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July 2004

Cahier n° 2004-016

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# Are neighbors welcome ? e-buyer search, price competition and coalition strategy in the Internet retailing<sup>1</sup>

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**Résumé:** On étudie les forces qui régissent l'agrégation des sites Web marchands (B-to-C) en concurrence sur un marché électronique différencié, où le coût de recherche pour les consommateurs est indépendant du coût d'adaptation/transport supporté lorsque le bien trouvé ne correspond pas à leur préférence. On s'intéresse à la possibilité pour les sites Internet de se coaliser, ce qui se traduit par une réduction du coût de recherche pour trouver d'autres sites à l'intérieur de la coalition. On effectue la statique comparative des structures de coalitions (en fonction du degré de différenciation de ses partenaires) lorsque les sites se font une concurrence en prix. On montre qu'un site préfère se coaliser avec un partenaire fortement différencié et fixe dans ce cas un prix unique plus bas.

**Abstract:** We study the forces that drive the phenomenon of aggregation of merchant Web sites (B-to-C) competing in a differentiated electronic market, where the search cost for the consumers is independent from the adaptation/transportation cost they incur when the good they find does not match with their preference. We focus on the possibility for Internet sites to coalesce, which results in a reduction of the search cost to find other sites within the coalition. We do the static comparative of coalition structures (depending on whether there is little or high differentiation between partners), when firms compete in price. We find that firms prefer to coalesce with highly differentiated partners, and set in this case lower prices.

**Mots clés :** e-commerce, coûts de recherche, coalition

**Key Words :** e-retailing, search costs, coalition

**Classification JEL:** L11, D83, D40.

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# 1 Introduction

When electronic commerce started to grow, understanding the novelty brought by Internet to business economics became a major issue, considering relations with suppliers and clients as well as among competitors. “Brick and mortar” firms operating in the distribution had to decide whether to enter the Internet retailing channel or not, at what pace and whether to be with or without partners. Firms located in the production area wondered if they had to wait and see, bearing the risk of having to face new powerful intermediaries concentrated on a distribution layer, that would gain market power against them, or even worse, to be foreclosed on a fast growing segment of demand by early vertically integrated competitors in Internet retailing. Competition law authorities contemplated the mushrooming of “B-to-B” coalitions (e-procurement and e-distribution cooperatives, new electronic market places put in place by “pure” players) and had to understand, beyond the announced reduction of transaction costs, if new forces were not about to structure e-markets towards e-monopolies. Increasing returns with their expected “winner takes all” phenomena were indeed supposed to be the rationale of the capital investment boom in the dotcom companies. Several years after the bubble, the consequences of these increasing returns on online retailing, as well as their very explicitation, remains to be done.

We believe that industrial organization can say something on two main topics raised by the rise of electronic commerce, which could drive excessive concentration from a welfare point of view (or economic rents for strategizing firms):

(i) the re-structuring of the vertical relationships within a given industrial sector when a new efficient distribution channel appears. This topic refers to the B-to-B e-trade (clients are distributors and not end-users)<sup>4</sup>. The new possible way of procurement (through a Web-site rather than within a usual shop) could for instance be directly exploited by producers (or at least by coalitions of complementary producers) as an opportunity to integrate downstream. But the subsequent bypass of their historical distributors on the remaining traditional channel complexifies the game, and often delays producers’ investments in e-retailing. Conversely, new entrants focussed on Web-retailing, benefiting from scale and scope economies as well as other

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<sup>4</sup>Which is supposed to represent more than 90 % of the e-commerce in the USA in 2001. *Source:* Census Bureau.

ingredients of increasing returns, could monopolize online distribution and further integrate upstream, with a threat of foreclosure of the e-distribution markets. Vertical relationships models with product differentiation have been used for studying concentration at the production and distribution levels in relation with brand marketing of the firms (brand names versus private labels). These models can be applied without major technical changes (differentiation will be associated here to the old and new channels) to the issues of changes of market power throughout the supply chain and will therefore remain out of the scope of the present paper (see Giraud-Héraud *et alii* 2001). (ii) the nature of competition within the online distribution channel, once this channel will become a significant way for firms to access customers. We enter here the world of B-to-C retailing, which even though playing a minor role in 2003, enjoys a high growth rate and already accounts for a large share of sales in some sectors<sup>5</sup>.

We will put aside in what follows problems related to vertical relationships and competing distribution channels, and we will focus on the mechanisms of competition among integrated (production-distribution) mono-product firms selling through Internet to end-users. Our goal is to contribute to the understanding of the differences induced on the structuring of markets between selling through physical stores and selling through Web-sites. We will restrict our analysis to the competition among firms within a given sector of goods or services (products are more or less substitutes). As long as delivery to customers does not alter the relative costs of competing firms, these goods do not need to be digital ones.

When the business press or the economics and management literature deals with the new features of the Web-retailing, “one to one” marketing opportunities on the sellers’ side and exchanges of informations among potential buyers and their network effects (communities) seem to be the most significant factors that could change the rules of the competition game. Having a look at the development of the e-sales and the most important B-to-C sites is yet far from confirming the prominent role of these two phenomena. We do not have plenty of Amazon-like sites in other fields than the entertainment and edition sector, but instead a huge growth for example of the e-tourism sales. In addition to this, the reasons of the Amazon’s success remain themselves to

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<sup>5</sup>For example, in France for 2001, Travels and hotels weight 300 million Euros over the 680 million Euros generated by the B-to-C, followed by Computers and multi-media (90 million Euros), food and drinks (80 million Euros), etc. *Source*: Benchmark Group, july 2001.

be elucidated: the hyper-mediatization of the site, the quasi-exhaustiveness and easy access to their database of books could be as important as the availability of customers' comments or the profile-related suggestions and other targeted dynamic marketing devices. Communities suppose discipline and frequent visits to the Web-sites, while occasional transactions by customers looking for some type of goods at a given moment represent a great share of the B-to-C sales (think for instance about the Christmas peak); in such a case interactions among users are obviously less likely to appear. What is then left of the Internet attractiveness that could foster this new distribution channel? According to us, simply a more efficient search without incurring transportation costs.

Direct sales through catalogs and mail or phone already avoided transport before the Internet era but we gained through the Web both a huge broadening of the scope of search and a higher quality of information on the goods offered. Of course it is always possible to get a higher quality in a physical store through a face to face with the goods and the sellers, but enlarging the scope of search leads then the transportation costs and the opportunity cost of time to rapidly explode.

