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Statistical modelling of turbulence and applications to Large-Eddy Simulation

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Introduction

The subject of this presentation is turbulence modelling and applications to Computational Fluid Dynamics (CFD). A special attention is paid to the spectral models of turbulence, also called statistical two-point closures. The main ideas underlying the derivation of the usual two-point closures are reviewed. The Direct Interaction Approximation (DIA) and related closures, such as the Eddy-Damped Quasi Normal Markovian (EDQNM) model or the Closure Based on Fluid Particle Displacements (CBFPD), are briefly presented. Particular emphasis is given to the application of closures to high Reynolds number turbulence. The relations between two-point closures and other types of turbulence models, in particular subgrid models for Large-Eddy Simulations (LES), are discussed. It is stressed that two-point closures constitute useful tools to understand the interaction between large and small scales that is essential for the problem of subgrid modeling in LES.

General overview of the two-point closure approach

Two-point closures, also called spectral models or analytical theories of turbulence, are statistical approaches. They are based on ensemble averages and the Reynolds decomposition. In this respect, they can be considered to belong to the family of RANS models (Reynolds Averaged Navier Stokes). One might further categorize them as "sophisticated" RANS models. On the other hand, two-point closures can also claim the status of "theories" in the sense that they are self-consistent approaches free of ad hoc parameters or empirical constants that must be determined by comparing to experiments. They are based on the Navier Stokes equations and an acceptable number of reasonable assumptions. They can be regarded as expansions about Gaussianity of the probability density function of the velocity fluctuation (Kraichnan^[1]).

Two-point closures were proposed as early as in the late 50's. As far as isotropic turbulence is concerned, most of the results of closures were obtained in the 60's or 70's (see for example Lesieur and Schertzer^[2]). From the theoretical point of view, the LHDIA (Kraichnan^[3]) appears as the major achievement in the closure strategy and the limitations of closures are well identified: they fail to take internal intermittency into account. It has to be stressed that closures have nevertheless contributed considerably, even recently, to our understanding of

turbulence. This is particularly true in the case of high Reynolds number turbulence or in the case of anisotropic turbulence and its interaction with "external" effects such as the presence of a mean shear or a solid body rotation. The oral presentation will illustrate this aspect of closures.

Basic quantities and guidelines for closure derivation

One characteristic of two-point closures is that their precise formulation generally requires rather heavy and intricate equations. It is beyond the scope of the present paper to reproduce equations that can nowadays be found readily in textbooks (Leslie^[4]), (Lesieur^[5]), (Mathieu and Scott^[6]).

The basic object in the two-point description of turbulence is the velocity correlation at two points. In the case of single-time closures like EDQNM (Orszag^[7]), this correlation involves only velocity fluctuations at one time, whereas for two-time theories such as DIA, the correlation has to be defined at two times. In the case of homogeneous turbulence, two-point correlations are functions of a single vector: the separation between the two points. It is then convenient to work in the spectral space, Fourier transforming the equations.

In the Direct Interaction Approximation (Kraichnan^[8]), a second important quantity is introduced: the response function, defined as the response at time t to a perturbation introduced at t' . The closed set of equations is not reproduced here. It is a set of integro-differential equations for the two-point two-time correlations and the response function.

It is known that the DIA equations are not invariant under a random Galilean transformation, and that, as a consequence, they do not predict the $K^{-5/3}$ Kolmogorov scaling for the turbulent kinetic energy spectrum in the inertial range. They lead to a spurious $K^{-3/2}$ spectrum instead. It is beyond the scope of the present paper to go into the details of the Lagrangian History version of the Direct Interaction Approximation (LHDIA), but it must be stressed that recasting the theory in a properly Galilean invariant framework led Kraichnan^[3] to propose a theory that has the advantage of predicting both the $K^{-5/3}$ inertial range and an acceptable value of the associated Kolmogorov constant.

One-time two-point theories are simpler and require less computational effort to be numerically integrated. Their most popular one-time closure is the Eddy Damped Quasi-Normal Markovian model (EDQNM, Orszag^[7]).

There are two different ways to derive the EDQNM model. It can be presented as a simplification of a two-time DIA formalism in which one simply assumes the two-time correlation to be an exponentially decreasing function of the time separation. It can also be obtained directly; by working with one-time quantities only.

