function of the party with the Kantian equilibrium concept, which has normative appeal as a cooperative solution concept. One may object that it is not clear how the party would implement this cooperative solution – how it would find the Kantian equilibrium in contributions; a similar statement is often made with respect to Lindahl equilibrium in a public-goods economy, and we have noted that Kantian equilibrium is a special case of Lindahl equilibrium.

In defense of our concept, however, it must be noted that even Walrasian equilibrium is only a normative concept, despite frequent claims to the contrary, because we have no robust theory of how ‘the market’ finds the Walrasian equilibrium<sup>5</sup>. *If* the market somehow finds a Walrasian equilibrium, then there is reason for it to be stable (because all markets clear under optimizing behavior of traders). Similarly, *if* parties find the Kantian equilibrium in contributions, then contributions are stable, in the sense

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<sup>5</sup> In other words, a purely positive concept of market equilibrium, I am proposing, would describe *how* the equilibrium is arrived at, as well.



of unanimous agreement of members about proposals to change proportionally all contributions.

Indeed, we must say that the present model is only plausible given the assumption of an atomless set of citizen types. This equilibrium is not the limit of a similar sequence of equilibria in finite type economies, as the number of types increases, because in those economies, we would have to allow members to optimize individually on their campaign contributions<sup>6</sup>.

## 5. A characterization of private political equilibrium and the computation of equilibrium

### A. General computation of equilibrium

We first compute the first-order conditions for a Kantian equilibrium in contributions. This involves using the expression (3.7a) and setting the derivative of  $U^h(rM^h, rm^h; m^R)$ , with respect to  $r$ , equal to zero at  $r=1$ . The F.O.C.s are:

$$\text{for all } h \text{ in } L, \quad m^h = \frac{\partial \square}{\partial m^L} m^L \square v^h(t^L, t^R)$$

$$\text{for all } h \text{ in } R, \quad m^h = \frac{\partial(1 \square \square)}{\partial m^L} m^R \square v^h(t^R, t^L).$$

These two equations can be conveniently expressed together as:

$$(\text{for } J = L, R)(\square h \square J)(m^h = \frac{\partial \square}{\partial m^J} m^J \square v^h(t^J, t^{\bar{J}})), \quad (5.1)$$

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<sup>6</sup> It should here be noted that Walrasian equilibrium is not logically plausible for a finite economy *either*, because in such economies, individuals have the power to influence prices, and it is therefore inconsistent that they should treat prices as given, as they do in Walrasian equilibrium. Truly competitive equilibrium usually fails to exist in finite economies, as well! For discussion, consult Makowski and Ostroy (2001).

where it is understood that the function  $\square$  is evaluated at  $(t^L, t^R, m^L, m^R)$ . Using the fact

that  $m^J = \square \int_{h \in J} m^h d\mathbf{F}(h)$ , we can integrate equations (5.1)  $d\mathbf{F}(h)$  and divide by  $m^J$ ,

yielding:

$$\text{for } J=L,R: \quad \frac{1}{\square} = \frac{\partial \square}{\partial m^J} \int_{h \in J} v^h(t^L, t^R) d\mathbf{F}(h). \quad (5.2a,b)$$

We next compute the first-order conditions for  $(t^L, t^R)$  to be a policy equilibrium.

From (3.6), we may write:

$$\begin{aligned} t^L &= \arg \max_t \left[ \frac{m^L}{\square} \log \square(t, t^R, m^L, m^R) + \int_{h \in L} m^h \log \square v^h(t, t^R) d\mathbf{F}(h) \right], \\ t^R &= \arg \max_t \left[ \frac{m^R}{\square} \log (1 \square \square(t^L, t, m^L, m^R)) + \int_{h \in R} m^h \log \square v^h(t, t^L) d\mathbf{F}(h) \right]. \end{aligned}$$

Differentiating these expressions w.r.t.  $t$ , and setting the derivatives equal to zero

produces:

$$\frac{m^L}{\square} \frac{\partial \square}{\partial t^L} + \square \int_L \frac{m^h}{\square v^h(t^L, t^R)} \frac{\partial v^h}{\partial t}(t^L) d\mathbf{F}(h) = 0 \quad (5.3a)$$

$$\frac{\square m^R}{1 \square \square} \frac{\partial \square}{\partial t^R} + \square \int_R \frac{m^h}{\square v^h(t^R, t^L)} \frac{\partial v^h}{\partial t}(t^R) d\mathbf{F}(h) = 0 \quad (5.3b)$$

We now use the continuum of equations (5.1) to substitute for  $m^h$  in expressions

(5.3a,b), which simplifies the latter to:

$$\frac{1}{\square} \frac{\partial \square}{\partial t^L} + \square \frac{\partial \square}{\partial m^L} \int_L \frac{\partial v^h}{\partial t}(t^L) d\mathbf{F}(h) = 0 \quad (5.4a)$$

$$\frac{1}{1 \square \square} \frac{\partial \square}{\partial t^R} + \square \frac{\partial \square}{\partial m^R} \int_R \frac{\partial v^h}{\partial t}(t^R) d\mathbf{F}(h) = 0 \quad (5.4b)$$

(yes, the signs are correct!)

Now notice that equations (5.2a,b) and (5.4a,b) comprise four equations in the four unknowns  $(t^L, t^R, m^L, m^R)$ . How do we determine the  $(L, R)$  partition? We simply note that

$$\begin{aligned} L &= \{h \mid v^h(t^L) > v^h(t^R)\} \\ R &= \{h \mid v^h(t^R) > v^h(t^L)\} \end{aligned} \quad (5.5a,b)$$

Therefore, (5.2 a,b), (5.4a,b) and (5.5 a,b) comprise six equations in six unknowns (the last two being  $L$  and  $R$ ) which, hopefully, will possess a solution. If we can solve them, then the contributions of individual types are immediately computed from (5.1).

#### B. An interesting characterization of equilibrium

Using equations (5.2a,b), solve for  $\frac{d\pi}{dm^L}$  and  $\frac{d\pi}{dm^R}$ , and substitute these expressions into equations (5.4a,b), which produces, after some minor re-writing:

$$\frac{1}{\pi} \frac{\partial \pi}{\partial t^L} + [\sum_L \pi v^h(t^L, t^R) d\mathbf{F}(h)] \pi \sum_L \frac{dv^h}{dt}(t^L) d\mathbf{F}(h) = 0 \quad (5.6a)$$

$$\frac{\pi}{1 - \pi} \frac{\partial \pi}{\partial t^R} + [\sum_R \pi v^h(t^R, t^L) d\mathbf{F}(h)] \pi \sum_R \frac{dv^h}{dt}(t^R) d\mathbf{F}(h) = 0 \quad (5.6b)$$

Now define the functions:

$$V^L(t) \equiv \sum_L v^h(t) d\mathbf{F}(h), \quad V^R(t) \equiv \sum_R v^h(t) d\mathbf{F}(h).$$

Then (5.6a,b) can be written, after some minor algebraic manipulation:

$$\frac{\partial \pi}{\partial t^L} (V^L(t^L) - V^L(t^R)) + \pi \frac{dV^L}{dt}(t^L) = 0 \quad (5.7a)$$

$$\frac{\partial(1 - \pi)}{\partial t^R} (V^R(t^R) - V^R(t^L)) + (1 - \pi) \frac{dV^R}{dt}(t^R) = 0 \quad (5.7b)$$

But these equations are equivalent to the following first-order conditions:

$$\frac{d}{dt}[\alpha V^L(t) + (1-\alpha)V^L(t^R)] = 0 \text{ at } t = t^L \quad (5.8a)$$

$$\frac{d}{dt}[\alpha V^R(t^L) + (1-\alpha)V^R(t)] = 0 \text{ at } t = t^R \quad (5.8b)$$

Note that the expression in brackets in (5.8a) is the *expected utility* of a party that has a utility function  $V^L$  on the policy space, if the lottery it faces is one over the policies  $t^L$  and  $t^R$ , with probabilities  $\alpha$  and  $(1-\alpha)$ , respectively. An analogous statement is true for (5.8b), with respect to a party endowed with utility function  $V^R$ . Therefore, equations (5.8a,b) say that  $(t^L, t^R)$  is a Nash equilibrium of a game played between two parties, equipped with utility functions  $V^L$  and  $V^R$ , where the strategy space is  $T$ , and each party wishes to maximize its expected utility!