Thus, our objective is to give a diametrically opposed, and therefore fully complementary, view of the forces that drive the evolution of the Internet landscape, compared to the world of Internet users enjoying to spend their time crawling through the Web, chatting and willing to belong to some communities, and then eventually captured by unexpected temptations through advertising or advises. Our proposed view originates in the sole e-retailing area and considers opportunistic Web-users looking for some good to buy and having a high opportunity cost for the time spent searching that good. Unlike some rare exceptions (such as Amazon and its few competitors in the edition sector), such an e-customer does not a priori know where is the right door to ring the bell. Then searching on the Web is not *per se* an easy walk and can very quickly become tedious and time consuming. Though Internet often remains more attractive than other distribution channels regarding this point, search costs of e-customers may well be one of the main competitive key drivers of the demand each Internet shop will meet at the end of the day. The more puzzling empirical fact from our e-customer's point of view is that it generally does not find *ex post* a convenient Web-site where it could easily compare all the available offers close to its preferences and then fine-tune its choice, using a specialized search engine. Why? While Internet may remain more attractive than other distribution channels for lowering search costs,

why is it still so painful for a customer who ignores the potential offerings, the true availability of the goods and services among providers, the price and the precise characteristics of the offers, to find out and buy what it is looking for, though all the needed technology exists? Let us consider tourism and travel sites: suppose you are looking for a stay in a winter-ski station in Europe for a week during a given period (school holidays), knowing that you have three children, one of them being a baby requiring baby-sitting, the others needing to attend ski-school; on top of that you would strongly appreciate a big enough ski domain and you have an upper limit for the entire cost, including travel and ski expenses. You will not find a portal site with a dedicated search engine gathering the offers of most of the ski stations (hotels and rentals) in France, Swiss, Austria and Italy. What you can actually find is some sites for one single location ([www.valdisere.com](http://www.valdisere.com)) where at best a reservation engine may exist for some part of the accommodation offering, or proprietary multi-location hotel clubs ([www.clubmed.fr](http://www.clubmed.fr)). Otherwise, and most of time, you may find portals and e-travel agency sites ([www.degriftour.com](http://www.degriftour.com)) with a limited offer for ski-holidays but a large scope of types of stays at that period (including cruises in Caribbean's). The value of quick finding (otherwise said, the cost of searching) is all the best exemplified here that, as time elapses, the stays that would have fit your preferences may disappear (no more rooms in hotels, no more train or airplane tickets for the period). Of course you can go to a brick & mortar travel agency, and by chance<sup>6</sup> in Paris most of them are located in the same district (Opéra). But they are not opened 24 hours a day, nor situated on your home-office way and finally once you are inside one of these shops you have to wait for your turn and then you will meet a vendor who will put pressure for making you buy what he has and not what you want.

Coalitions of Internet sites are far more easily built (portal with search engine and/or electronic linkage) than in the physical world (the street of the fish-shops), and the importance of the coalition phenomenon on the Web has already been many times evidenced. Whereas in the physical world the size of the required specific investments, the limitation of available geographical locations and the widespread location of customers put the brake on the coalitions of independent firms (localization in the same area) and may instead favorize true mergers and specialized chain stores developments, the coalition of independent mono-product firms on the Web is by far the easiest

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<sup>6</sup>it is of course not the fruit of hazard.

aggregation move. So we put aside mergers that, of course, also happen on the Web for many reasons, most of them being not Web-specific, and turn back to the two main questions that are behind our e-customer somehow disappointing experiment:

- What is the motivation for a Web-retailer to enter a coalition and what is the reason why it would be accepted or not by insiders?
- What is the coalition structure that is likely to appear: firms offering close substitutes grouped together or firms offering a basket of differentiated products?

The closest literature to these subjects deals with the aggregation of “true” shops in some locations (Schulz and Stahl 1996, Henkel and Stahl 2000) in the presence of search costs. Two main forces are considered in these papers: on one side, the lowering of search costs for consumers who may ignore the prices and characteristics of goods but know *a priori* the number of shops/products that are offered in a given location (or “marketplace”), and on the other side, the increased competition due to the proximity of shops, on the other side. The first force drives aggregation of shops by directly increasing the demand through the number of consumers that want to economize on search costs. The second one limits this aggregation when the decrease of price with the number of firms - which may happen more or less quickly according to the differentiation of products - is no more counterbalanced by the increased market accruing to each shop. Up to a limit number of already existing shops, it can become more profitable for a shop to choose a distant location<sup>7</sup>. Finally, entry sunk costs in a given location determine the number of viable firms.

Though the seminal papers on search costs (Stigler 1961) emphasized the subsequent choice of spatial monopolies by firms (maximum dispersion), the empirical evidence of concentration phenomena (Nelson 1970, Stuart 1979) led to the introduction of differentiation among the products offered by competing firms (Stahl 1982) or by the focus on coordination on volume or prices (organization of the market place, mergers, collusion). The main interesting result is that, due to the search costs of consumers, prices may increase with the number of firms in a given location (entrants are more than welcome). Not surprisingly, when the entry or coalition formation process is part of the model, the more the products are substitutes for customers, the less the “big

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<sup>7</sup>except when price coordination is made possible on the marketplace, for instance through mergers or more or less tacit collusion.

coalition” on a unique marketplace is likely to appear.

In all those models, every product has the same level of differentiation compared to the other products (the value of this level being a parameter) and the questions under study are only the number of products offered in a marketplace, or the number of marketplaces<sup>8</sup>. As a consequence, if we understand well the forces driving aggregation/disaggregation of firms in the physical world - and we will discuss later on the differences on the Web- we don't have yet insights or discussions on our second question: what is the coalition structure of the firms located in the same place? Do groups of close substitutes prefer to aggregate on the same locations or the opposite? Competition among firms that are more or less close in the sense of search but also more or less close in the sense of product offering is hardly intuitively predictable. This idea of coalition structure, related to the relative differentiation of products taking part of the coalition, exists in another stream of the literature, where alas search costs are absent from the scope of the models. For instance, Giraud-Héraud *et alii* (2003) look for the optimal product range in a coalition game where mono-product firms can merge in the sense that they will coordinate their pricing decisions. As these authors do not consider a bi-dimensional space (spatial location and product characteristics), the only sense aggregation can take is a merge among firms<sup>9</sup>. They find that the coalition at the equilibrium includes close-substitute products, in order to capture captive customers with relatively high prices in the core offering and then be more aggressive on the border of the product range.

In order to modelize the forces driving aggregation of shops on Internet and the outcome in terms of coalition structure (geography of the Web-retailing in a given sector), we need to include both search costs of imperfectly informed consumers and a measure of differentiation among goods offered by firms. Moreover, we must clearly distinguish search costs *per se* and adaptation costs of consumers to the goods they find (which do not perfectly fit with their preferences)<sup>10</sup>. We choose a circular differentiation model similar to

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<sup>8</sup>The tractability of such models in a context of imperfect competition is of course an issue and explains this assumption of symmetry.

