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► **To cite this version:**

Claire David. A note on the Compound Burgers-Korteweg-de Vries Equation with higher-order nonlinearities and its traveling solitary waves. 2008. ⟨hal-00218330⟩

HAL Id: hal-00218330

<https://hal.science/hal-00218330v1>

Preprint submitted on 26 Jan 2008

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A note on the Compound Burgers-Korteweg-de Vries Equation with higher-order nonlinearities and its traveling solitary waves

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Abstract

In this paper, we study a compound Korteweg-de Vries-Burgers equation with a higher-order nonlinearity. A class of solitary wave solutions is obtained by means of a series expansion.

1 Introduction

Consider either the a Korteweg-de Vries-Burgers-type equation of the following form:

$$u_t + \alpha u^p u_x + \beta u^{2p} u_x + \gamma u_{xx} + \mu u_{xxx} = 0 \quad (1)$$

where α , β , γ , μ and s are real constants, while p is a positive number, or, more generally:

$$u_t + P(u) u_x + \beta u^{2p} u_x + \gamma u_{xx} + \mu u_{xxx} = 0 \quad (2)$$

where P is a generalized polynom, i.e. of the form:

$$P(u) = \sum_{r \in \mathbb{R}^+} \alpha_r u^r \quad (3)$$

2 Traveling solitary wave solutions

Assume that equation (1) has the solution of the form:

$$u(x, t) = u(\zeta) \quad , \quad \zeta = x - vt \quad (4)$$

where v is the velocity.

Substituting it into (1) yields:

$$-v u'(\zeta) + \alpha u^p u'(\zeta) + \beta u^{2p} u'(\zeta) + \gamma u''(\zeta) + \mu u^{(3)}(\zeta) = 0 \quad (5)$$

Performing one integration, we have then:

$$-v u(\zeta) + \frac{\alpha}{p+1} u(\zeta)^{p+1} + \frac{\beta}{2p+1} u(\zeta)^{2p+1} + \gamma u'(\zeta) + \mu u^{(2)}(\zeta) + d = 0 \quad (6)$$

where d is an integration constant.

For sake of simplicity, we shall take $d = 0$.

Set:

$$a = \frac{\alpha}{\mu(p+1)} \quad , \quad b = \frac{\beta}{\mu(2p+1)} \quad , \quad c = \frac{\gamma}{\mu} \quad , \quad r = \frac{v}{\mu} \quad (7)$$

Equation (6) can thus be written as:

$$-r u(\zeta) + a u(\zeta)^{p+1} + b u(\zeta)^{2p+1} + c u'(\zeta) + u^{(2)}(\zeta) = 0 \quad (8)$$

Consider the surface S_u in the three-dimensional euclidean space:

$$-r X + a X^{p+1} + b X^{2p+1} + c Y + Z = 0 \quad (9)$$

$u, u', u^{(2)}$ are traced on this surface. The knowledge of a parametrization of this surface will thus lead to the determination of u .

2.1 The integer case

Contrary to previous works ([1]), there is no useful information available about the solutions or their profiles. Thus, we choose to search the solution as the following series expansion:

$$u(x, t) = \sum_{k=0}^{+\infty} U_k e^{kz} \quad (10)$$

where the U_k 's are constants to be determined.

Substitution of (10) into equation (1) leads to an equation of the following form:

$$\sum_{k=0}^{+\infty} P_k(U_k, a, b, c, r) e^{kz} = 0, \quad (11)$$

where the P_k ($k = 0, \dots, +\infty$), are polynomial functions of the U_k and of a, b, c, r . The solution is obtained equating the P_k ($k = 0, \dots, +\infty$) to zero.

With the aid of mathematical softwares such as Mathematica, the previous system can be solved.

In the following, we present the solution obtained in the case $p = 1$, for $r = 1$, $a = 0.4$, $b = 0.01$, $c = 0.2$. The series (10) is truncated at $n = 3$:

$$u(x, t) = \sum_{k=0}^3 U_k e^{kz} \quad (12)$$

The values of the coefficients are:

$$\begin{cases} B_0 = & -0.2523887531009444 \\ B_1 = & 7.920512040580792 + 16.296799786819456 i \\ B_2 = & 24.87642134042838 - 31.6589105912486 i \\ B_3 = & -59.69562063336409 - 12.94860377480183 i \end{cases}$$

Figure 1 displays the real part of the solitary wave solution.

2.2 The non-integer case

In the specific case where the real number p is not an integer, the exact determination of the traveling solitary wave solution of (1) becomes impossible. Yet, by means of numerical methods of surfaces reconstruction, one can approximate u . Also, plotting the surface S_u can yield interesting informations on u .

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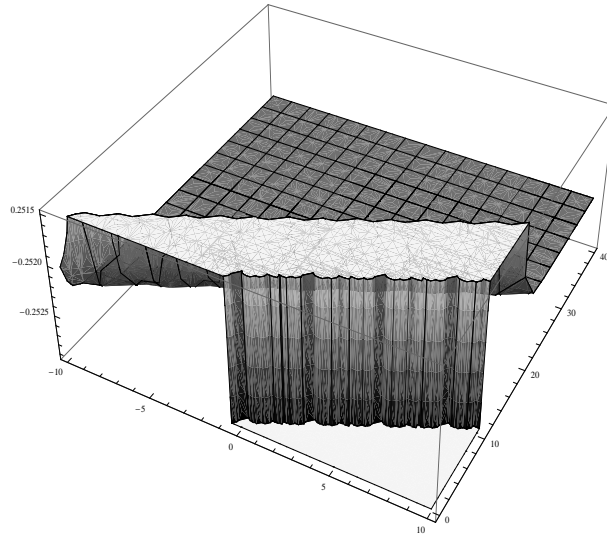


Figure 1: The real part of the solitary wave solution in the case surface in the case $p = 1$, for $r = 1$, $a = 0.4$, $b = 0.01$, $c = 0.2$.

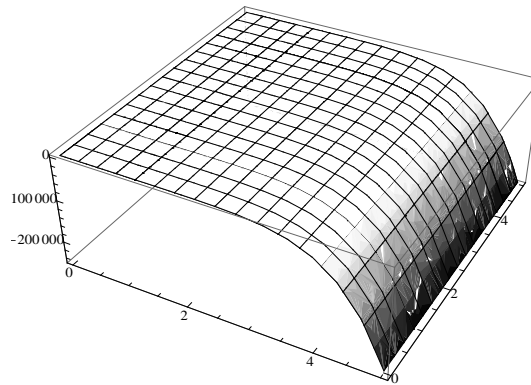


Figure 2: The surface in the case $a = 1$, $b = 2$, $c = 3$, $r = 1$, $p = \pi$.

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