In other words, at a political equilibrium with private campaign finance, the policy equilibrium is exactly what the policy equilibrium would be in a game played between two ‘virtual parties’ equipped with utility functions each of which is simply the *average utility function* of its members, an average in which each member-type’s utility function enters with its type’s *population* weight, not its *contribution* weight. This is also known as an *endogenous-party Wittman equilibrium* (for discussion of that concept, which has nothing to do, in its general form, with campaign contributions, see Roemer [2001, chapter 5]).

I call the fact that political equilibrium with private campaign finance is equivalent to an endogenous- party Wittman equilibrium the *aggregation principle*.

What is remarkable is that the financial contributions have fallen out of the picture – a ‘coincidence’ that is a consequence of the Kantian equilibrium property of contributions and the Nash bargaining property of the policy equilibrium. One is led to

ask: where, then, is the ‘distortion’ in policies due to the fact that types are represented in parties according to their contributions, rather than according to their numbers? The answer is that that distortion is reflected in the probability function. In equations (5.8a,b), the derivatives are with respect to policies only, but the campaign contributions enter, of course, into the probability functions, and therefore the values of those functions are (very) different from what they would be, were every member to contribute the same amount to her party.

From our knowledge of the behavior of endogenous-party Wittman equilibrium, we can make a prediction about the nature of political equilibrium with private campaign finance, in a polity where the policy concerns redistribution, and a citizen’s type is his income or wealth. We know the equilibrium must be characterized by equations (5.8a,b) and (5.2a,b) which can be written, now, as:

$$\frac{1}{\square} = \frac{\partial \square}{\partial m^L} m^L \square V^L(t^L, t^R)$$

$$\frac{1}{\square} = \frac{\partial (1 \square \square)}{\partial m^R} m^R \square V^R(t^R, t^L) .$$

In such an environment, we can expect that the parties will endogenously form to represent the upper and lower parts of the wealth distribution, with some cut point. The fact that the rich will contribute more, loosely speaking, to their party than the poor will have the consequence that the party representing the upper part of the distribution will be small and the other party will be large. This will be the consequence of the distortion of the probability function entailed by disproportional contributions of the rich and the poor. Thus, we predict that *we will observe a political equilibrium with a small party representing the very rich, and a large party representing all others.*

Before proceeding with the analysis of an example to check whether this prediction is borne out, a warning to the reader is in order. We have assumed two things about utility functions: first, that they are von Neumann- Morgenstern, and second, that they are quasi-linear in contributions. No interpersonal comparability of utility has been assumed. What does this mean about the average functions  $V^J, J=L,R$  ? It means that we cannot interpret  $V^L(t)$  as a meaningful average welfare level of the members of party  $L$ . For to do so, the individual utility functions would have to be *cardinally unit comparable* – that is, the utility they measure would have to in units that are interpersonally comparable in a meaningful way<sup>7</sup>. Consider again an example where a person's type is her wealth. With the quasi-linear family of utility functions we have chosen, the marginal disutility of contributing \$1 is the same for all citizens. Clearly, in interpersonally comparable units, this statement is false in actuality: we believe it is much less costly for a rich person to contribute a dollar than a poor person. Therefore, the right family of utility functions for purposes of interpersonal comparability is *not* the quasi-linear family. Another way of saying this is that it would be an error to interpret  $V^L$  and  $V^R$  as *utilitarian* functions – because the aggregation of individual utilities they perform is not interpersonally meaningful.

Thus, the aggregation principle is a *formal property* of political equilibrium. It must not be thought to imply that the virtual parties of equations (5.8a,b) are maximizing the average expected *welfare* of their members<sup>8</sup>.

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<sup>7</sup> The measurement and comparability of utility is a topic of social choice theory. For a discussion, the reader is referred to Roemer[1996, Chapter 1].

<sup>8</sup> Social choice theorists will recognize that this point occurs as well in the debate over Harsanyi's theorems on utilitarianism. For a discussion, the interested reader is again referred to Roemer[1996, Chapter 4].

## 6. An example

We now specialize to the case, for computational purposes, where  $T=H=R_+$ , and assume Euclidean preferences:

$$v^h(t) = \frac{\alpha^h}{2} (t - h)^2.$$

Think of  $h$  as income or wealth and  $t^h=h$ , as the ideal policy for type  $h$ . Since utility is quasi-linear in contributions, the constants  $\alpha^h$  will allow us to model the idea that the trade-off between policy and contributions is different for types of different wealth.

Given two policies  $(t^L, t^R)$ , the indifferent type is  $h^*(t^L, t^R) = \frac{t^L + t^R}{2}$ , and the associated partition of types into parties is  $L = \{h < h^*\}$ ,  $R = \{h > h^*\}$ . (Here, we have identified party  $L$  with the ‘poor’ types; hence the nomenclature *Left* and *Right*.) If  $t^L < t^R$ , then  $\Pi(t^L, t^R) = \{h < h^*\}$ , and it follows that  $\mathbf{F}(\Pi(t^L, t^R)) = F(h^*)$ , where, recall,  $F$  is the C.D.F. of  $\mathbf{F}$ .

Denote  $\bar{t} = \frac{t^L + t^R}{2}$ . Then  $\Pi v^h(t^L, t^R) = \alpha^h \Pi(t(\bar{t}) + h)$ , where  $\Pi t = t^L \Pi t^R$ . Now

define:

$$\begin{aligned} \alpha_0^L &= \frac{\int_0^{h^*} \alpha^h d\mathbf{F}(h)}{F(h^*)}, \quad \alpha_0^R = \frac{\int_{h^*}^\infty \alpha^h d\mathbf{F}(h)}{1 - F(h^*)}, \\ \alpha_1^h &= \frac{\int_0^{h^*} h \alpha^h d\mathbf{F}(h)}{F(h^*)}, \quad \alpha_1^R = \frac{\int_{h^*}^\infty h \alpha^h d\mathbf{F}(h)}{1 - F(h^*)}. \end{aligned}$$

Then we may write (5.2a,b) as:

$$\frac{1}{\square F(h^*)} = \frac{\partial \square}{\partial m^L} \square t(\square \tilde{t} \square_0^L + \square_1^L) \quad (6.1)$$

$$\frac{1}{\square(1 \square F(h^*))} = \frac{\partial \square}{\partial m^R} \square t(\square \tilde{t} \square_0^R + \square_1^R) \quad (6.2)$$

Compute that:

$$\begin{aligned} \frac{\partial \square}{\partial m^L} &= \frac{\square}{2\sqrt{m^L}} \frac{.5 + \square(F(h^*) \square 1)}{1 \square \square}, \\ \frac{\partial \square}{\partial m^R} &= \frac{\square}{2\sqrt{m^R}} \frac{\square F(h^*) \square .5}{1 \square \square}, \\ \frac{\partial \square}{\partial t^L} &= \frac{\partial \square}{\partial t^R} = \frac{1 + \square(\sqrt{m^L} + \sqrt{m^R})}{1 \square \square} \frac{\square f(h^*)}{2} \end{aligned}$$

Substituting these derivatives into (5.4a,b) yields:

$$\frac{1 + \square(\sqrt{m^L} + \sqrt{m^R})}{\square F(h^*)} f(h^*) + \frac{\square(1 + 2\square(F(h^*) \square 1))}{2\sqrt{m^L}} (\square t^L \square_0^L + \square_1^L) = 0 \quad (6.3)$$

$$\frac{1 + \square(\sqrt{m^L} + \sqrt{m^R})}{(1 \square \square)(1 \square F(h^*))} f(h^*) + \frac{\square(2\square F(h^*) \square 1)}{2\sqrt{m^R}} (\square t^R \square_0^R + \square_1^R) = 0 \quad (6.4)$$

Equations (6.1)-(6.4) comprise a simultaneous system in the four unknowns

$(t^L, t^R, m^L, m^R)$ . Indeed the system is separable. By using the expressions for  $\frac{\partial \square}{\partial m^j}$ , we

can solve (6.1) and (6.2) for  $m^L$  and  $m^R$  in terms of the policies, and substitute these expressions for the contributions into (6.3) and (6.4), which then become a pair of simultaneous equations in  $(t^L, t^R)$ .