<sup>9</sup>Most of the analysis concerning the formation of coalitions consider that the primary objective of companies consists on agreeing on the prices and quantities sold on the market. All the works built on the models of Salant, Switzer and Reynolds (1983), Perry and Porter (1985), Deneckere and Davidson (1985), Farrell and Shapiro (1990) belong to this vein.

<sup>10</sup>In the mono-product models with spatial differentiation and imperfectly informed consumers about prices (see for instance Gabszewicz and Garella 1986), search cost is a



the one of Bakos (1997) rather than the classical linear one, first proposed by Hotelling (1929), since, in our model, it is more convenient that all the locations in the differentiated market are *a priori* equivalent. In this way, the choice of a partner to form a coalition with depends only on whether the partner site is highly differentiated from the location of the initiator or not, and not on whether the partner's location is a privileged one, which is the case for the firms that are located in the extremities of the linear city. Let us precise now what is a coalition. As in the works of geographical economy with the presence of search costs (and contrary to Giraud-Héraud *et alii* (2003)), coalition will mean a low search cost to discover all the products offered by the members of the coalition, once you have paid to access it. Of course, it is now more related to spatial location, and search cost is not a transportation cost, but once again, firms belonging to a coalition remain free to choose their prices. In other terms we exclude price coordination that arises when firms merge or when a single firm chooses more than one location on the circle (multi-product firm). This allows us to isolate the effect of lowering search costs for the consumer, in the optimal choice of the coalition structure.

*Online booking.* We can illustrate, by using the example of “online booking”, our motivation for building a specific model for Internet retailing where search costs and adaptation costs are clearly separated and where firms coalesce without price coordination. Let us describe the search procedure of a (rich) traveller willing to book online in a luxury hotel in Paris. All luxury hotels usually have a Web-site and also present for us the advantage of being not too many which makes our presentation easier. Luxury hotels are mainly located in a few neighborhoods of Paris (Champs-Élysées, Opéra, Bastille, etc.) that will be a criterion of differentiation for consumers. Firstly because the system of stars guarantees a type of service which makes that a consumer searching on the Internet (rather than directly making reservations in a hotel chosen for its reputation for example) can be considered to be indifferent between two hotels in terms of quality. Secondly the fact that travellers might have different reasons to come to Paris gives them preferences in terms of location (proximity to a person, a meeting, a conference center). Thus, both the hotel characteristics and the consumers' preferences are defined in terms of location. A choice of hotel category (number of stars) also corresponds to an anticipated price that the consumer is ready to pay, say 300 Euros per transportation cost (or an adaptation cost, the preferred location of a firm for a customer being its own location).

night, but discovering a lower price could be a motivation to adapt to a hotel whose location is more distant than the maximum acceptable distance for a price of 300 Euros. The search procedure begins by a typing a request in a search engine (such as google.com, altavista.com, excite.com, etc.). To a request equivalent to ‘luxury hotels in Paris’, the search engine will return a great number of sites, so an amount of time and energy is spent by the consumer for picking up a suitable site among the list and visiting this site in order to get the information about its price and location. Therefore a search cost is incurred in order to get complete information about one site (one hotel). If the hotel is “close enough” to the consumer’s preference, the transaction can take place, otherwise it is preferable to perform a new search due to the perspective of finding a better alternative. The decision depends on the characteristics of the current hotel, on the priors on the characteristics of future hotels to discover, and on the cost of searching further. The results coming from the search engine are of different kinds. The consumer gets sites of a single hotel (Bristol, Le Crillon, Meurice, Ritz, Scribe, Westminster, Hilton, Intercontinental, etc.), but also sites that refer to several hotels. In their more simple version these collective sites, to which we will refer as “coalitions”, only have an electronic link to the sites of single hotels: Le Crillon and Bristol for example appear in lodgingfrance.com with two other luxury hotels. Another kind of collective site have also the facility to reduce the search cost of the consumer through the development of specialized search engines. By sorting the results of the site by category, it is possible to find “in one clic” a number of luxury hotels that may vary from a site to another (none in hotel-paris.com or holelus.com, 2 in parishotelreservation.com, 4 in paris.book-online.org, 5 in hotel-paris-tobook.com and hotelclub.org, 6 in 0800paris-hotels.com). By visiting such a site, the consumer can have access to more results corresponding to its search by incurring a lower cost than the one required to extract the information from the entire Web. The reduction of search costs is considered to be the result of the coordinated efforts of the coalesced sites to develop more efficient search tools in order to facilitate the finding of the goods closest to the tastes of Internet consumers. But on the hotels’ side, the interest of forming a coalition is to increase the probability of being visited, thereby increasing the expected demand. Developing a site for a group of hotels (or agreeing to appear in a portal site made by a specialized firm), has also the advantage to represent a very low cost if we compare it to the cost of merging in the physical world. In these kinds of coalitions, hotels remain independent (in particular in their price policy) and only take profit

of the increase in visits of those consumers that book online.

We observe that in most of the cases, inside a coalition site, hotels belong to different neighborhoods. For instance in the 0800paris-hotels.com coalition, the 6 luxury hotels proposed by the site are located in 6 different districts of Paris.

The results we obtain with the model presented in the next section are consistent with these findings: firms prefer to coalesce with differentiated partners. But we obtain more when we look at the Nash equilibrium prices of highly differentiated firms belonging to a coalition compared to the prices of close-substitute ones grouped in a coalition: in the configuration we modeled, prices are lower within a coalition of highly differentiated firms than in the other case. This breaks the intuitive explanation for the preference for highly differentiated partners that would highlight the fear for increased competition, since there is perfect information for consumers once they have reached a coalition and no price coordination between coalesced firms.

The rest of the paper is organized as follows: after a presentation of the modelization of the differentiated market and the search procedure of the consumer, we describe the different coalition structures. We first examine the preferred coalition structure when prices are fixed at the level given by the game without coalitions and, second, when sites compete in price. We finish by some concluding remarks on the obtained results.

## 2 The model

As in Bakos (1997) model, we consider a market with a continuum of Internet consumers and  $m$  B-to-C sites.  $m$  is supposed to be common knowledge. Each site  $j$  sells a unique good at price  $p_j$  and the characteristics  $x_j$  of the goods are differentiated along the unit circle as in Eaton's (1976) pioneering work and subsequent models (D'Aspremont, Gabszewicz, Thisse (1979), Salop (1979), Novshek (1980), Eaton (1982), Stahl (1982), Economides (1989)). The tastes  $x_i$  of the consumers are heterogeneous and uniformly distributed along the same circle. By buying a unit of good that does not match exactly with its preference, the consumer incurs an adaptation cost  $t$  per unit of distance ( $t > 0$ ) between its location (*i.e.* its preferred product) and the location on the circle of the chosen site (*i.e.* the good offered) for the transaction. Therefore, the utility function if consumer  $i$  buys a unit offered by site  $j$  is:  $U(i, j) = r - p_j - t|x_i - x_j|$ , where  $r$  is the reservation utility of

each consumer.