The equations of the EDQNM model are here recalled in the case of isotropic turbulence where their formulation remains simple. Isotropy leads to a description in terms of the turbulent kinetic energy spectrum as a function of wave-number alone: $E(K)$. $E(K)$ is the well known turbulent kinetic energy spectrum, which is governed by the Lin equation:

$$\frac{\partial E(K,t)}{\partial t} = -2\nu K^2 E(K,t) + T(K,t) \quad (1)$$

in which T is the non linear transfer term whose expression depends on the closure. In the case of the EDQNM model, T is given by:

$$T(K,t) = \int_{\Delta(k)} \theta_{K P Q} \frac{xy + z^3}{Q} E(Q,t) \{K^2 E(P,t) - P^2 E(K,t)\} dP dQ \quad (2)$$

in which the integral is over all the triads such as $\bar{K} \bar{P} \bar{Q}$ can form a triangle. x , y and z are coefficients depending on the geometry of the triad.

$$\theta_{K P Q} \propto \frac{1}{\mu} \quad (3)$$

is a spectral time scale inversely proportional to the damping term μ .

The EDQNM model requires the heuristic specification the spectral time scale, or of the damping term. The damping is specified to be compatible with the existence of a Kolmogorov inertial range in the energy spectrum. It has the signification of a damping of the triple correlation in the direct derivation of EDQNM, whereas, when deducing EDQNM from DIA, it appears as the inverse of the time scale associated with the two-time correlation. Different formulations of the damping coefficient compatible with the Kolmogorov scaling can be used, but in any case, the prefactor in the damping formulation remains an adjustable constant. It has to be expressed heuristically in order to lead to a satisfactory value of the Kolmogorov constant. In that respect, EDQNM is not a self-consistent theory: the existence of a $K^{-5/3}$ inertial range and the value of the Kolmogorov constant are not predicted by EDQNM. They are injected as assumptions in the model instead.

Recently Bos and Bertoglio^[9] have proposed a new single-time two-point closure that is no longer relying on the introduction of heuristic assumptions. This new theory, Closure Based on Fluid Particle Displacement (CBFPD), leads to equations that are very similar to the ones corresponding to the EDQNM model (equations (1) and (2) are still obtained). Only the specification of the damping coefficient, μ , differs. The CBFPD derivation takes advantage of the analogy between particle displacement and scalar fluctuation in isotropic

turbulence subjected to a mean scalar gradient. Then, extending the closure treatment to this auxiliary scalar field, a Lagrangian time-scale can be evaluated and used to specify μ (see Bos and Bertoglio^[9]).

Examples of results for isotropic turbulence

An example of results obtained with the CBFPD closure in the case of decaying isotropic turbulence is given in Figs. 1 and 2. Spectra are shown at $Re_\lambda=150$, 500 and 1500. The results in Fig. 1 show that a $K^{-5/3}$ inertial range is obtained for the energy spectrum. The resulting value of the Kolmogorov constant is 1.73, as can be observed in Fig. 2 where the spectra are shown in compensated form. It has to be reminded that in the case of the EDQNM closure, this value is not a prediction of the model but has to be specified by a proper choice of an ad-hoc constant.

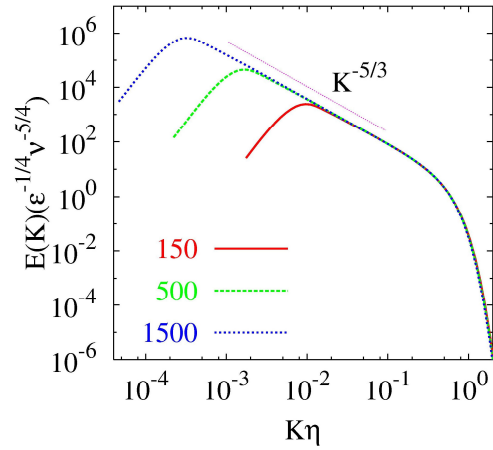


Fig. 1 Energy spectra obtained with the CBFPD closure, for different Reynolds numbers.

Reynolds number effects: closures and scalings

When closures were first proposed and applied, in the 50's and 60's, there was no alternative to predict the details of the behavior of turbulence, and in particular of the turbulent spectrum. Obviously the situation has drastically changed, and today most of the situations to which closures can be applied (homogeneous turbulence for example) can easily be treated with Direct Numerical Simulation. Coding the equations requires less effort for the Navier Stokes equation than for the integro-differential set of equations resulting from theories like DIA, and the results can also be considered free of any uncertainty since no approximations are made. The interest of closures nowadays is related to the fact that they can be applied to high Reynolds number turbulence, which is not the case for DNS. The strategy is then to apply a closure to a given situation, validate the results of the closure at small or moderate Reynolds numbers by comparisons with DNS, and then use the closure to explore the high Reynolds number domain.