For the model to behave ‘properly,’ it must be the case that the derivatives

$\frac{\partial \square}{\partial m^L}$  and  $\frac{\partial \square}{\partial m^R}$  be positive and negative, respectively. From the equations for these

two derivatives, this requires that:

$$\begin{aligned} .5 + \square(F(h^*) \square 1) &> 0, \\ \text{and } \square F(h^*) \square .5 &< 0, \end{aligned}$$



which reduce to :

$$\alpha < \min\left[\frac{.5}{1-\alpha F(h^*)}, \frac{1}{2F(h^*)}\right]. \quad (6.5)$$

We now present a specific example, parameterizing the model as follows:

$\alpha = 0.1$ ,  $\alpha^h = \frac{h^\alpha}{100}$ ,  $F$  is the lognormal distribution with mean 40 and median 30. We choose, initially,  $\alpha = 1$ . We think of type  $h$  as the type with annual income of  $h$  thousands of dollars; then the  $F$  looks like the US income distribution in the early 1990s. Note that the marginal rate of substitution between contributions and policy ( $\frac{dt}{dm}$ ) for type  $h$  is  $\alpha^h(t \alpha^h)$ , so making  $\alpha^h$  an increasing function of  $h$  means that this MRS is larger, for a small change in policy near a type's ideal point, for the rich than for the poor – thus, we would expect the rich to contribute more to campaigns than the poor. The larger is the exponent  $\alpha$ , the faster will the MRS of policy against campaign contributions increase with  $h$ .

In table 1, I report the values of political equilibria for various values of  $\alpha$ , at  $\alpha=1$ <sup>9</sup>. The first five columns are self-explanatory. Columns 6 and 7 give the centile of the type, in the income distribution, whose ideal policy is the Left policy and the Right policy, respectively. Column 8 gives the centile in the income distribution of the type which defines the cut-point between membership in the two parties. In other words, the fraction of the polity represented by the Left party is exactly the value in column 8. Column 9 gives the probability of victory of Left. Column ‘exp pol’ is the centile of the

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<sup>9</sup> All computations were programmed in Mathematica; the programs are available from the author, upon request.

type whose ideal policy is the *expected* policy,  $\bar{t}^L + (1 - \bar{\alpha})t^R$ . And the last column, ‘com’, reports the fraction of voters who finally, after advertising, are committed to one of the parties, that is,  $1 - (1 - \bar{\alpha})(1 - \bar{\alpha}^L - \bar{\alpha}^R)$ . (In other words, the votes of the fraction ‘1-com’ of the population are governed by the random variable  $X$ .)

[table 1 here]

Perhaps the two most important statistics are those in the  $F[\bar{t}]$  and ‘exp pol’ columns. We see that, for small values of  $\alpha$ , our prediction is true: equilibrium entails a small party of the Right, representing the top 20 percent of the wealth distribution, and a large party representing all others. The expected policy is quite right-wing: for example, at  $\alpha=0.4$ , it is the ideal policy of the type at the 70<sup>th</sup> centile of the wealth distribution. At  $\alpha=0.4$ , the Left (Right) party proposes the ideal policy of the type at the 61<sup>st</sup> centile (89<sup>th</sup> centile) of the income distribution. We see that, in this equilibrium, Left spends a little more than Right in the election – but it spends only about one-fourth the amount *per party member*, since there are four times as many members in Left as in Right. Thus, the large individual expenditures of Right members on the campaign enable a minority Right party to survive – in the sense of having a positive probability of victory.

As the population becomes more informed ( $\alpha$  increases), politics become less skewed, so that at  $\alpha=.775$ , the Left represents 55% of the population, and the Right 45%. The expected policy is a little to right of the ideal policy of the median wealth holder. Campaign spending decreases quite radically as the population becomes more informed: this occurs because there are fewer voters to be convinced by campaign ads. Note also that for large values of  $\alpha$ , the Right spends much more than Left: at  $\alpha=.775$ , Right is

spending 47 times as much as Left. We cannot attach any dollar meaning to the per capita campaign chests of the two parties, as we do not know the units of money. The ratio of the campaign contributions is, however, relevant.

We can understand the sense in which politics become right-wing when  $\alpha$  is small by invoking the aggregation principle. When  $\alpha=0.4$ , the Right party represents the richest 20% of the income distribution. The Left party also has a significant number of fairly rich people in it, because it represents the bottom 80% of the income distribution. We can thus conjecture that it will propose policies that are not too far left. So both parties will propose quite ‘conservative’ policies.

In other words, in a population whose party partition has a high cut point  $h^*$ , politics will be fairly right-wing, and in a partition with a low cut point, politics will be fairly left-wing, by analogous reasoning. We note that the equilibrium partition is always to the right of the median income type in the equilibria of Table 1.

Why is a Right party that is so small at  $\alpha=0.4$  politically feasible? Because the Right is spending much more per member than is Left and so the probability of Right victory is not as small as it ‘should’ be, given the policy of the Right. If all voters were informed, 80% of the polity would vote for the Left policy, and the Right would lose the election for sure. But if all voters were informed, this would not be the equilibrium partition.

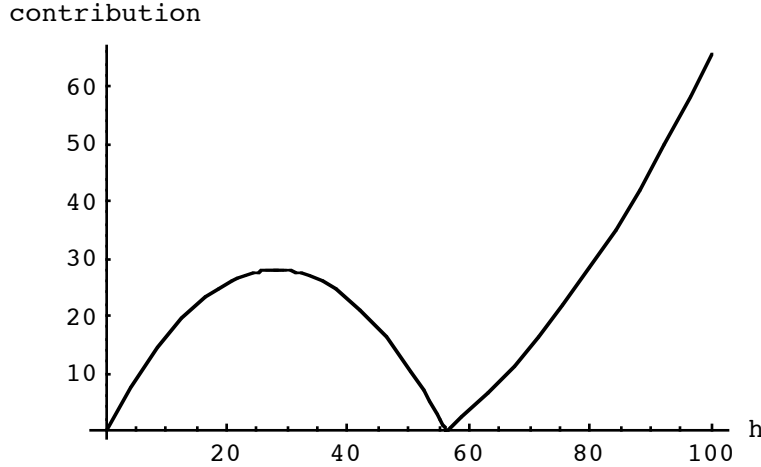


Figure 1: Contributions by type at the equilibrium of the example for  $\beta=0.4$

In Figure 1, I plot campaign contributions as a function of  $h$ . Of course, it follows from (5.1) that the indifferent type (the pivot) contributes zero. Contributions approach infinity, as  $h$  approaches infinity.

Finally, we note that inequality (6.5) holds for all these equilibria, so we are in the region where the model is well-behaved.

The next experiment is to observe what happens as we increase the value of the exponent  $\beta$ . I fix  $\beta=0.4$ , and increase  $\beta$  from 1 to 1.6: the results are reported in Table 2.

