Discussion of "Equilibrium Near-Bed Concentration of Suspended Sediment" by Z. Cao

David Hurther, Ulrich Lemmin

EQUILIBRIUM NEAR-BED CONCENTRATION OF SUSPENDED SEDIMENT[a]

Discussion by D. Hurther[2] and U. Lemmin[3]

The author proposes an expression for the equilibrium near-bed concentration C_a based on scales of turbulent bursts. This is a significant improvement over existing empirical formulations (VanRijn 1984; Zyserman and Fredsoe 1994). He gives a relation for the bed sediment-entrainment function E in which the outer-scale law is used for the determination of the normalized turbulent bursting period. However, as pointed out by the author, a direct validation of the proposed near-bed sediment entrainment function was not possible, because quantitative data for sediment entrainment under different particle size and hydraulic conditions is lacking. Instead, the author has undertaken a calibration to determine the bursting parameter T_B^+/A_C based on existing data.

Recently, the discussers have investigated a method to estimate the dynamics of suspended sediment transport capacity from instantaneous mass flux profile measurements by applying a conditional sampling technique (Hurther and Lemmin, unpublished manuscript, 2000). Based on the measured bursting characteristics, the discussers will herein verify and discuss the author's expression of the near-bed equilibrium concentration.

The data set used to discuss these points was obtained by Cellino (1998) in open-channel laboratory experiments using the Acoustic Particle Flux Profiler (Shen and Lemmin 1999a). Detailed information concerning the experimental setup is given in Cellino (1998). Uniform suspension flows under capacity charge conditions were investigated as shown in Table 5.

In the entrainment flux equation [(12)], the author uses the bursting period estimated by conditionally sampling the instantaneous velocity field with a half-value shear stress threshold, as defined by Nezu and Nakagawa (1993). The discussers will refer to this sampling condition as ''NN50'' herein.

The author assumes an equilibrium condition [(17)] between the entrainment flux E [(12)] and the deposition flux D [(14)]. It is important to note that the entrainment flux E is conditionally sampled while the deposition flux D is not. As will be seen, this difference between the two terms has a number of important implications.

In writing (12), (14), and (17), the author assumes that the particle flux other than the vertical is negligible. Coherent structures are observed to be three-dimensional patterns (Grass et al. 1993) in which nonuniformities can exist. This implies that turbulent mass fluxes other than the vertical one may have to be considered in the diffusion equation.

In order to quantify the suspended sediment transport capacity of the flow structures related to the bursting phenomenon, the discussers investigated experimentally the order of magnitude of the different terms of the conditionally sampled sediment advection-diffusion equation, written as

$$\frac{\partial \langle c \rangle_{H_1}}{\partial z} = \frac{1}{\nu_0}\left[\frac{\partial \langle cu \rangle_{H_1}}{\partial x} + \frac{\partial \langle cw \rangle_{H_1}}{\partial z}\right] \tag{25}$$

where $\langle\ \rangle_{H_1}$ denotes a temporal average over instantaneous values, during which the instantaneous shear stress was greater than H_1 times the local mean shear stress value. The ''hole size'' H_1 is defined in Nezu and Nakagawa (1993). Since low sediment concentration situations were investigated, the effect of the suspension on the particle settling velocity was not considered.

The profiles of the first and second term of the right-hand side of (25) as functions of the selection criterion H_1 are presented in Fig. 10. Even for $H_1 = 3$, which corresponds to the NN50 shear stress level, the longitudinal gradient of the horizontal mass flux is obviously negligible compared with the vertical gradient of the vertical mass flux. The discussers' measurements show that a relation between the conditionally sampled vertical turbulent mass flux and the deposition flux can be considered as a good approximation, thus confirming the author's assumption.