*Consumers' search procedure.* Consumer  $i$  acquires information on the location and the price of one of the  $m$  sites of the electronic market by incurring a constant search cost  $c > 0$ . We consider this search cost to be both the cost associated with the discovery of the site on the Web, for example through a search engine, and the cost of visiting the site to find out about its characteristics: sell price  $S$ , and distance  $D$ . The utility of the consumer in case of a transaction is  $U(S, D) = r - S - tD$ . If the consumer decides to search further and finds another site located at distance  $x$  and with price  $p$ , the utility in this case is  $U(p, x) = r - p - tx$ . Thus,  $(U(x, p) - U(S, D))^+ = (S + tD - xt - p)^+$  represents the increase of utility for the consumer if  $U(x, p) - U(S, D) > 0$  (otherwise it is 0). We suppose that the consumers are risk neutral. The calculation of the expected gain in utility based on the priors on the distributions of sites' locations and prices allows the consumer to decide on the opportunity to continue the search procedure. This defines the space of strategy of the consumer.

*Consumers' priors.* Concerning the priors on prices the consumers believe that at equilibrium all sites choose the same price  $p^*$ . More precisely the distribution of prices is such that  $f(p) = 1$  if  $p = p^*$ , and  $f(p) = 0$  if  $p \neq p^*$ . Concerning the priors on locations, the consumers believe that sites locate according to a uniform distribution over the unit circle. We also suppose that consumers find sites according to a random trial with replacement. These assumptions are related to the fact that consumers are considered to not change their priors on the distributions of locations or prices after finding each site.

*Stopping rule.* The expected gain in utility obtained in this case is:  $g(S, D) = \int_{x=0}^1 (\int_{\mathbb{R}} (S + tD - xt - p)^+ f(p) dp) dx$ . According to the priors of the consumers on the locations, we find like in Bakos (1997) that  $g(S, D) = (S + tD - p^*)^2 / t$ . Next, the consumer has only to compare its expected gain in utility with the search cost  $c$ . If  $g(S, D) > c$ , the consumer will prefer to continue its search. If  $g(S, D) < c$ , the consumer will choose to buy a unit of the good located at a distance  $D$  and at price  $S$ . At equilibrium with rationale expectations for the consumers,  $S = p^*$ . For each consumer  $i$  located in  $x_i$ , we have that  $g(p^*, D) < c$  on the interval  $[x_i - L, x_i + L]$ , where  $L = \sqrt{c/t}$ . Consequently, if the consumer discovers a site at a distance smaller than  $L$ , the transaction will take place. Symmetrically, from the point of view of a site, the more distant potential client is located at distance  $L$ . We obtain an interval of

length  $2L$  around any site, which will be referred to as “natural territory”<sup>11</sup>.

**Definition 1** *The natural territory of a site corresponds to the interval around its location in which consumers stop their search and buy from this site if they find it.*

Let us now describe the simplest framework needed to capture the effects we want to describe once it is possible for sites to coalesce. We consider that  $m = 4$ , that these sites are located according to the principle of maximum differentiation<sup>12</sup>, and that they sell at price  $p^*$ , which is also the price anticipated by the consumers. We restrict the study in terms of length of natural territories by supposing that  $\underline{L} < L \leq \bar{L}$  such that no consumer is priced out of the market and the natural territory of a site only intersects with those of its neighbors. In the case of 4 sites, we have  $\underline{L} = 1/8$  and  $\bar{L} = 1/4$ . Let us compute the expected demand of each of the 4 sites. If a consumer belongs to the natural territory of only one site, then its search procedure will continue until this site is found. A consumer that belongs to the intersection of two natural territories will buy at the first of the two sites to appear during the search procedure. As a result, each site has an interval around its location in which it does not share the consumers (of length  $1/4 - L$ ), followed by the intersection of the natural territories of length  $2L - 1/4$  where the site shares consumers with one competitor. Since no consumers are priced out, the expected demand is  $D = 2(1/4 - L) + 2(2L - 1/4)/2 = 1/4$ .

**Proposition 2** *There is a unique symmetric equilibrium price with rational expectations in which the 4 firms choose  $p_j = p^* = t/4$*

**Proof.** We search for a non-cooperative price equilibrium where all sites sell at price  $p^*$ . Since the expected demand is  $1/4$ , the expected profit of a site is  $\Pi^* = p^*/4$ . Let  $\Pi^\delta$  be the profit resulting from a price deviation  $\delta p$  of one of the sites, *i.e.*  $p = p^* - \delta p$ , resulting in a variation of natural territory by  $\delta p/t$ :  $\Pi^\delta = p/4 + 2p\delta p/2t = p/4 + 2p(p^* - p)/2t$ , leading to:

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<sup>11</sup>It is clear that the length of the natural territory depends on consumers’ anticipations. For other scenarios on consumers’ anticipations in an agent-based environment, see Laye, Lina and Tanguy (2004).

<sup>12</sup>Given the assumptions of our model, we can prove that in a two-stage game in which sites choose locations in the first stage of the game and set prices in the second stage, sites choose the same price (symmetric equilibrium) and locate so as to maximize the distance between their locations.

$d\Pi^\delta/dp = 1/4 + 2p^*/2t - 4p/2t$ . The condition  $d\Pi^\delta/dp|_{p=p^*} = 0$  provides the equilibrium price:  $p^* = t/4$ . ■

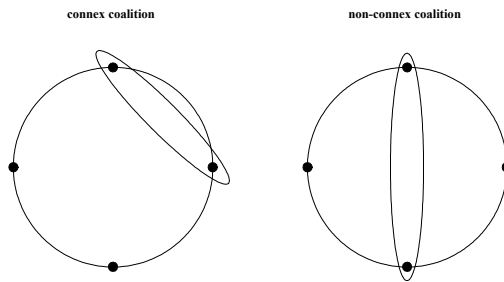
We see that the equilibrium price increases with the adaptation cost and does not depend on the search cost<sup>13</sup> as long as  $\underline{L} < L \leq \bar{L}$ , where  $L = \sqrt{c/t}$ . In reality  $L$  and  $p^*$  are two aspects of the same reality at equilibrium based on the assumption of rational expectations. In fact, we can show that there exists a bijection between the set of possible prices and the set of possible natural territories: since  $g(S, D) = (S + tD - p^*)^2/t$ , if  $S = p^*$ , then  $g(p^*, D) = tD^2$  and the transaction takes place if  $D < \sqrt{c/t}$ . If a site situated at a distance  $D$  offers a different price  $p' = p^* - \delta p$ , then:  $g(p', D) = (-\delta p + tD)^2/t = (t(D - \delta p/t))^2/t$ . This expected gain has to be compared with the search cost:  $g(p', D) < c \iff D - \delta p/t < \sqrt{c/t} \iff D < \sqrt{c/t} + \delta p/t$ . If we define  $l = L + (p^* - p)/t$ , where  $l$  is half the length of the natural territory that results from a price  $p$ , the corresponding space of strategy for  $l$  is the interval  $[0, 1/2]$ .