[table 2 here]

As  $\beta$  becomes larger, citizens will want to spend more on campaigns, and the effect will be magnified as  $h$  increases. We should therefore expect that the political equilibrium will be even more skewed to the right, as  $\beta$  increases. We observe that this is indeed so from Table 2: consult the ‘exp pol’ column. Curiously, the size of the Right party is not monotonic in  $\beta$ , although it is always close to 20% of the polity. The most dramatic observations from Table 2 are the extreme increase in Right spending as  $\beta$  increases, the

movement of Left's equilibrium policy to the right, and the decreasing probability of Left victory.

## 7. Constrained private campaign finance

Our second financing institution is private finance with a cap on contributions. To define political equilibrium for this institution, we must first generalize the definition of Kantian equilibrium to this environment.

Consider the environment of section 2, and the utility functions  $v^i$  of equation (2.1). We now restrict the contributions  $x_i$  to be bounded above by some number  $M_0$ . There are various conceivable generalizations of the notion of Kantian equilibrium to this environment: our motivation for the choice below will soon be apparent.

**Definition 6** Let  $x = (x_1, x_2, \dots, x_N)$  be a vector of contributions for the 'constrained' environment with a contribution cap of  $M_0$ . Define the set  $C = \{i \mid x_i = M_0\}$ . We say:

- (1) If  $C = \emptyset$ , then  $x$  is a Kantian equilibrium just in case it satisfies definition 1;
- (2) If  $C \neq \emptyset$ , then  $x$  is a constrained Kantian equilibrium iff:
  - (a) no unconstrained agent  $j \notin C$  would like to increase or decrease the contributions of *all* agents by *any* factor  $r$ ;
  - (b) no constrained agent  $j \in C$  would like to decrease the contributions of *all* agents by any factor  $r$ .

It is important to understand that, in part 2(a) of the definition,  $r$  can be greater than one. Thus, the condition allows unconstrained agents to contemplate infeasible vectors of contributions.

Given the concavity and differentiability of the functions  $v^i$ , we can characterize a Kantian equilibrium by the first-order conditions:

$$\begin{aligned} \forall j \in C \quad \frac{d}{dr} v^j(r x_1, \dots, r x_N) &= 0 \text{ at } r = 1 \\ \forall j \in C \quad x_j &= M_0 \text{ and } \frac{d}{dr} v^j(r x_1, \dots, r x_N) \geq 0 \text{ at } r = 1. \end{aligned} \quad (7.1)$$

In words, we can say that a constrained Kantian equilibrium is an allocation of contributions such the contributors who are not constrained are unanimously pleased with the vector of contributions (in the sense of 2(a)), while the constrained agents would like everyone to increase his contribution, an action that is infeasible.

I claim this is the right generalization of Kantian equilibrium to the constrained environment for two reasons: first, it will engender a locally unique equilibrium allocation. This is easily seen. In case (1) we are back in the world of section 2. In case (2), note that there will be  $N - |C|$  first – order conditions from part 2(a), and  $|C|$  equations of the form  $x_j = M_0$  -- thus  $N$  equations in  $N$  unknowns. Second, Kantian allocations, so defined, are again Pareto efficient, a fact which we now demonstrate.

**Proposition 2** *Let  $x^* = (x_1^*, \dots, x_N^*)$ ,  $x^* > 0$ , be a Kantian equilibrium in the constrained environment. Then  $x^*$  is Pareto efficient.*

To prove the proposition, we use the following lemma:

**Lemma 3.** Let the utility functions  $\{v^i\}$  be concave. Let  $x$  be a vector of constrained contributions, with  $C = \{i \mid x_i = M_0\}$ . If there exist positive numbers  $\{\alpha_i \mid i = 1, \dots, N\}$  and non-negative numbers  $\{\beta_i \mid i \in C\}$  such that:

$$\sum_i \alpha_i v^i = \sum_{i \in C} \beta_i e^i, \quad (7.2)$$

where  $e^i$  is the  $i$ th unit vector in  $\mathbf{R}^N$ , then  $x$  is Pareto efficient.

Proof of Lemma:

By concavity, we need only show that  $x$  is locally Pareto efficient. Let  $d \in \mathbf{R}^N$  be a feasible direction at  $x$ : then

$$\text{for all } i \in C, \quad d \cdot e^i \geq 0, \quad (7.3)$$

because  $x_i = M_0$  for  $i \in C$ . Suppose moving in direction  $d$  produces a Pareto improvement: this means that

$$\lambda v^i(x) \cdot d \geq 0 \text{ for all } i, \text{ with strict inequality for some } i.$$

But taking the scalar product of equation (7.2) with  $d$  produces:

$$\sum_i \lambda_i v^i(x) \cdot d \geq 0,$$

by invoking (7.3), and so, since all the  $\lambda$ 's are positive, we must have

$$\lambda v^i(x) \cdot d = 0 \text{ for all } i,$$

a contradiction which proves the lemma. ■

Proof of Proposition 2:

1. Let  $C = \{i \mid x_i^* = M_0\}$ . Denote the complement of  $C$  by  $C^c$ . Define the numbers

$$\lambda_j = \frac{\frac{\partial Q(x^*)}{\partial x_j} x_j^*}{\sum_{j \in C^c} \frac{\partial Q(x^*)}{\partial x_j} x_j^*}$$

and note that the  $\{\lambda_j\}$  are positive numbers that add up to one.

2. Our task is to produce numbers  $\{\lambda^j \mid j = 1, \dots, N\}$  and  $\{\lambda^j \mid j \in C\}$  fulfilling Lemma 3, which will prove that  $x^*$  is Pareto efficient. Let us note that the vector equation (7.2) of the lemma can be written as the scalar equations:

$$\sum_i \lambda_i \frac{\partial v^i(x^*)}{\partial x_j} = 0 \text{ for } j \in C \quad (7.2a)$$

$$\sum_i \lambda_i \frac{\partial v^i(x^*)}{\partial x_j} \geq 0 \text{ for } j \in C \quad (7.2b)$$

3. The first F.O.C. of (7.1) says that

$$\sum_{j \in C} \lambda_j c^j(x_j^*) x_j^* + h^j(Q(x^*)) (\sum Q(x^*) \cdot x^*) = 0 \quad (7.4a)$$

and the second F.O.C. says that

$$\sum_{j \in C} \lambda_j c^j(x_j^*) x_j^* + h^j(Q(x^*)) (\sum Q(x^*) \cdot x^*) \geq 0 \quad (7.4b)$$

4. We have the derivatives:

$$\frac{\partial v^i}{\partial x^i}(x^*) = \lambda_i c^i(x_i^*) + h^i(Q) \frac{\partial Q}{\partial x_i}(x^*),$$

$$\frac{\partial v^i}{\partial x^j}(x^*) = h^i(Q) \frac{\partial Q}{\partial x_j}(x^*).$$

From these we may compute that

$$\text{for any } j \in C \quad \sum_i \lambda_i \frac{\partial v^i(x^*)}{\partial x_j} = \sum_i \lambda_i h^i(Q(x^*)) \frac{\partial Q(x^*)}{\partial x_j} \sum_j \lambda_j c^j(x_j^*),$$

and hence, multiplying both sides by the positive number  $x_j^*$ :

$$\sum_i \lambda_i \frac{\partial v^i(x^*)}{\partial x_j} x_j^* = \sum_i \lambda_i h^i(Q(x^*)) \frac{\partial Q(x^*)}{\partial x_j} x_j^* \sum_j \lambda_j c^j(x_j^*) x_j^*. \quad (7.5)$$