However, in (17), the author implies that the total deposition

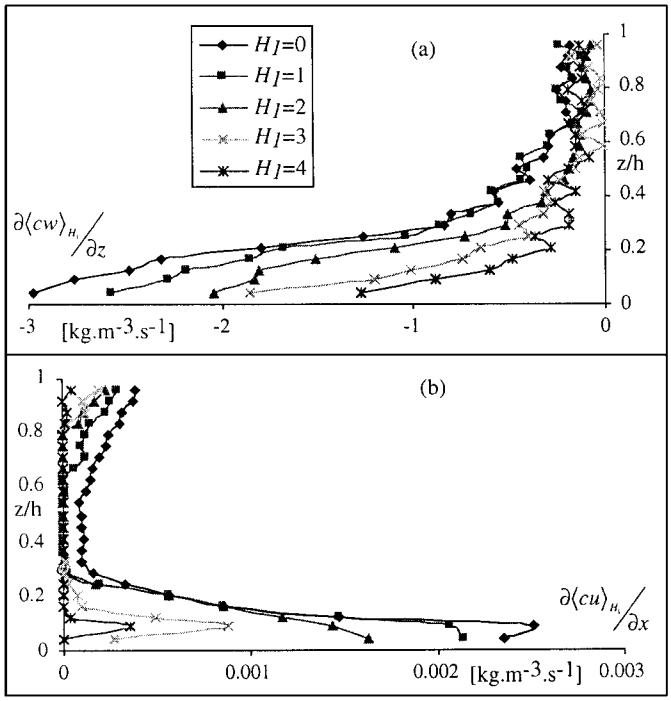

FIG. 10. Conditionally Sampled Terms of Sediment Diffusion Equation for Run Q50: (a) Vertical Mass Flux Gradient; (b) Horizontal Mass Flux Gradient

TABLE 5. Hydraulic Parameters for Experiments

Run	Q (m^3/s)	h (cm)	U (cm/s)	u_* (cm/s)	S ($\times 10^{-3}$)	R_h ($\times 10^3$)	F_h	k_S^+	d (mm)	ρ_S (kg/m^3)	ν_0 (mm/s)	θ_c ($\times 10^{-2}$)	θ ($\times 10^{-1}$)	C_a (kg/m^3)	T_b^+
Q50	0.058	12	80.1	3.9	1.00	274.3	0.74	8.9	0.23	2,650	21	3.5	4.07	21.31	3.4
Q55	0.060	12	83.3	4.4	1.50	285.5	0.77	10.1	0.23	2,650	21	3.5	5.19	28.07	3.6
Q57	0.060	12	83.6	4.5	1.75	286.4	0.77	9.9	0.23	2,650	21	3.5	5.42	24.77	3.5
Q60	0.061	12	85.0	4.9	2.00	291.1	0.78	10.3	0.23	2,650	21	3.5	6.43	23.29	3.6
Q65	0.062	12	86.5	5.1	2.25	296.4	0.80	10.8	0.23	2,650	21	3.5	6.97	34.36	3.7
Q70	0.057	12	86.8	5.4	2.50	297.4	0.80	11.0	0.23	2,650	21	3.5	7.81	33.83	3.8

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[2]Res. Asst., Lab. de Recherches Hydrauliques, Ecole Polytechnique Fédérale de Lausanne, CH-1015 Lausanne, Switzerland.

[3]Res. Assoc., Lab. de Recherches Hydrauliques, Ecole Polytechnique Fédérale de Lausanne, CH-1015 Lausanne, Switzerland.

flux at the equilibrium near-bed level is caused by the conditionally sampled entrainment due to bursts that are stronger than NN50. A priori, that hypothesis is not obvious, since there is no physical reason that weaker bursts should not contribute to the deposit. To justify this assumption it would be necessary to prove that the contribution of the weaker bursts is insignificant.

Therefore, the entrainment capacity of those flow structures which are delimited by the threshold level NN50 has to be quantified with respect to the total entrainment flux. In order to investigate this point, we start by calculating the dependence of different relative parameters on the threshold parameter H_1 at depth $z/h = 0.08$, which is close to the bottom but above the author's equilibrium depth. The parameters R1, R2, and T5 represent the mean shear stress contribution, the mean vertical mass flux, and the time fraction of the unselected flow part, respectively.