One has also to mention that closures can be helpful in suggesting the relevant parameters on which to build theoretical scaling. Often, the analytical formulations of closures naturally provide a route to identifying the

relevant parameters.

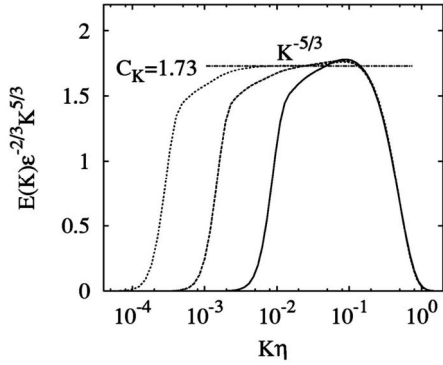


Fig. 2 Compensated energy spectra obtained with the CBFDP closure, for different Reynolds numbers; the position of the horizontal line corresponds to the predicted value of the Kolmogorov constant.

An example of how closures can be used to explore the effect of the Reynolds number is provided by the recent study of Bos et al^[10]. In this study, the problem that is addressed is the behavior of the so-called normalized dissipation rate of turbulence as a function of the Reynolds number, and whether or not this quantity tends to a universal value in the limit of very high Reynolds number. The normalized dissipation rate is defined as:

$$C_\varepsilon = \frac{\varepsilon L}{U^3} \quad (4)$$

in which ε is the viscous dissipation of turbulent kinetic energy, L is the integral length-scale and U stands for the root-mean-square velocity.

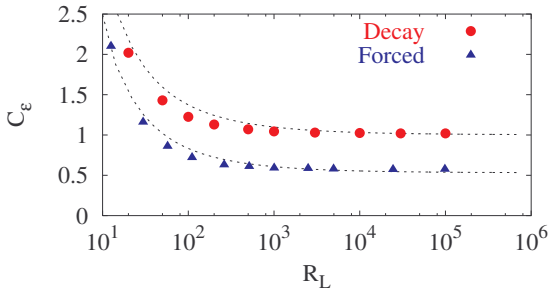


Fig. 3 Normalized dissipation rate as a function of the Reynolds number. Comparison between decaying isotropic turbulence and turbulence maintained statistically stationary by a forcing (EDQMN closure).

In Fig. 3 C_ε is plotted as a function of the Reynolds number for the case of decaying isotropic turbulence and for the case of turbulence maintained statistically stationary by injecting energy in the large scales (forced turbulence). It can clearly be seen that there is a large difference between the values of C_ε obtained in the two cases, and that this difference persists at high Reynolds number. In Fig. 4, the results of the EDQMN closure are compared with DNS and LES data. It is seen that the agreement with DNS is satisfactory at small Reynolds and that the closure accounts for the Reynolds number effects. It also appears that the closure permits to explore a much larger range of parameters than the DNS.

Other illustrations of the interest of two-point closures to explore high Reynolds number turbulence can

be mentioned (see for example Bos et al^[11] for the case of isotropic turbulence with a mean scalar gradient, or Fauchet and Bertoglio^[12] for compressible turbulence). Some significant results will be discussed during the oral presentation.

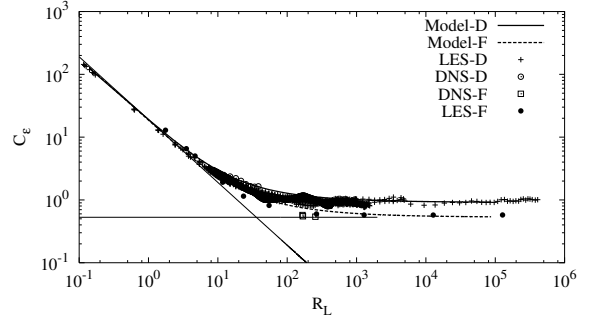


Fig. 4 Normalized dissipation rate as a function of the Reynolds number, for decaying isotropic turbulence and statistically stationary turbulence. Comparisons between results of the EDQMN closure, LES and DNS.