Now, invoking (7.4a), we have:

$$(\sum_{j \in C} \lambda_j \frac{\partial v^j(x^*)}{\partial x_j} = 0) \implies \sum_i \lambda_i h^i(Q(x^*)) \frac{\partial Q(x^*)}{\partial x_j} x_j^* = \sum_j \lambda_j h^j(Q) (\sum Q(x^*) \cdot x^*)$$

where the last equation to the right of the implication sign can be written :

$$\lambda_j = \frac{\sum_i \lambda_i h^i(Q)}{\sum_i \lambda_i h^j(Q)}. \quad (7.6)$$



5. Now consider  $j \in C$ . Then, invoking (7.4b), we see that (7.5) – which is true for all  $j$  – implies :

$$\sum_i \alpha_i \frac{\partial v^i(x^*)}{\partial x_j} \geq 0 \text{ if } \sum_i \alpha_i h^i(Q) \frac{\partial Q}{\partial x_j} x_j^* = \sum_i \alpha_i h^i(Q) (Q(x^*) \cdot x^*).$$

In other words,

$$j \in C \implies \left( \sum_i \alpha_i \frac{\partial v^i(x^*)}{\partial x_j} \geq 0 \text{ if } \alpha_j = \frac{\sum_i \alpha_i h^i(Q)}{\sum_i \alpha_i h^i(Q)} \right). \quad (7.7)$$

6. Steps 2,4, and 5 thus tell us that the conditions of Lemma 3 are satisfied if there is a set of positive  $\alpha$ 's such that

$$\text{for all } j \quad \alpha_j = \frac{\sum_i \alpha_i h^i(Q)}{\sum_i \alpha_i h^i(Q)}.$$

But the existence of such positive  $\alpha$ 's now follows directly from Lemma 2. (Again, let  $a^i = h^i(Q(x^*))$ .) It follows that  $x^*$  is Pareto efficient. ■

For completeness, we state the definition of political equilibrium with a cap on contributions:

**Definition 7** A *political equilibrium with contributions capped at  $M_0$*  consists of:

(1) a partition  $H = L \sqcup R$ ,  $L \cap R = \emptyset$

(2) vectors of contributions  $M^L = \{m^h \mid h \in L\}$ ,  $M^R = \{m^h \mid h \in R\}$  from the informed members of types to their parties, such that  $m^h \in M_0$  for all  $h$ ,

(3) policies  $t^L$  and  $t^R$  of the two parties,

such that:

(4)  $(t^L, t^R)$  is a policy equilibrium at contribution vectors  $M^L, M^R$ , and

(5)  $M^L$  and  $M^R$  comprise a constrained contribution equilibrium at  $(t^L, t^R)$ .

Here, the definition of *constrained contribution equilibrium* just mimics Definition 4, but using the concept of ‘constrained Kantian equilibrium’ in place of ‘Kantian equilibrium.’

We proceed to calculate political equilibrium with a cap for the example of section 6. There, we observed that the contributions of party  $L$  were bounded, but the contributions of members of party  $R$  were not. We therefore expect that, if the bound  $M_0$  is not too small, only members of the  $R$  party will be constrained by the cap. Further, we noted that, for members of the  $R$  party, contributions are an increasing function of  $h$ . We therefore expect that there will be a type  $h_R$  in party  $R$  such that all  $h \geq h_R$  will be contributing at the cap, and no one else will be constrained.

It follows that, in an equilibrium of this kind, the equations characterizing the contributions of members of  $R$  will be:

$$\begin{aligned} h \leq h_R \quad m^h &= \frac{\partial \square}{\partial m_R} m_R \square v^h(t^L, t^R) \\ h > h_R \quad m^h &= M_0 \end{aligned} .$$

Integrating and dividing by  $m_R$  gives:

$$\frac{1}{\square} = \frac{\partial \square}{\partial m_R} \int_{h^*}^{h_R} \square v^h(t^L, t^R) d\mathbf{F}(h) + \frac{M_0}{m_R} \mathbf{F}(R \setminus H^R) \quad (7.8)$$

where  $H^R = [h^*, h^R]$  is the set of types in party  $R$  who are not constrained by the cap.

The first-order condition characterizing policy equilibrium is again given by (5.3b), which now reduces not to (5.4b) but to:

$$\frac{1}{1 \square \square} \frac{\partial \square}{\partial t^R} + \square \frac{\partial \square}{\partial m^R} \int_{h^*}^{h_R} \frac{dv^h(t^R)}{dt} d\mathbf{F}(h) \square \frac{\square M_0}{m^R} \int_{h_R} \frac{1}{\square v^h(t^R, t^L)} \frac{dv^h(t^R)}{dt} d\mathbf{F}(h) = 0 \quad (7.9)$$

The equations for the  $L$  party, whose members are unconstrained, will be exactly as in (5.2a) and (5.4a), which we here reproduce for convenience:

$$\frac{1}{\square} = \frac{\partial \square}{\partial m^L} \int_{h \in L} \square v^h(t^L, t^R) d\mathbf{F}(h) \quad (7.10)$$

$$\frac{1}{\square} \frac{\partial \square}{\partial t^L} + \square \frac{\partial \square}{\partial m^L} \int_L \frac{dv^h}{dt}(t^L) d\mathbf{F}(h) = 0 \quad (7.11)$$

The final two equations are:

$$h^* = \frac{t^L + t^R}{2} \quad (7.12)$$

$$M_0 = \frac{\partial \square}{\partial m^R} m^R \square v^{h_R}(t^L, t^R) \quad (7.13)$$

the first of which says that the type  $h^*$ , who is the pivot between the parties, is indifferent between the two equilibrium policies, and the second of which states that  $h_R$  is the supremum of types who are unconstrained in the Right party, at equilibrium.

Equations (7.8)-(7.13) comprise six equations in the six unknowns  $(t^L, t^R, m^L, m^R, h^*, h_R)$ .

This time, the equations do not separate the determination of the  $m$ 's and the  $t$ 's; fortunately, we are able to solve them, nevertheless. Table 3 reports a set of constrained political equilibria for various values of the cap. In all environments, I chose  $\square=1$  and  $\square=0.4$ .

{table 3 here}

Refer to the first line of Table 1 to compare these constrained equilibria with the unconstrained equilibrium for the same environment. In the unconstrained equilibrium, the Right party comprised the richest 20% of the population; total campaign contributions

were 13.3 units of money per capita, and about 61% of the polity were eventually committed to one party. In Table 3, we see that, in the region of the cap reported in the table, the Right party represents about 22% of the population. Total contributions are only 11 per capita, with the consequence that 55% of the polity become committed to a party.

In the range of the cap in Table 3, only members of the Right are constrained – and not many of them find the constraint binding. Only those types  $h > h^*$  contribute the maximum, which is about 6% of the income distribution in line 1 of Table 3.

The comparison between Table 3 and the second line of Table 1 (which reports unconstrained private equilibrium for the same value of  $\bar{c}$ ) is interesting. The expected policy has moved only slightly to the left with the cap (from centile .684 to .663).

What changes dramatically is total contributions, which are only one-third as large with the cap. Thus, although the cap only constrains a small fraction of contributors in equilibrium, there is much less spent on campaigns with the cap. Evidently, the cap prevents an ‘arms’ race’ between the parties.

## 8. Public financing of campaigns

We now ask: what would policies be in equilibrium if campaigns were publicly financed? We will study two institutions with public financing. The first consists in the government’s giving every citizen a voucher worth  $k$  dollars to be donated to the party of her choice, should she so wish. The second consists of the government’s matching private contributions to parties.

### A. Public financing in proportion to party size

We begin with the first institution. The voucher institution is meant to be an approximation of systems in which parties receive funds from the government in proportion to the votes they have received, perhaps in some recent election. The voucher is a simple institution to study, which achieves approximately that outcome (although not exactly).

I will assume that only the informed citizens make use of the voucher. Uninformed citizens are not sufficiently politically involved, or politically committed, to contribute to parties, even though that act is personally costless.

We will treat the size of the voucher as exogenous (not itself subject to a political decision). The funds spent through the voucher system would be raised by taxation.