Concerning the mean contributions of shear stress and vertical mass flux, the relative vertical mass flux R2 is found to be lower than the relative shear stress contribution R1 (Fig. 11). As discussed in Hurther and Lemmin (unpublished manuscript, 2000), this difference originates from the decorrelation between downward mass fluxes and sweeps moving towards the wall, while ejection events are always highly correlated with ascendant mass flux events over the entire flow depth. This observation indicates that, in the vicinity of the bottom of the flow, the sediment entrainment may arise from pressure fluctuations at the bed, whereas the sediment resuspension process is highly correlated with shear stress ejection events. As indicated in Fig. 11, the NN50 sampling condition corresponds to a value of 3 for H_1. This is in good agreement with the results given by Nezu and Nakagawa (1993). The data in Fig. 11 demonstrate further that the entrainment at the near-bed level, related to those bursts that obey the NN50 condition as the burst delimiting value, corresponds to only 40% of total the vertical mass flux. It is interesting to note that this 40% entrainment occurs during only 10% of the total time. This is further discussed in Hurther and Lemmin (unpublished manuscript, 2000).

In order to determine the percentage of the total entrainment at the author's near-bed equilibrium depth, the discussers calculated the concentration profiles relative to the near-bed equilibrium concentration for H_1 equal to 3 corresponding to the NN50 sampling condition and for all runs (Fig. 12). The value of C_a was obtained from independent suction samples (Shen and Lemmin 1999b). Extrapolating our results to the near-bed equilibrium depth, only about 60% of the total entrainment is found at that level independent of the hydraulic conditions. Individual experiments scatter around this mean value, due to the very strong gradient of the curves in this depth range.

This is in contradiction to the author's assumption in (17).

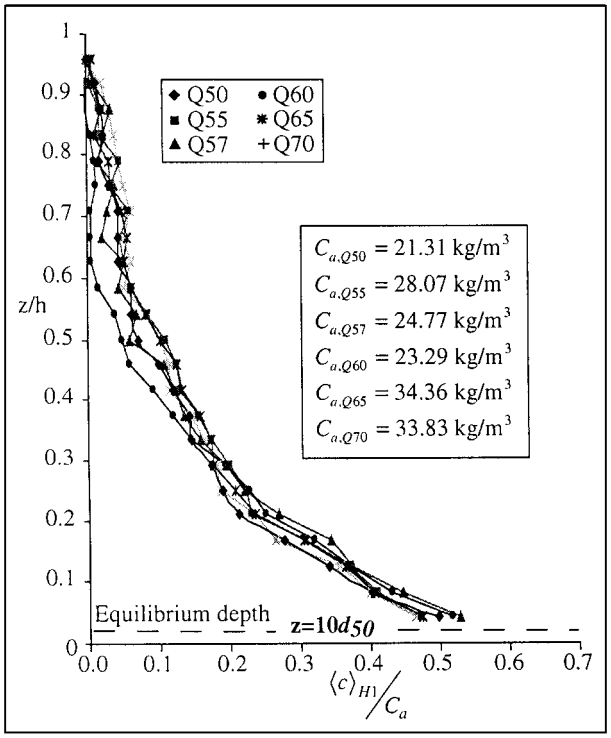

FIG. 12. Transport Capacity of Bursts Sampled at Shear Stress Half-Value Threshold Level $H_1 = 3$

Under the NN50 sampling condition that the author imposes, our measurements discussed above show that the corresponding deposition flux accounts for only 60% of the total deposition flux. As a consequence, based on the results of our measurements, the outer-scale formulation of the volumetric near-bed concentration for low sediment concentration [the author's (19)] should be corrected with a factor of 0.6 as follows:

$$0.6C_a \cong \frac{A_C}{T_B^+}\frac{C_0 d}{w_0 \theta_c}\frac{(\theta - \theta_c)U_\infty}{h} \tag{26}$$

Obviously, (18) should be corrected in the same way.