## 2.1 Coalition structures

The setting we described is also the minimal setting required to differentiate coalition structures: a site willing to coalesce can choose two kind of partners defining the two different categories of coalitions.

**Definition 3** *A coalition will be called “connex” if the natural territories of its members intersect, otherwise the coalition will be called “non-connex”.*

For  $\underline{L} < L \leq \bar{L}$ , a coalition is connex if its members are located consecutively on the circle (little differentiation), and non-connex otherwise (high differentiation).



<sup>13</sup>More precisely  $p^*$  is independent from  $c$  in each interval of study, but increases with  $c$  through  $L = \sqrt{c/t}$  as follows: if  $L < 1/8$ , then  $p^* = \sqrt{ct} < t/8$ ; if  $1/8 \leq L < 1/4$  then  $p^* = t/4$ ; if  $1/4 \leq L < 3/8$  then  $p^* = 3t/8$  and if  $3/8 \leq L < 1/2$  then  $p^* = 3t/4$ .

From the point of view of the consumers, visiting a coalesced site allows the consumer, which has only incurred the search cost  $c$  for a search on the entire Web, to visit other sites by incurring a lower cost  $c' < c$  within the coalition. We normalize  $c'$  to zero. We suppose that the consumer benefits only *ex post* from the reduction of the search cost: the consumer do not anticipate the presence of a coalition on the market and if one coalition is discovered the search procedure continues without modifying its priors accordingly. Modifying the priors would be equivalent to consider that after discovering a coalition (without finding a site to make the transaction), the search cost is lower for the rest of the search procedure. In other words, the expected gain is increased since there is more chance to find a site that matches the preferences of the consumer from now on, in a market with coalitions. Therefore after the first discovering of a coalition and for the rest of the search procedure we would be in an equivalent situation than without this simplifying assumption. Therefore the results are not affected qualitatively by this assumption.

From the point of view of a coalesced site, a coalition is a possibility to increase its expected demand. To analyze the incentives to choose a connex or a non-connex partner, we do first the static comparative of the two coalition structures, putting aside price competition.

**Proposition 4** *With fixed prices, a site willing to coalesce has more incentives to choose a non-connex partner.*

**Proof.** Without loss of generality, the price is fixed at the equilibrium price without coalition  $p^*$ . As we have already shown, without coalitions, the expected demand of each of the 4 sites is  $1/4$ . Depending on the coalition structure the expected demand will be modified. In the connex case, if  $I$  denotes the length of the intersection of the natural territories, the expected demand is  $1/4 - I/2 + 2I/3 = 1/4 + I/6$  for each coalesced site and  $1/4 - I/6$  for the non-coalesced ones. In the non-connex case, the expected demand is  $1/4 - I + 4I/3 = 1/4 + I/3$  for coalesced sites, and  $1/4 - I/3$  for independent sites. And  $1/4 + I/6 < 1/4 + I/3$ . ■

For  $m > 4$ , the equilibrium price is different and the values of the  $\underline{L}$  and  $\overline{L}$  so that no consumer is priced out of the market and the natural territory of a site only intersects with those of its neighbors also change. The length

of the intersection of natural territories will be modified but the qualitative result holds. For any  $m$ , if we do not suppose that  $\underline{L} < L \leq \bar{L}$ , then either the natural territories have no intersection, either the territory of a site first intersects with those of its neighbors but also with the one of at least two other sites. In the first case ( $L < \underline{L}$ ), some consumers are priced out and will never accept to make the transaction, while the others belong to the natural territory of only one site. The search procedure of a consumer will continue until this site is found, and the demand is deterministic. The coalition has no effect on the expected demand which makes that this situation is not interesting to study. In the second case ( $L > \bar{L}$ ) not only the immediate neighbors are potential connex partners, but it is still a non-connex partner (if it exist since if  $L$  is big enough or if  $m$  is small enough, all the sites can be connex) that will be preferred by the initiator of a coalition. It is still the fact that the natural territories intersect or not that will drive the choice of a partner to coalesce.

## 2.2 Coalition strategy with price competition

In the previous section, all sites sell at a fixed price  $p^*$ , equilibrium price without coalitions. Given the strength of this assumption, it is necessary to study in what way strategic pricing for the sites influences the choice of between the coalition structures. For each coalition structure, we find that, at equilibrium, sites deviate from  $p^*$ . We only provide here the expressions of the prices ( $p_{nc}$  for non-connex and  $p_c$  for connex) in the intervals in which there is no multiplicity of equilibria. See the annex for details about the multiple equilibria.

**Proposition 5** *In the **non-connex** coalition case, there exists  $\tilde{\varepsilon}$  such that if  $L \in [\underline{L} + \tilde{\varepsilon}, \bar{L}]$ , we obtain a unique and symmetric price equilibrium:*

$$\begin{cases} p_i = p_{-i} = p_{nc} = p^* + \Delta/3 \\ \pi_i = \pi_{-i} = \pi_{nc} = p^* - \Delta/6 \end{cases}$$

*if  $L \in [\underline{L}, \underline{L} + \tilde{\varepsilon}]$ , there is a multiplicity of symmetric equilibria<sup>14</sup>.*

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<sup>14</sup>In this case, for a price chosen by a type of sites (coalesced or non-coalesced), the other type will choose a price so that their natural territories are adjacent.



In the **connex** coalition case, there exists  $\widehat{\varepsilon}$  such that if  $L \in [\underline{L} + \widehat{\varepsilon}, \overline{L}]$ , we obtain a unique price equilibrium:

$$\begin{cases} p_i = p_{-i} = p_c = p^* + 14\Delta/69 \\ \pi_i = \pi_{-i} = \pi_c = p^* - 10\Delta/69 \end{cases}$$

if  $L \in [\underline{L}, \underline{L} + \widehat{\varepsilon}]$ , there is a multiplicity of eventually non-symmetric equilibria but there always exist symmetric equilibria<sup>15</sup>.

where  $\Delta = p^* - tL > 0$  for  $L \in [\underline{L}, \overline{L}]$

**Proof.** See the annex.

■

The proposition shows<sup>16</sup> that for both coalition structures (connex and non-connex), coalesced sites have an incentive to lower their prices from the one obtained without coalitions ( $p^* = t/4$ ) in order to increase their natural territories (and therefore the length of the intersection of natural territories). The opposite tendency is observed for the non-coalesced sites: they increase their price in order to decrease the length of this intersection. Furthermore, we see that the non-connex coalition is more aggressive than the connex one. The fact that non-connex partners decrease more their prices than is they were in a connex coalition shows that **it is not the increase in the competition between the coalesced sites that drives the price decrease**. Decreasing the price reflects only the opportunity to gain market share from the non-coalesced sites. When the coalition is connex, a coalesced site gains market shares from the territory shared with a non-coalesced site (on one side of its location) and shares equally the rest of the consumers with its partner (on the other side of its location), which brings no additional demand. The gain of market shares is far better exploited when the coalition is non-connex since it occurs on both sides of coalesced sites' locations, without interacting with their partner<sup>17</sup>.