The notion of political equilibrium with this institution is quite simple. Internal bargaining within parties continues to take place, but this time members have bargaining powers proportional to their *numbers*, because every individual contributes the same amount,  $k$ , to the party. We therefore have the first-order conditions for policy equilibrium:

$$\frac{1}{\square} \frac{\partial \square}{\partial t^L} F(h^*) + \int_L \frac{1}{\square v^h(t^L, t^R)} \frac{dv^h}{dt}(t^L) d\mathbf{F}(h) = 0 \quad (8.1)$$

$$\frac{1}{1 \square \square} \frac{\partial \square}{\partial t^R} (1 \square F(h^*)) + \int_R \frac{1}{\square v^h(t^R, t^L)} \frac{dv^h}{dt}(t^L) d\mathbf{F}(h) = 0 \quad (8.2)$$

The last three equations simply state that the campaign contributions are proportional to membership of parties, and that the cut-point is the indifferent type:

$$m^L = k \square F(h^*) \quad (8.3)$$

$$m^R = k \square (1 \square F(h^*)) \quad (8.4)$$

$$h^* = \frac{t^L + t^R}{2} \quad (8.5)$$

This system has five equations in the five unknowns  $(t^L, t^R, m^L, m^R, h^*)$ .

The public campaign funds can be raised in an arbitrary way by income taxation. Because utility functions are quasi-linear in money, this will have no effect on the forthcoming political equilibrium: each voter's income is simply reduced by the amount of his tax. To endogenize the political determination of the value of the voucher would take us into a realm beyond our present scope.

We now solve this model for the parameterization of section 6, with  $\beta=1$ . The comparative statics we study vary the value of  $\alpha$ . We begin at  $\alpha=0.4$ . At each value of  $\alpha$ , I set the value of public campaign subsidies,  $k$ , to approximately 1.4 times the total expenditures in the *private* model, at that level of  $\alpha$ . The consequence is that total campaign spending (which is a fraction of  $k$ ) is approximately the same in the private and public models, at each value of  $\alpha$ . This is, if you will, a 'revenue-neutral' institutional comparison.

The equilibrium values are reported in Table 4.

[table 4 here]

We see that in all equilibria, both parties propose policies very close to the ideal policy of the median voter (which is  $t=30$ ). Each party represents approximately one-half the population; campaign expenditures are approximately equal for the two parties. The fractions of voters who are eventually committed to one party are approximately the same in tables 1 and 3.

Clearly, politics have moved considerably to the left with the institution of public campaign finance. *Under public finance, each party represents approximately one-half the informed polity.* Politics are duller (little variety in party proposals), but the party

structure is more representative in the sense that each party represents approximately one-half the polity.

#### B. Private contributions with matching public funds

We next model a financing institution in which citizens contribute privately to parties, and the government matches private contributions. One can think of this as a model where each dollar an individual contributes to a party costs her only a dollar, but is worth two dollars to the party.

What is the appropriate concept of Kantian equilibrium here? It is exactly as in section 2, except that the utility function for the individual, in the model of section 2, becomes:

$$v^i(x_1, \dots, x_N) = c^i(x_i) + h(Q(2x_1, \dots, 2x_N)).$$

The analysis proceeds just as in section 2; a Kantian equilibrium is still Pareto efficient.

When we apply this to our political model, we define a *political equilibrium with matching public funds* just as in Definition 5. I am therefore brief:

**Definition 8** A *political equilibrium with matching public funds* is a partition

$H = L \sqcup R$ , a pair of policies  $(t^L, t^R)$ , a schedule of private contributions to parties

$M^L = \{m^h \mid h \in L\}$ ,  $M^R = \{m^h \mid h \in R\}$ , per capita private contributions to parties  $m^L$  and

$m^R$ , and total party campaign chests  $z^L$  and  $z^R$  such that:

$$(1) \quad \begin{aligned} m^J &= \int \int_{h \in J} m^h d\mathbf{F}(h), \quad J = L, R \\ z^J &= 2m^J, \quad J = L, R \end{aligned}$$

(2)  $(M^L, M^R)$  is a contribution equilibrium, given  $(L, R)$  and  $(t^L, t^R)$

(3)  $(t^L, t^R)$  is a policy equilibrium, given  $(L, R)$  and  $(z^L, z^R)$ .

The reader can now deduce that the equations for this equilibrium are almost like (5.2a,b) and (5.4 a,b):

$$\frac{1}{\square} = 2 \frac{\partial \square}{\partial z^L} \int_L \square v^h(t^L, t^R) d\mathbf{F}(h) \quad (8.6a)$$

$$\frac{1}{\square} = 2 \frac{\partial \square}{\partial z^R} \int_R \square v^h(t^L, t^R) d\mathbf{F}(h) \quad (8.6b)$$

$$\frac{1}{\square} \frac{\partial \square}{\partial t^L} + \square \frac{\partial \square}{\partial z^L} \int_L \frac{dv^h}{dt}(t^L) d\mathbf{F}(h) = 0 \quad (8.6c)$$

$$\frac{1}{1 \square \square} \frac{\partial \square}{\partial t^R} + \square \frac{\partial \square}{\partial z^R} \int_R \frac{dv^h}{dt}(t^R) d\mathbf{F}(h) = 0 \quad (8.6d)$$

The numeral ‘2’ appears in (8.6a,b) because the F.O.C.s for Kantian equilibrium, with this institution, are:

$$m^h = 2 \frac{\partial \square}{\partial z^L} \int_L \square v^h(t^L, t^R) d\mathbf{F}(h),$$

(I illustrate for the Left), which integrates to (8.6a).

We compute the equilibria for the same set of environments as that described in Table 1. The statistics are presented in Table 5. The interesting comparison is with Table 1, the case of unconstrained private contributions. We see that, for every value of  $\square$ , *politics move to the right* with public matching, in the sense that the expected policies are uniformly more favorable to the wealthy in Table 5 than in Table 1. The proximate cause of this result can be gleaned from looking at the equilibrium private contributions. For instance, in Table 1, at  $\square=0.4$ , Left and Right party campaign chests were about equal: but in Table 5, the Right campaign chest is almost three times the Left’s. It appears that, for the relatively poor, public funding acts as a *substitute* for private funding, while for the rich, it acts as a *complement*. So the matching funds institution exacerbates the distortion caused by private campaign finance.



One lesson of this section is that the *nature* of public financing makes a tremendous difference in the policy equilibrium. Our institution of equal per capita subsidies, which is meant to model real-world systems in which parties receive federal subsidies in proportion to their votes, engenders the most left-wing outcomes we have studied, while the matching institution engenders the most right-wing.

## 9. Conclusion

We have studied a model of private campaign spending which displays the seven enumerated features set out as desiderata in the introduction. There is a continuum of voter types. A party is an empty vessel which becomes the forum for bargaining among its contributors over what the party's policy should be, when faced with an opposition party and policy. In addition, parties are cooperative ventures with respect to raising funds for the election campaign, and as such, they organize contributors to a campaign-contribution schedule that is Pareto efficient for their members. Thus, even with a continuum of contributors, where no single type can influence outcomes, contributions play a double role – as determinants of intra-party influence, and as the foundation of party campaign chests, needed to reach uninformed voters. Finally, the model generates locally unique equilibria, which are computable and unique in a canonical example<sup>10</sup>.