Homogeneous anisotropic turbulence is simple enough to be treated with two-point closures. It is however much richer than isotropic turbulence, since interesting practical effects, like turbulence production by mean shear, can be studied. Homogeneous anisotropic turbulence might well be the situation in which two-point closures have revealed the most useful results. Models like EDQMN have proven to lead to valuable insights on the interaction of turbulence with the mean flow. They also provide interesting information on terms that must be modeled at the level of simple engineering models, like the rapid part of the pressure strain correlation. Closures, such as EDQMN, combine the advantage of exactly accounting for the rapid terms, as Rapid Distortion Theory does, with taking into account the non-linear transfer of energy to the small scales in a quite realistic way. Since the energy transfer is present in the theory, there is no need to introduce an arbitrary equation for the dissipation rate as in one-point models. The advantage of two-point closures is clear : once the triple correlations are modeled, no other assumptions have to be made. The first attempts to extend the EDQMN closure to turbulence in the presence of a mean shear were made by Cambon et al^[13] and Bertoglio^[14]. Later, the EDQMN closure was revealed to be very helpful in the prediction and understanding of homogeneous turbulence subjected to solid body rotation (Cambon and Jacquin^[15]) and stable stratification (Staquet and Godeferd^[16]). Finally, the extensions of two-point models to inhomogeneous turbulence were addressed (see for example Parpais et al^[17]).

Application to subgrid models for Large-Eddy Simulation

Since two-point closures reproduce the cascade of energy to the small scale dissipative range of the spectrum that is missing in Large-Eddy Simulations, they were among the first tools used to model the subgrid terms. Kraichnan^[18] was the first to derive a subgrid model based on a two-point closure and to propose an

expression for a subgrid eddy viscosity. The most popular subgrid model deduced from a closure is certainly that of Chollet and Lesieur^[19], in which the subgrid viscosity is expressed as a function of the energy spectrum at the filter cut-off wave-number K_C :

$$\nu_t^0 = C \sqrt{\frac{E(K_C)}{K_C}} \quad (5)$$

in which C is a constant. The EDQNM closure shows that the subgrid viscosity is indeed a function of the wave-number:

$$\nu_t^0 = \nu_t^0 f(K/K_C) \quad (6)$$

but that f is approximatively constant and equal to unity over a large range of wave-numbers ("plateau"). Only in the vicinity of the cut-off does f increase ("cusp" effect) (an example of the spectral dependence of the subgrid viscosity is given in Kida and Goto^[20]). When applied to subgrid modeling, two-point theories also suggest the presence of "backscatter" from small scales to large scales. Using a stochastic model for EDQNM, Bertoglio^[21] proposed modeling the subgrid backscatter by adding a random force term to the classical subgrid viscosity term.

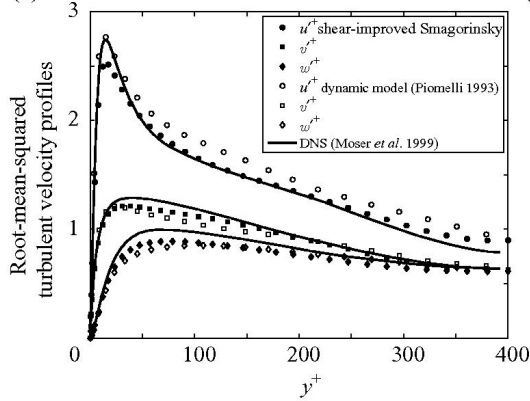


Fig. 5 Turbulent intensity profiles at $Re_\tau = 395$. Comparison between the new subgrid model and DNS and LES data obtained with the dynamic Smagorinsky model.

When a strong mean shear is present, as is the case near a wall, two-point closures show that the transfer of energy to the small scales results from two different and well identified mechanisms. The first one is the classical non-linear cascade, whereas the second one is a rapid contribution, directly related to the presence of mean velocity gradients. This observation suggests that the mean shear, S , must be present in the formulation of the subgrid viscosity in LES. A model along these lines was recently proposed by L ev eque et al^[22]. This model is an extension of the Smagorinsky model:

$$\nu_T = (C_s \Delta)^2 (|\bar{S}| - |\langle \bar{S} \rangle|) \quad (7)$$

When applied to the LES of a channel flow, it leads to satisfactory results, as appears in Fig. 5.

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