VALIDATION OF PROPOSED CORRECTION

As mentioned above, in the discussed paper, the parameter T_B^+/A_C has been determined through calibration. The correction factor of 0.6, which the discussers propose here in (26) and which shows that the condition $E = D$ is not correct, can only be determined through direct measurements of mass flux and the burst characteristics. Because of the wrong equilibrium assumption [(17)], the error is carried over into the author's verification procedure. Consequently, he did not observe a sig-

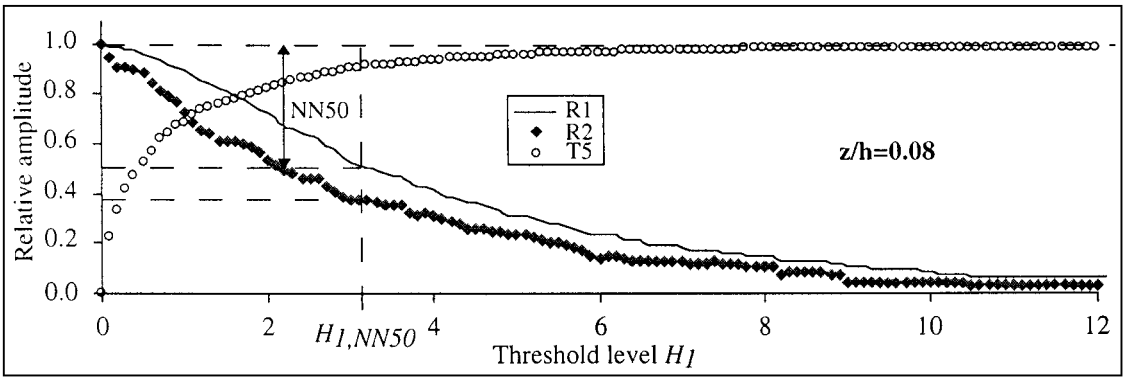

FIG. 11. Relative Shear Stress R1, Relative Vertical Mass Flux R2, and Time Fraction T5 of Unselected Events versus Threshold Value H_1 for Run Q50

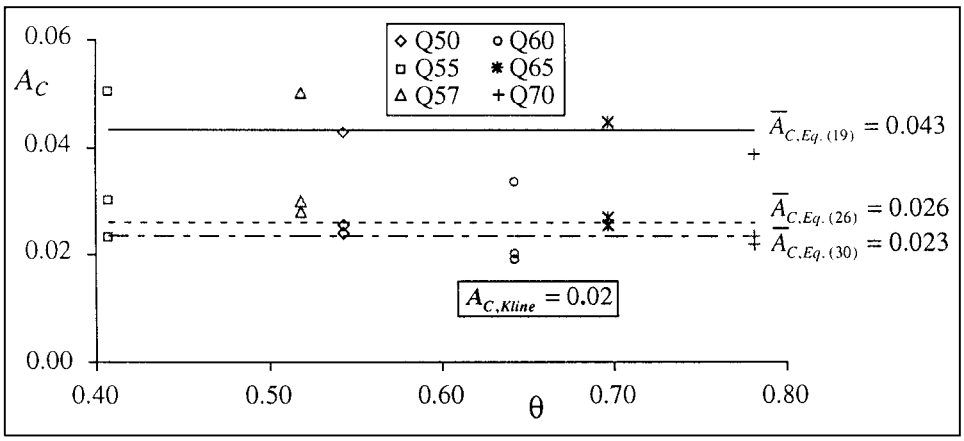

FIG. 13. Comparison of Surface Portion per Unit Area of Nursts versus Shields Parameter at Equilibrium Depth, Calculated with Cao's [Eq. (19)] Model and Herein Corrected Models [Eqs. (26) and (30)]

nificant discrepancy between the results from his model based on burst scales and the results from other models.