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<sup>15</sup>These strategies correspond to prices leading to adjacent natural territories for the non-coalesced sites.

<sup>16</sup>For both coalition structures, the analysis is made over the analytical expressions of the symmetric equilibria (unique or multiple).

<sup>17</sup>We can remark in the non-connex case that the price adjustment of the non-coalesced site  $\Delta/6$  is half the one of the coalesced sites  $\Delta/3$ . On the other hand the probability to attract consumers belonging to the intersection of natural territories for a non-coalesced site is also the half of the probability to attract them for a coalesced site.

We can also remark in the connex case that the ratio of price adjustment due to the presence of a coalition is equal to the ratio of the probability to obtain, for each type of sites, a consumer belonging to the intersections of natural territories (this ratio is 7/5).

Finally, by comparing the profit of the coalesced sites depending on the coalition structure, it is now possible to decide which coalition structure is preferred by a site willing to initiate a coalition thanks to the following proposition:

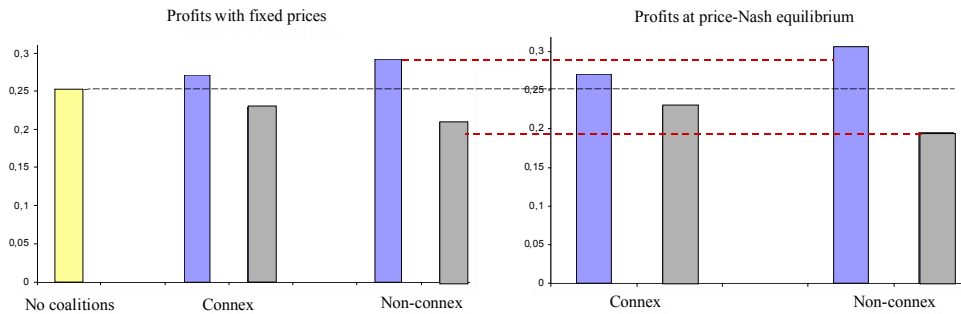
**Proposition 6** *For a search cost  $c$  and an adaptation cost  $t$  such that  $\sqrt{c/t} = L \in [\underline{L}, \bar{L}]$ , the profit of a coalesced site of a non-connex coalition is greater than the profit of a coalesced site of a connex coalition, i.e.  $\Pi_{connex} \leq \Pi_{non-connex}$*

**Proof.** See the static comparative of the coalition structures in the annex.

Consequently, with price competition, the result we obtain with fixed prices still holds: the non-connex structure is always preferred by the initiator of a new coalition.

### 3 Concluding remarks

In addition to the relative level of prices explicited in the previous section (no coalition, firms within or outside the connex and the non-connex coalition), the main results are summarized in the following chart<sup>18</sup>, which makes possible the comparison of profits in the different situations (fixed prices, price competition), for each coalition structure (connex or non-connex), and for the two type of sites (coalesced sites, which are represented first, and non-coalesced sites):



<sup>18</sup>For the parameters  $t = 4$ , and  $c = 9/16$ , leading to  $L = \sqrt{c/t} = (\underline{L} + \bar{L})/2 \in [\underline{L}, \bar{L}]$ , and to  $p^* = 1$ , we obtain a profit for each of the sites of  $1/4$ , when there is no coalition.

Although the analytical model is built with only 4 firms, some qualitative results can be drawn for more general situations, within the same corpus of assumptions (no community effect on the demand side, no price coordination on the supply side):

- if a site initiates a coalition for gaining visibility, it will prefer a differentiated partner (whose natural territory does not intersect with its own one) rather than its neighbors.
- if an individual site wonders whether or not to belong to a portal with its already coalesced members, the answer will be yes: to be in a coalition is better than staying alone.
- but if an insider firm has a close offering, otherwise said, if its natural territory intersects with the one of the potential entrant in the coalition, this incumbent will be worse off, and, thus, will reject the candidate, if it was endowed with the right to do it.
- finally, we expect firms within coalitions to price lower than single Web-sites, and firms within non-connex coalitions to be even more aggressive than firms within connex coalitions.

Of course, even if starting from the static comparative with 4 Web-sites allows reasonable conjectures to be made on the consequences of introducing dynamics and a huge number of sites<sup>19</sup>, we have not yet taken into account the various processes of coalition formation, the opportunity to belong to many coalitions, or the effects of competition among several coalitions.

Nevertheless, we have already captured some mechanisms that could be fundamental in the differences between the structuring of e-retailing markets and the structuring of traditional ones.

Firstly, although the entry process is out of the scope of the model, we must keep in mind that entry costs on the Web-retailing can be very low compared to the costs of building store chains, for the same potential market. Secondly,

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<sup>19</sup>Within the same framework, a simulation model with  $n$  sites and  $m$  customers has been built with some natural rules of the game for the formation of coalitions (multiple initiators, random choice of partners, individual profit criterion and veto right). The results obtained by simulation confirm the qualitative trend suggested by the analytical model in terms of coalition structures: few connex components in the coalitions present at a stabilized regime, although some may appear as a response to competition with many coalitions. The rejection criterion of a new member based on the fact that, after having joined the coalition, at least one of the previous members is worse off, is sufficient for stopping the coalition formation process after a finite number of simulation steps, the final outcome being typically one or a few “big” non-connex coalitions, many small ones and the remaining individual sites. For more details, see Laye, Lina, Tanguy, 2004.

entering a coalition, in the sense we used here, is a decision that can be completely separated from the decision of entering the market; the costs of entering, or quitting a coalition are again very low. In the usual retail markets, building a shop and choosing a location are the same decision, which involve significant entry costs, which are themselves mainly sunk. It is therefore not surprising that we observe a lot of individual, say mono-product, Web-sites (the supply can be very fragmented on the differentiation axis) and also a lot of coalitions whose structure can evolve rapidly<sup>20</sup>.

