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Abstract

In this paper, we study a compound Korteweg-de Vries-Burgers equation with a higher-order nonlinearity. A class of solitary wave solutions is obtained by means of a series expansion.

1 Introduction

Consider either the a Korteweg-de Vries-Burgers-type equation of the following form:

$$u_t + \alpha u^p \, u_x + \beta \, u^{2\,p} \, u_x + \gamma \, u_{xx} + \mu \, u_{xxx} = 0 \tag{1}$$

where α, β, γ, μ and s are real constants, while p is a positive number, or, more generally:

$$u_t + P(u) \, u_x + \beta \, u^{2\,p} \, u_x + \gamma \, u_{xx} + \mu \, u_{xxx} = 0 \tag{2}$$

where P is a generalized polymon, i.e. of the form:

$$P(u) = \sum_{r \in \mathbb{R}^+} \alpha_r \, u^r \tag{3}$$

2 Traveling solitary wave solutions

Assume that equation (1) has the solution of the form:

$$u(x,t) = u(\zeta) \ , \ \zeta = x - v\,t \tag{4}$$

where v is the velocity.
Substituting it into (1) yields:

$$-v\,u'(\zeta) + \alpha u^p\,u'(\zeta) + \beta\,u^{2\,p}\,u'(\zeta) + \gamma\,u''(\zeta) + \mu\,u^{(3)}(\zeta) = 0 \tag{5}$$

Performing one integration, we have then:

$$-v\,u(\zeta) + \frac{\alpha}{p+1} u(\zeta)^{p+1} + \frac{\beta}{2p+1} u(\zeta)^{2p+1} + \gamma\,u'(\zeta) + \mu\,u^{(2)}(\zeta) + d = 0 \tag{6}$$

where d is an integration constant.
For sake of simplicity, we shall take $d = 0$.
Set:

$$a = \frac{\alpha}{\mu(p+1)} \ , \ b = \frac{\beta}{\mu(2p+1)} \ , \ c = \frac{\gamma}{\mu} \ , \ r = \frac{v}{\mu} \tag{7}$$

Equation (6) can thus be written as:

$$-r\,u(\zeta) + a\,u(\zeta)^{p+1} + b\,u(\zeta)^{2p+1} + c\,u'(\zeta) + u^{(2)}(\zeta) = 0 \tag{8}$$

Consider the surface S_u in the three-dimensional euclidean space:

$$-r\,X + a\,X^{p+1} + b\,X^{2p+1} + c\,Y + Z = 0 \tag{9}$$

u, u', $u^{(2)}$ are traced on this surface. The knowledge of a parametrization of this surface will thus lead to the determination of u.

2.1 The integer case

Contrary to previous works ([1]), there is no useful information available about the solutions or their profiles. Thus, we choose to search the solution as the following series expansion:

$$u(x,t) = \sum_{k=0}^{+\infty} U_k\,e^{k\,z} \tag{10}$$

where the $U_k's$ are constants to be determined.

Substitution of (10) into equation (1) leads to an equation of the following form:

$$\sum_{k=0}^{+\infty} P_k(U_k, a, b, c, r)\, e^{k\,z} = 0, \tag{11}$$

where the P_k ($k = 0$, ..., $+\infty$), are polynomial functions of the U_k and of a, b, c, r. The solution is obtained equating the P_k ($k = 0$, ..., , $+\infty$) to zero.

With the aid of mathematical softwares such as Mathematica, the previous system can be solved.

In the following, we present the solution obtained in the case $p = 1$, for $r = 1$, $a = 0.4$, $b = 0.01$, $c = 0.2$. The series (10) is truncated at $n = 3$:

$$u(x,t) = \sum_{k=0}^{3} U_k\, e^{k\,z} \tag{12}$$

The values of the coefficients are:

$$\begin{cases} B_0 &= -0.2523887531009444 \\ B1 &= 7.920512040580792 + 16.296799786819456\, i \\ B_2 &= 24.87642134042838 - 31.6589105912486\, i \\ B_3 &= -59.69562063336409 - 12.94860377480183\, i \end{cases}$$

Figure 1 displays the real part of the solitary wave solution.

2.2 The non-integer case

In the specific case where the real number p is not an integer, the exact determination of the traveling solitary wave solution of (1) becomes impossible. Yet, by means of numerical methods of surfaces reconstruction, one can approximate u. Also, plotting the surface S_u can yield interesting informations on u.

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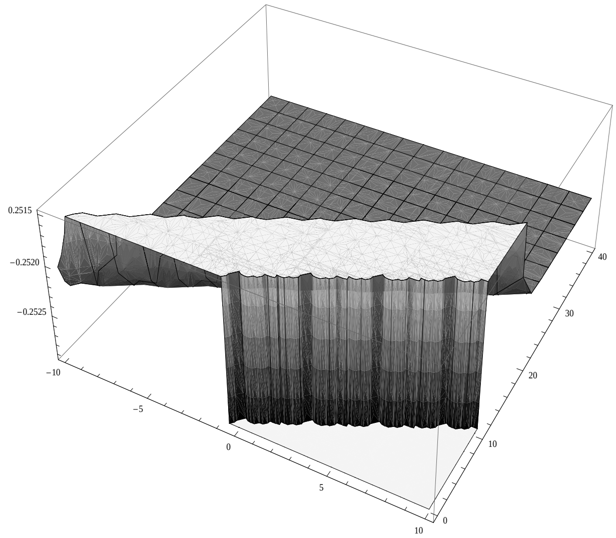

Figure 1: The real part of the solitary wave solution in the case surface in the case $p = 1$, for $r = 1$, $a = 0.4$, $b = 0.01$, $c = 0.2$.

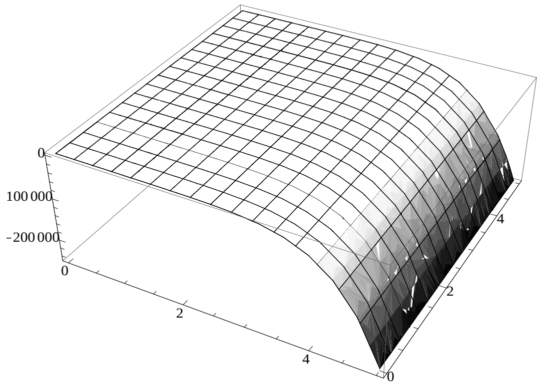

Figure 2: The surface in the case $a = 1$, $b = 2$, $c = 3$, $r = 1$, $p = \pi$.

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