We may now compare this model to that of Grossman and Helpman [2001, Chapter 10], hereafter, G-H. The models are similar in some respects: both postulate informed and uninformed voters, and both try to accommodate the double role of

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<sup>10</sup> It is not difficult to extend the model in this paper to cover multi-dimensional policy spaces, by grafting the PUNE model (see Roemer [2001]) onto the one here. In addition, this permits one, in a natural way, to include the opportunistic element in the party, that is, the influence upon party policy of political

campaign contributions. The G-H model, however, distinguishes between parties that represent *constituents*, and special interest groups (SIGs) that contribute to campaigns. The constituent-representing aspect of G-H is exogenously given, not modeled. Thus, their parties are, from the formal viewpoint, what I called ‘ideological,’ not ‘empty vessels.’ In our model, there are no SIGs, but every citizen is a potential contributor. With a large number of SIGs, the G-H model predicts that SIGs only contribute with an eye to influencing party policy – the ‘cooperative’ function of the party, that I have modeled, does not exist in G-H.

From a technical viewpoint, the equilibria in the present model are much simpler than those of G-H. They are, in particular, unique or at least few in number, for a given environment, while in G-H there is a large multiplicity of equilibria as the number of SIGs becomes large. This enables us to make quite strong statements of comparative statics with the present model, something which is more difficult to do with a large multiplicity of equilibria.

We studied a canonical example, meant to model a polity in which the electoral issue concerns redistribution among a citizenry with a distribution of wealth, with the characteristic feature that median wealth or income is less than mean wealth. Our analysis indicates that, with unconstrained private campaign finance, the policies of both parties will be biased towards the wealthy, even when every type contributes to at most one party. This aspect is more extreme, the smaller is the fraction of informed voters, and the larger is the rate of increase of the marginal rate of substitution of policy against contributions, as the wealth of the citizen increases. As the electorate becomes more

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entrepreneurs who are concerned only with winning office. I think that any effort to calibrate a model like ours to actual US politics should work with (at least) a two dimensional policy space.

informed, there is a lesser role for advertising to play, and it is not surprising that parties become more evenly sized. In a comparative-static computation where we alter preferences, as it becomes decreasingly costly for the rich to finance campaigns, we are also not surprised that politics become increasingly skewed to the right, in the sense that *both* parties propose increasingly right-wing policies.

One might conjecture that, if monied interests understood this theory, they would prefer that the electorate remain uninformed, thus shifting equilibrium policies in the conservative direction. To attribute polity-ignorance-preserving actions to the wealthy, however, would be a functionalist error, absent historical evidence, and the identification of a mechanism, such as control of the press by the wealthy.

We then adapted our model to study equilibrium under three other financing institutions: private financing with a cap on contributions, and two institutions with public financing of elections. With a public financing system in which each citizen receives a voucher for a fixed amount, both parties propose policies very close to the median voter's ideal point. Each party represents roughly one-half the informed polity. Politics become less interesting, than in the private model, but also to the left of the private-finance outcome.

Under private financing with a cap, we observed that the expected policy remains very close to what it was in the private finance model, but total contributions fall sharply, despite the fact that very few are, in equilibrium, constrained by the cap. We suggested that the cap prevents an arms' race between the parties.

Recently, Ansolabere, Figueiredo, and Snyder (in press) have written that the small amount spent in American electoral campaigns is a puzzle. Total campaign

spending, they argue, seems too small, given the prize of government allocation of public resources that is at stake. They suggest, as an explanation, that political contributions are not governed by self-interested considerations of the usual sort, but by a joy-of-participation motive. I wish to suggest that their conclusion may be premature.

Ansolaere et al write: “Perhaps the most surprising feature of the PAC world is the fact that the constraints on contributions are not binding. Only 4% of all PAC contributions to House and Senate candidates are at or near the \$10,000 limit (p. 7)” We have shown, however, that at an equilibrium under a financing system with a cap, where voters are of the usual self-interested sort, a very small fraction of voters are constrained by the cap. Secondly, we have shown that the existence of the cap, although not binding for the great majority of contributors, does reduce total contributions a great deal from what it would be in an unconstrained system. This, too, could help explain why contributions seem small in comparison to the prize to be allocated. Ansolaere et al show that contributions are increasing in the income of donors, and in the competitiveness of elections. This, too, is consistent with our results. If an election is close, then the importance of reaching uninformed voters is greater, and hence it is collectively rational (in the sense of Kantian equilibrium) for contributions to increase. Ansolaere et al show that virtually all campaign contributions in the United States come from individuals, and they suggest that the role of PACs may be to coordinate giving by individuals. In our framework, this means that the PAC structure could be the instruments through which something like a Kantian equilibrium is achieved. We suggest that it would be worthwhile to study whether campaign contributions in US elections do satisfy the conditions of a Kantian equilibrium. If so, then the apparently small total of contributions would be consistent

with rational behavior, without invoking a joy-of-participation motive. (Of course, in actuality, both a joy-of-participation and a ‘rational’ motive may exist together.)

Under a public system that matches private funds, the non-representative aspect of the private system is magnified: the expected policy moves to the right from the private finance equilibrium. In our example, public finance simply replaces some of the private financing for the poor, but it *augments* private financing for the rich, thus exacerbating the distortion caused by private finance.

The American system appears to be best approximated as a combination of private financing with a cap, and public matching funds<sup>11</sup>. Clearly, this system could bring us quite close to the public- voucher institution, if the cap were small. On the other hand, it could look deliver equilibria very much like the public-matching institution, if the cap were large. Thus, the ‘American’ system, viewed generically as a combination of private contributions with a cap and public matching, has the potential to run the gamut between the most representative and the least representative of our ideal types of institution. Calibrating the American system to the models of this paper, to discover exactly where it lies on that continuum, would be a worthwhile project, but one for the future. (Indeed, at first glance it appears that matching funds are a fairly small fraction of total campaign finance, and so the US system may be closer to the ‘private contributions with a cap’ model. See Ansolabehere et al for details.)

We finally present a welfare comparison of the private and the more egalitarian of the public campaign finance institutions. In the public-finance model, we now suppose that the public budget for expenditures on campaigns is raised from the citizenry

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<sup>11</sup> Ansolabehere et al (in press) provide a useful overview of US campaign finance law.

according to proportional taxation. If the total expenditure on the campaigns is  $y$  per capita at equilibrium, then citizens of income  $h$  are taxed in amount  $\frac{h}{\bar{h}} y$ , where  $\bar{h}$  is mean income (in our example, 40). In Table 6, we compare the welfare (expected utility) of informed voters at the equilibria of the two institutions, for the various values of  $\bar{h}$ , and always with  $\bar{h}=1$ . ( Thus, we are comparing the welfare of citizens in Tables 1 and 4.)

[table 6 here]

Unsurprisingly, the poor always do better under the public institution. In Table 6, all informed members of types with  $h < \text{'root'}$  have higher welfare at the public finance equilibrium, and all  $h > \text{'root'}$  have higher welfare with (unconstrained) private financing. The fourth column of Table 6 reports the fraction of the population who prefer public financing. For these values of  $\bar{h}$ , the majority always fares better under public finance, though the size of the majority decreases as the polity becomes more informed.

Nevertheless, it must be said that *welfare* comparisons of these two institutions should not necessarily be decisive with regard to our evaluation of them. If a democracy should be evaluated with respect to how *representative* its institutions are, and if we take ‘one-man-one-vote’ in the intra-party bargaining process to be a necessary condition of good representation, then, even if the majority has lower expected utility with public financing, we might well decide that public financing is the better (more democratic) institution. I do not claim that representation is the only criterion by which democratic institutions should be judged: welfare, I think, should count, too. Believing that both welfare and representation count must implies that we cannot reject a more representative institution even if, in some cases, majorities disprefer it. The minority, in this case, can

claim that representation is a democratic right, and rights, as we know, are by definition protected against majorities.

It is, however, not simple, as a matter of political theory, to characterize what the ‘right to representation’ requires, and it is beyond this paper’s scope to consider the question more carefully. My own instinct is that a system that produces, in a two-party system, one party that is very small and another that is very large, violates an axiom of good representation. If this is so, then we have provided some basis for advocating the public, egalitarian financing of political parties, as opposed to either unconstrained or constrained private financing, or public finance through matching private contributions.