The second main difference, when we associate Web coalitions to geographical locations, is the nature of search costs. In the “real” world, we have clearly two dimensions for horizontal differentiation, which must be taken into account for explaining the structuring of the supply side: distance between customers and shops (transportation costs) and characteristics of products (adaptation costs). When the search cost of consumers is an issue, transportation cost is at least a big part of it, and, moreover, consumers a priori know the location of the market places as well as the importance of the product offering (number of products is the proxy used in the models)<sup>21</sup>. In the Web-retailing world, the distance component of the differentiation space disappears, the search cost becomes independent of any “location” issue for the aggregation phenomenon. Moreover, we assumed that consumers ignore the existence of coalitions: when the Web-user unwittingly enters in some

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<sup>20</sup>Portals specialized in a given sector, or sub-parts of multi-sectors portals best exemplify the coalitions we are speaking about. But the results and discussion can be extended to several other forms of coalitions, such as sites that play the role of an intermediary (as Anyway.com for travel services), and while taking a percentage of the sales, do not distort the prices proposed by their suppliers or “partners”. Finally, we can also consider the situation of on-line multi-product distributors, whenever their price policy is constrained by suppliers: having been referred to at this distribution site, a supplier indeed appears not far from other competitors (as in a coalition), and keeps some control over the pricing of its products as it is the case in the “selective” distribution channels where minimum or “advised” prices can be legally enforced by the suppliers.

<sup>21</sup>In spatial competition models without search costs, firms are generally spread over the differentiation line at the equilibrium. Coalition is therefore often associated to price coordination between firms belonging to different locations. To our knowledge, if double horizontal differentiation has already been used, for instance for studying pricing issues between 2 distant firms offering 1 or 2 differentiated products (with price coordination), the structure of the supply has not been treated per se so far, certainly not in a world of imperfect informed consumers (search costs). Where do independent mono-product firms locate and what product do they choose to produce remains as an open question in the real world.

“city”, the search cost for the characteristics of the products offered there is suddenly dramatically reduced.

All the specific features of the Internet retailing we incorporated in the model are necessary for obtaining the main intriguing result, which is the lower prices in the non-connex coalition (within which firms do not face direct competition) compared to the connex one (where firms compete against each other). When eliminating search costs within the coalitions, the possible gain of market shares against non-coalesced sites is therefore the main force that drives the lowering of prices. This force is dominating the increased competition among firms which is due to the frictionless price comparison. Actually, this result can be obtained because we made a distinction between the situation in which both neighbors, close in terms of product differentiation, are “far away” because of search costs (non-connex case) and the situation in which one of the two neighbors is also close from the search cost point of view (connex case). Obtaining the same type of topology with transportation costs as the main component of search cost remains to be done and we hardly see the empirical facts that could be matched with such situations: in the connex case, the firm must compete both with close substitutable firms in the same city and other ones spread alone in the countryside, in the non-connex case, the city shops are highly differentiated, each one competing with isolated shops out of the city.

Finally and may be still more interestingly, the model and its results presented should be seen as an inquiry device on the evolving landscape of the Web-retailing sector, while questioning the relevancy of the assumptions that have been voluntarily left out of the model. As long as tough competition for market shares on a fast growing market will occur, and new entries of individual sites are possible, we expect that price coordination will not hold within coalitions and we also expect that sites grouped in coalitions will go on competing with single sites. Consequently, we remain in the competition framework of the model. So we do not expect the emerging of a global coalition, nor of several big connex coalitions in those sectors that drive the development of e-retailing. The consequence is that an under-optimization of the search process for e-buyers should continue unless other mechanisms take place, such as:

- strong communities effect that alter the utility function of Web-users. These ones would then reject all the non-connex coalitions where potential buyers are too much different and therefore provide poor additional gains in terms of information exchanges, driving in turn connex sites to get together for

obtaining higher demand;

- search costs of Web-users that are dramatically altered by massive advertising or word of mouth about sites that are “must” in each sector and directly capture the e-buyer. The assumption of imperfectly informed consumers could also be progressively irrelevant with the maturation of the sector (few new entrants or landscape changes), a clever use of bookmarks and other learning effects.

Of course, with the maturation of the e-retailing and decreasing forces towards the challenge of market shares, the opportunity of price coordination and best taking advantage of that within connex coalitions could reverse the trend. If we add in the picture the costs of developing efficient search engines equipped with the right level of sophistication according to the size of the coalition, free riding on this common good is less likely to happen in a well organized coalition where discussions on prices would also take place. At this stage, it is still unclear how far our e-customer would be worse off with highly efficient search and higher prices.

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## 4 Annex

*Notations.* In order to compute the price equilibrium it is more suitable to use an equivalent strategic variable (half natural territories):  $\lambda_i$  corresponding to price  $\pi_i$  is set by a coalesced site;  $\lambda_{-i}$  is set by the other coalesced site (partner in the coalition);  $l_i$  corresponding to price  $p_i$  is set by a non-coalesced site;  $l_{-i}$  is set by the other non-coalesced site. We also define an auxiliary variable  $K = L/2 + 1/8$  in order to simplify the expressions at equilibrium. The first step of the demonstration consists in showing that in both the connex and non-connex case, price strategies corresponding to natural territories that exceed  $1/2$  (i.e. with  $l > 1/4$ ) are dominated. In order to compute the Nash equilibrium strategy of a coalesced site, we suppose that the strategies of the other sites are fixed and we search the best reply strategy of this site. We can remark that its profit for  $l \in [0, 1/4]$  depends on the configuration of the overlapping with the natural territories of its neighbors that determines



the probability to capture consumers, thus the expected demand. We can put in evidence three different intervals depending on whether there is an intersection with none, one or both neighbors' natural territories. In each of these intervals the expected demand is a continuous linear function of  $l$ , therefore the profit on each intervals is the restriction of a continuous and concave function. As a result we can reduce the study of the best reply to finding the maximum of three points, each of them corresponding to the unique optimum of the profit function on each interval. For each coalition structure (connex or non-connex) there are many cases to be treated depending on how the strategies of the remaining sites are fixed. The method consist in treating them exhaustively.

In the **non-connex** case, the profit function is concave-like. In this case we know that the maximum of the profit function in  $[0, 1/4]$  is unique. Since the two coalesced sites face the same situation, they choose this unique optimal strategy. The same happens for the non-coalesced sites. Thus  $\lambda_i = \lambda_{-i} = \lambda$  and  $l_i = l_{-i} = l$ . The best reply functions verify  $\lambda^{BR}(l) = \max(1/4 - l, K - 1/16 + l/4)$  and  $l^{BR}(\lambda) = \max(1/4 - \lambda, K - 1/4 + \lambda)$ . Solving this system shows that if  $K \geq 5/26$ , there is a *unique symmetric equilibrium* in which the coalesced sites have a larger natural territory than the non-coalesced sites:  $l_i = l_{-i} = 8K/3 - 5/12$  and  $\lambda_i = \lambda_{-i} = 5K/3 - 1/6$ . And if  $K < 5/26$  there is a *multiplicity of symmetric equilibria* that correspond to adjacent territories (sites avoid to intersect and to price out consumers) given by:  $l_i = l_{-i} \in [K/2, 1/4 - 4K/5]$  and  $\lambda_i = \lambda_{-i} = 1/4 - l_i$ . By applying the bijection  $l = L + (p^* - p)/t$ , we find the price equilibrium and the definition of  $K$  gives  $\tilde{\varepsilon} = 1/104$ .