However, in order to determine whether the proposed correction factor provides a better representation of the physical processes, we can analyze the results of the author's calibration. Since the values of T_B^+/A_C obtained by (22*a*) are based on the calculations using (19), we can compare them directly to the parameter T_B^+/A_C determined with the herein proposed (26) for our data set.

First, the normalized bursting period T_B^+ is directly estimated from our measurements. For the results summarized in Table 5, a scaling on the outer flow variables was used, applying the author's method based on the NN50 condition. For all runs, we find a bursting period that varies between 3.4 and 3.8. This range is in good agreement with the experimental results previously given by Laufer and Narayanan (1971) and Jackson (1976).

As mentioned by Cao (1997), (26) can be written as follows:

$$0.6C_a \cong \frac{A_i}{T_B^+}\frac{C_0 d}{w_0}\frac{U_\infty}{h} \tag{27}$$

with

$$A_1 = A_C\frac{\theta - \theta_c}{\theta_c} \tag{28}$$

or

$$A_2 = A_C\frac{\theta}{\theta_c} \tag{29}$$

where A_i = mean surface portion of the bursts per unit bed area of those bursts containing enough energy to lift up sediments from the bed (expressed by the NN50 condition); and A_c = mean surface portion of all bursts per unit bed area. Cao (1997) proposed (28) and (29) to relate the last two parameters and found that (28) gives better results [i.e., the combination of (16) with (28) results in (26)]. If (29) is used instead of (28), the following expression is found for the mean near-bed equilibrium concentration:

$$0.6C_a \cong \frac{A_C}{T_B^+}\frac{C_0 d}{w_0 h}\frac{\theta U_\infty}{\theta_c} \tag{30}$$

The validation of the proposed corrections can be achieved by comparing the surface portion of the bursts per unit bed area, A_C, estimated with (19), (26), and (30). Based on the results of Kline et al. (1967) and Kim et al. (1971), Cao (1997) calculated a value of A_C roughly equal to 0.02.

Fig. 13 shows the values of A_C calculated with T_B^+ measured and using (19), (26), and (30) for all investigated runs. It can be seen that results using (30), which includes the correction, are in close agreement with the expected value of 0.02, while those obtained using (19) give a value of A_C that is significantly higher.

SUMMARY AND CONCLUSION

The present discussion is concerned with the formulation of the near-bed equilibrium concentration C_a developed by the author considering turbulent bursts scales in the near-wall flow region.

The discussers have used direct instantaneous mass flux measurements in several suspension flow conditions to verify different aspects of the author's analysis. We have shown that the conditionally sampled vertical entrainment flux is by far the dominant contributor to entrainment fluxes, in agreement with the author's assumption. However, when the sampling condition NN50 is applied to the vertical entrainment fluxes, the resulting entrainment flux is not in equilibrium with the total deposition flux as assumed in the author's (17).

Therefore, the discussers propose to introduce a correction factor in his near-bed equilibrium concentration formulation. Finally, the validity of our proposed correction has been checked by calculating the burst surface portion values for one particle size but for different hydraulic conditions. The corrected predictions are found to be in very good agreement with results given in the literature.

An important point in this context is whether the correction the discussers propose is universal. Here we have only worked with one particle size and with different hydraulic conditions. The author's analysis indicates that the parameter T_B^+/A_C is a strong function of particle size, particularly for small-sized particles. However, his Fig. 6(a) shows low scatter between the experimental results and the fitted curve based on (22*a*). From this observation, one may expect that the value of the correction factor will not vary too much. Obviously our data do not fit onto the curve in Fig. 6(a). Therefore, (22*a*) will also have to be modified in order to take into account the correction once more data for other particle sizes are available.

ACKNOWLEDGMENT

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Closure by Z. Cao[4]

The writer is grateful to the discussers for their interest in and discussion of the research reported in the paper.