Table 1 Private campaign finance, various values of  $\rho$ , always  $\beta=1$ 

Out[74]/TableForm=

$\rho$	tL	tR	mL	mR	F[tL]	F[tR]	F[tbar]	$\pi$	exp pol	com
0.4	36.789	75.5	6.911	6.482	0.606	0.888	0.796	0.803	0.697	0.605
0.425	35.842	71.287	5.313	6.36	0.593	0.873	0.778	0.793	0.684	0.612
0.45	34.924	67.387	4.045	6.212	0.579	0.857	0.759	0.783	0.671	0.621
0.475	34.043	63.783	3.055	6.053	0.566	0.84	0.74	0.773	0.657	0.63
0.5	33.203	60.46	2.292	5.895	0.553	0.822	0.721	0.763	0.644	0.641
0.525	32.413	57.403	1.712	5.747	0.541	0.804	0.703	0.752	0.63	0.653
0.55	31.679	54.598	1.276	5.612	0.529	0.785	0.684	0.742	0.617	0.667
0.575	31.005	52.031	0.95	5.492	0.517	0.766	0.666	0.73	0.604	0.681
0.6	30.396	49.686	0.707	5.382	0.507	0.747	0.648	0.719	0.592	0.696
0.625	29.854	47.546	0.528	5.277	0.497	0.728	0.631	0.707	0.581	0.712
0.65	29.381	45.594	0.394	5.171	0.489	0.709	0.616	0.694	0.571	0.729
0.675	28.976	43.813	0.295	5.054	0.482	0.691	0.601	0.682	0.561	0.746
0.7	28.639	42.187	0.221	4.917	0.476	0.673	0.587	0.669	0.552	0.764
0.725	28.37	40.699	0.165	4.751	0.471	0.656	0.574	0.656	0.544	0.782
0.75	28.168	39.336	0.123	4.544	0.467	0.64	0.562	0.642	0.537	0.8
0.775	28.03	38.082	0.091	4.288	0.464	0.623	0.551	0.628	0.53	0.818



Table 2 Private campaign finance, various values of  $\beta$ , always  $\alpha=0.4$ 

Out[82]/TableForm=

$\delta$	tL	tR	mL	mR	F[tL]	F[tR]	F[tbar]	$\pi$	exp pol	com
1.	36.789	75.5	6.911	6.482	0.606	0.888	0.796	0.803	0.697	0.605
1.1	37.286	73.55	10.602	16.611	0.613	0.881	0.791	0.795	0.701	0.654
1.2	38.134	72.108	17.026	41.62	0.624	0.876	0.789	0.78	0.71	0.708
1.3	39.453	71.218	28.872	101.281	0.641	0.873	0.79	0.758	0.724	0.764
1.4	41.319	70.838	51.496	237.824	0.664	0.871	0.795	0.729	0.744	0.816
1.5	43.743	70.874	95.233	536.646	0.69	0.871	0.803	0.697	0.765	0.86
1.6	46.659	71.225	179.314	1163.66	0.72	0.873	0.813	0.665	0.787	0.896

M0	tL	tR	mL	mR	hRstar	F[tL]	F[tR]	F[tbar]	prob	expol	com
30.	33.404	65.106	2.586	1.455	98.837	0.556	0.846	0.743	0.751	0.663	0.551
28.	33.329	64.907	2.523	1.394	97.241	0.555	0.846	0.742	0.749	0.663	0.55
26.	33.248	64.69	2.455	1.33	95.575	0.554	0.844	0.741	0.747	0.662	0.548
24.	33.159	64.45	2.383	1.263	93.83	0.553	0.843	0.739	0.745	0.661	0.546
22.	33.06	64.185	2.304	1.194	91.995	0.551	0.842	0.738	0.743	0.66	0.544

Table 3 Constrained private campaign finance,  $\beta=0.425$ ,  $\beta=1$ , varying the cap

k	$\rho$	tL	tR	mL	mR	F[tL]	F[tR]	F[tbar]	$\pi$	expol	com
28.	0.4	29.13	32.703	5.777	5.423	0.485	0.545	0.516	0.519	0.515	0.593
26.	0.425	29.127	32.676	5.697	5.353	0.484	0.545	0.516	0.521	0.514	0.609
24.	0.45	29.124	32.647	5.565	5.235	0.484	0.544	0.515	0.522	0.514	0.624
22.	0.475	29.119	32.615	5.382	5.068	0.484	0.544	0.515	0.523	0.514	0.64
20.	0.5	29.113	32.58	5.146	4.854	0.484	0.543	0.515	0.524	0.513	0.654
18.	0.525	29.105	32.542	4.86	4.59	0.484	0.543	0.514	0.526	0.513	0.669
16.	0.55	29.097	32.501	4.522	4.278	0.484	0.542	0.514	0.527	0.512	0.683
14.	0.575	29.087	32.456	4.132	3.918	0.484	0.541	0.513	0.528	0.512	0.697
12.	0.6	29.075	32.408	3.692	3.508	0.484	0.541	0.513	0.529	0.511	0.71
10.	0.625	29.063	32.357	3.202	3.048	0.483	0.54	0.512	0.53	0.511	0.723

Table 4 Public campaign finance, equal citizen subsidies, varying  $k$ , always  $\beta=0.4$ ,  $\beta=1$

Table 5 Private finance with matching public funds

Out[62]/TableForm=

$\rho$	tL	tR	mL	mR	F[tL]	F[tR]	F[tbar]	$\pi$	exp pol	com
0.4	37.104	62.676	3.054	8.577	0.61	0.834	0.749	0.692	0.703	0.639
0.425	36.309	59.5	2.335	8.28	0.599	0.817	0.731	0.682	0.69	0.646
0.45	35.537	56.581	1.775	7.959	0.588	0.799	0.714	0.673	0.676	0.654
0.475	34.795	53.904	1.345	7.63	0.577	0.78	0.697	0.664	0.662	0.662
0.5	34.09	51.452	1.016	7.298	0.567	0.762	0.68	0.655	0.649	0.672
0.525	33.429	49.211	0.767	6.969	0.557	0.743	0.664	0.646	0.636	0.683
0.55	32.817	47.167	0.58	6.643	0.547	0.725	0.648	0.637	0.623	0.694
0.575	32.254	45.304	0.439	6.32	0.538	0.707	0.632	0.629	0.61	0.707
0.6	31.744	43.609	0.333	5.997	0.53	0.689	0.618	0.62	0.598	0.72
0.625	31.287	42.066	0.253	5.672	0.522	0.672	0.604	0.612	0.587	0.734
0.65	30.88	40.661	0.192	5.34	0.515	0.656	0.592	0.604	0.577	0.748
0.675	30.524	39.382	0.146	4.998	0.509	0.64	0.58	0.596	0.567	0.763
0.7	30.216	38.216	0.111	4.644	0.504	0.625	0.569	0.588	0.558	0.778
0.725	29.953	37.151	0.084	4.276	0.499	0.611	0.559	0.58	0.55	0.794
0.75	29.736	36.179	0.064	3.892	0.495	0.598	0.549	0.573	0.542	0.81
0.775	29.56	35.289	0.047	3.491	0.492	0.585	0.541	0.565	0.535	0.826

Table 6 Welfare comparison: Private unconstrained with public voucher institutions

Out[94]/TableForm=

k	$\rho$	root	F[root]
28.	0.4	44.2037	0.695326
26.	0.425	42.5248	0.677227
24.	0.45	40.9976	0.659735
22.	0.475	39.618	0.643046
20.	0.5	38.3847	0.62738
18.	0.525	37.299	0.61298
16.	0.55	36.366	0.600136
14.	0.575	35.5959	0.589197
12.	0.6	35.006	0.580608
10.	0.625	34.6241	0.574948

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