In the **connex** case, let us identify a player with its strategic variable. When we consider a coalesced site  $\lambda_i$ , we denote by  $\lambda_{-i}$  its neighbor and partner in the coalition, and by  $l_i$  its non-coalesced neighbor. In the same way, when we consider a non-coalesced site, we denote it as  $l_i$ , and its neighbors as  $\lambda_i$  and  $l_{-i}$ . We compute the three candidates for optimality and we eliminate strongly dominated strategies in the different configurations of natural territories ( $\lambda_{-i} \geq l_i$ ,  $\lambda_{-i} \leq l_i$ ,  $l_{-i} \geq \lambda_i$ ,  $l_{-i} \leq \lambda_i$ ). We find that the Nash equilibrium must verify simultaneously the two systems:

$$(E) \begin{cases} \lambda_i = K - 5/56 + 3\lambda_{-i}/14 + l_i/7 \\ \lambda_{-i} = \max(K - 5/56 + 3\lambda_i/14 + l_{-i}/7, 1/4 - l_{-i}) \\ l_i = \max(K - 7/40 + 3l_{-i}/10 + 2\lambda_i/5, 1/4 - l_{-i}) \\ l_{-i} = \max(K - 7/40 + 3l_i/10 + 2\lambda_{-i}/5, 1/4 - l_i) \end{cases} \quad (I) \begin{cases} \lambda_i > 1/8 \\ \lambda_{-i} > 1/8 \\ l_i \geq 1/8 \end{cases}$$

We check all the possible cases by replacing each expression containing a “max” by an equality with one of the two terms and the corresponding inequality. More precisely, in case  $k$ ,  $(E)$  will be replaced by a linear system  $(E_k)$  of equalities containing no “max” and a system of inequalities  $(I_k)$ , and the Nash equilibria verify simultaneously  $(E_k)$ ,  $(I_k)$  and  $(I)$ .

*Case 1:* we suppose that  $\lambda_{-i} = K - 5/56 + 3\lambda_i/14 + l_{-i}/7$ , and  $l_i = K - 7/40 + 3l_{-i}/10 + 2\lambda_i/5$  and  $l_{-i} = K - 7/40 + 3l_i/10 + 2\lambda_{-i}/5$ . These assumptions impose the inequalities  $K - 5/56 + 3\lambda_i/14 + l_{-i}/7 > 1/4 - l_{-i}$ ,  $K - 7/40 + 3l_{-i}/10 + 2\lambda_i/5 > 1/4 - l_{-i}$ , and  $K - 7/40 + 3l_i/10 + 2\lambda_{-i}/5 > 1/4 - l_i$ . The set of solution is non-empty and *if*  $K \in [3/16 + 7/664, 1/4]$ , there is a *unique equilibrium* in which the coalesced sites have a larger natural territory than the non-coalesced sites, defined by:  $l_i = l_{-i} = 166K/69 - 97/276$  and  $\lambda_i = \lambda_{-i} = 118K/69 - 49/276$ .

*Case 2:* we suppose  $\lambda_{-i} = K - 5/56 + 3\lambda_i/14 + l_{-i}/7$ , and  $l_i = 1/4 - l_{-i}$ . The corresponding system has an infinity of solutions. More precisely *if*  $K \in [3/16, 3/16 + 7/664]$ , there is a *multiplicity of eventually non-symmetric equilibria*, (even though there exists a unique symmetric equilibrium), parameterized by  $\delta \in [0, 4471/18744 - 2822K/2343]$ : with  $\lambda_i = 14K/11 - 1/11 + 2\delta/17$ ,  $\lambda_{-i} = 14K/11 - 1/11 - 2\delta/17$ ,  $l_i = 1/8 + \delta$ , and  $l_{-i} = 1/8 - \delta$ .

*Remaining cases:* the set of solutions is empty in all the other cases.

By applying the bijection  $l = L + (p^* - p)/t$ , we find the price equilibrium and the definition of  $K$  gives  $\hat{\varepsilon} = 7/332$ .

### Static comparative of the coalition structures.

By comparing the profit of the coalesced sites depending on the coalition structure, it is now possible to decide which coalition structure is preferred by a site willing to initiate a coalition. We have shown that there always exists a symmetric equilibrium in both the connex and the non-connex case. This equilibrium is unique in the connex case. We provide the profit function at this equilibrium in each case in order to show that a non-connex coalition is preferred by the initiator of a coalition. It is easy to show that for the non-symmetric equilibria of the connex case, the non-connex coalition is preferable as well. In fact, the non-symmetric equilibria are close enough to the symmetric equilibrium and the profit function is almost the same.

In the non-connex case, for  $K < 5/26$ , we consider the values  $l = 5/52$  and  $\lambda = 1/4 - l = 2/13$ . The natural territories don't intersect. Thus, the profit function is given by:  $\Pi_{nc} = (2K - \lambda)2\lambda = 8K/13 - 8/169 \simeq$

$0.615K - 0.047$ . For  $K \geq 5/26$ , the unique equilibrium is given by  $l = 8K/3 - 5/12$  and  $\lambda = 5K/3 - 1/6$ . The natural territories intersect and the profit function is given by:  $\Pi_{nc} = (2K - \lambda)(2(1/4 - l) + 4/3(\lambda - 1/4 + l)) = 4/27(K^2 + K + 1/4) \simeq 0.148K^2 + 0.148K + 0.037$ .

In the connex case, for  $K < 3/16 + 7/664$ , we consider the unique symmetric equilibrium:  $l_i = l_{-i} = 1/8$ ,  $\lambda_i = \lambda_{-i} = 14K/11 - 1/11$ . The profit function is given by:  $\Pi_c = (2K - \lambda)(1/4 + 2/3(\lambda - 1/8)) = 56/363(4K^2 + K + 1/16) \simeq 0.617K^2 + 0.154K + 0.001$ . For  $K \geq 3/16 + 7/664$ , the unique equilibrium is given by  $l_i = l_{-i} = 166K/69 - 97/276$ ,  $\lambda_i = \lambda_{-i} = 118K/69 - 49/276$ . The profit function is given by:  $\Pi_c = (2K - \lambda)(1/8 + 1/4 - l + 2/3(\lambda - 1/4 + l)) = (1400K^2 + 1715K + 16807/32)/14283 \simeq 0.098K^2 + 0.120K + 0.036$ .

We see that for all  $K = L/2 + 1/8$ ,  $\Pi_c \leq \Pi_{nc}$ .