The discussers first raised a question that the turbulent mass fluxes in addition to the vertical component may have to be considered due to the three-dimensional structure of coherent motions. Using their experimental datasets, they then confirm the vertical mass flux is by far the dominant. This conclusion has essentially been well known, as in most previous formulations for sediment transport the streamwise and spanwise fluxes are neglected (for steady uniform flows).

The major concern of the discussers appears to be related to the equilibrium condition, (17), used by this writer. In this connection, it is necessary to emphasize that in equilibrium sediment-laden flows [(17)] must be satisfied, characterizing the balance between the time-averaged entrainment and deposition fluxes. Otherwise, there will be aggradation or degradation of the channel bed, which is not the problem being considered. Therefore, the discussers' concern should be interpreted more appropriately, as whether the dominated role of bursting in entraining sediment can be justified. The discussers show that, at the near-bed level $z/h = 0.08$ (far above the equilibrium reference level used by this writer), turbulent bursting contributes only 40% of the total entrainment flux and suggest that pressure fluctuations near the bed may contribute to entrainment. Further, the discussers extrapolate the sediment concentration profiles to the writer's reference level $a = 10d$ (equivalent to $z/h = 0.0192$) and mention that the contribution to entrainment is only 60%. Based on this extrapolation, a correction factor of 0.6 is introduced in (26). Unfortunately, the discusser's extrapolation to the equilibrium reference level $z/h = 0.0192$ is clearly unreliable because of the highly variable gradient of sediment concentration closer to the bed as well as the apparent scatter of their concentration data. Thus, their correction factor is questionable. At best it is a conjecture, though this writer would confess that weaker bursts should lead to entrainment to a much less extent than energetic bursts. The evidence of fractional contribution of bursting to entrainment at a higher level $z/h = 0.08$ can only be understood to be the situation locally. Moreover, it is noted that the percentage of contribution of bursting to the total entrainment tends to increase closer to the bed, as observed by the discussers. This trend at least qualitatively supports the premise that turbulent bursting contributes most to entrainment closer to the bed.

Starting from their extrapolation-based correction factor, the discussers further show that the value of unit bursting area A_C can be rendered to be closer to 0.02 than was estimated by this writer based on turbulent bursting of single-phase flows (Cao 1997a). However, it must not be forgotten that the discussers' experiments are for capacity flows. The near-bed concentrations are higher than 0.008 in volume (i.e., 21.31 kg/m^3). In these cases, the interaction between the fluid and the dispersed particle phases is influential (Crowe et al. 1996). It is recognized that the bursting-particle interaction is as yet poorly known, which is one of the major reasons that this writer has chosen to calibrate the bursting variables T_B^+/A_C using existing measured datasets rather than directly using a value of 0.02 for A_C and a value around 5.0 for T_B^+. The discussers use the value of $A_C = 0.02$ for clear-water flows to back up their extrapolation for capacity flows, which to this writer is unbelievable.

Quantifying bed sediment entrainment continues to be one of the fundamental impediments to refined modeling of sediment transport in turbulent flows. The last four decades have seen enhanced understanding of the physics of sediment entrainment. Turbulent bursting has been experimentally found to play a central role in picking up sediment. This background forms the physical basis of the research, in which bed sediment entrainment has been linked to the time-averaged characteristics of bursting. To the best of this writer's knowledge, this is the first formulation of its kind in the context of fluvial sediment transport. It must, however, be pointed out that the development along this line has had to be based on current understanding of the process and is still in its infancy. High-quality measurement, although extremely difficult close to the bed, is evidently needed. The writer very much appreciates the discussers' viewpoints and would be happy to see their experimental findings relevant to this research.

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[4]Prof. and Dir., Inst. of Hydr. Res., Tianjin Univ., Tianjin 300072, China; presently, Visiting Prof., Facu. of Sci. and Engrg., Ritsumeikan Univ., Noji-Higashi 1-1-1, Kusatsu, Shiga 525-8577, Japan. E-mail: zcv98138@se.ritsumei.ac.jp