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Seesaw and noncommutative geometry

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Abstract

The 1-loop corrections to the seesaw mechanism in the noncommutative standard model are computed. Other consequences of the Lorentzian signature in the inner space are summarised.

dedicated to Alain Connes on the occasion of his 60th birthday

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Understanding the origin of the standard model is currently one of most challenging issues in high energy physics. Indeed, despite its experimental successes, it is fair to say that its structure remains a mystery. Moreover, a better understanding of its structure would provide us with a precious clue towards its possible extensions.

This can be achieved in the framework of noncommutative geometry [1], which is a branch of mathematics pioneered by Alain Connes, aiming at a generalisation of geometrical ideas to spaces whose coordinates fail to commute. The physical interpretation of the spectral action principle [2] and its confrontation with present-day experiment still require some contact with the low energy physics. This follows from the standard Wilsonian renormalization group idea. The spectral action provides us with a bare action supposed to be valid at a very high energy of the order of the unification scale. Then, evolving down to the electroweak scale yields the effective low energy physics. This line of thought is very similar to the one adopted in grand unified theories. Indeed, in a certain sense models based on non commutative geometry can be considered as alternatives to grand unification that do not imply proton decay.

Ten years after its discovery [2], the spectral action has recently received new impetus [3, 4, 5] by allowing a Lorentzian signature in the internal space. This mild modification has three consequences: (i) the fermion-doubling problem [6] is solved elegantly, (ii) Majorana masses and consequently the popular seesaw mechanism are allowed for, (iii) the Majorana masses in turn decouple the Planck mass from the W mass. Furthermore, Chamseddine, Connes & Marcolli point out an additional constraint on the coupling constants tying the sum of all Yukawa couplings squared to the weak gauge coupling squared. This relation already holds for Euclidean internal spaces [7].

1 Constraints on gauge groups and representations

There are two ways to extract the gauge group G from the spectral triple. The first way defines the gauge group to be the unimodular (i.e. of unit determinant) unitary group [8, 5] of the associative algebra, which by the faithful representation immediately acts on the Hilbert space. The second way follows general relativity whose invariance group is the group of diffeomorphisms of M (general coordinate transformations). In the algebraic formulation this is the group of algebra automorphisms. Indeed $\mathrm{Aut}(\mathcal{C}^\infty(M)) = \mathrm{Diff}(M)$. We still have to lift the diffeomorphisms to the Hilbert space. This lift is double-valued and its image is the semi-direct product of the diffeomorphism group with the local spin group [9]. Up to possible central $U(1)$s and their central charges, the two approaches coincide.

There are other constraints on the fermionic representations coming from the axioms of the spectral triple. They are conveniently captured in Krajewski diagrams which classify all possible finite dimensional spectral triples [10]. They do for spectral triples what the Dynkin and weight diagrams do for groups and representations. Figure 1 shows the Krajewski diagram of the standard model in Lorentzian signature with one generation of fermions including a right-handed neutrino.

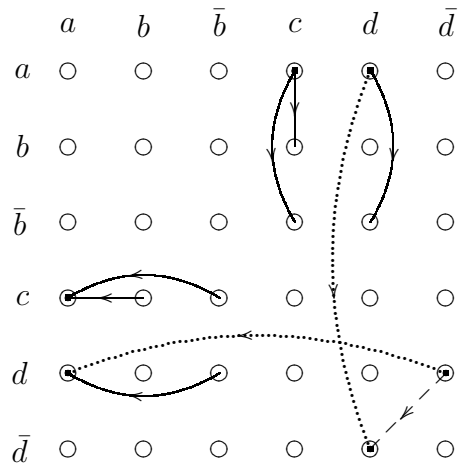

Figure 1: Krajewski diagram of the standard model with right-handed neutrino and Majorana-mass term depicted by the dashed arrow.

Certainly the most restrictive constraint on the discrete parameters concerns the scalar representation. In the Yang-Mills-Higgs ansatz it is an arbitrary input. In the almost commutative setting it is *computed* from the data of the inner spectral triple.

Here we present the inner triple of the standard model with one generation of fermions including a right-handed neutrino. The algebra has four summands: $\mathcal{A} = \mathbb{H}\oplus\mathbb{C}\oplus M_3(\mathbb{C})\oplus \mathbb{C} \ni (a,b,c,d)$, the Hilbert space is 32-dimensional and carries the faithful repesentation $\rho(a,b,c,d) := \rho_L \oplus \rho_R \oplus \bar{\rho}^c_L \oplus \bar{\rho}^c_R$ with

$$\begin{aligned} &\rho_L(a) := a \otimes 1_3 \oplus a,\ \ \rho_R(b,d) := b1_3 \oplus \bar{b}1_3 \oplus b \oplus d, \\ &\rho^c_L(c,d) := 1_2 \otimes c \oplus \bar{d}1_2,\ \ \rho^c_R(c,d) := c \oplus c \oplus \bar{d} \oplus \bar{d}. \end{aligned} \tag{1}$$

The Dirac operator reads

$$\mathcal{D} = \begin{pmatrix} 0 & \mathcal{M} & 0 & 0 \\ \mathcal{M}^* & 0 & 0 & S \\ 0 & 0 & 0 & \bar{\mathcal{M}} \\ 0 & S^* & \bar{\mathcal{M}}^* & 0 \end{pmatrix}, \quad S = \begin{pmatrix} M_M & 0 \\ 0 & 0 \end{pmatrix} \tag{2}$$

where S contains the Majorana mass for the right-handed neutrino and $\mathcal{M}$ contains the Dirac masses

$$\mathcal{M} = \begin{pmatrix} M_u & 0 \\ 0 & M_d \end{pmatrix} \otimes 1_3 \oplus \begin{pmatrix} M_\nu & 0 \\ 0 & M_e \end{pmatrix}. \tag{3}$$

This model is conform with the standard formulation of the axiom of Poincaré duality as stated in [1]. It seems to be closely related to the older bi-module approach of the Connes-Lott model [11] which also exhibits two copies of the complex numbers in the algebra. In the original version of the standard model with right-handed Majorana neutrinos Chamseddine, Connes & Marcoli [5] used an alternative spectral triple based

on the algebra $\mathcal{A} = \mathbb{H} \oplus \mathbb{C} \oplus M_3(\mathbb{C})$. But this spectral triple requires a subtle change in the formulation of the Poincaré duality, i.e. it needs two elements to generate KO-homology as a module over K_0. It should however be noted that right-handed neutrinos that allow for Majorana-masses always fail to fulfil the axiom of orientability [12] since the representation of the algebra does not allow to construct a Hochschild-cycle reproducing the chirality operator. For this reason we have drawn the arrows connected to the right-handed neutrinos with broken lines in the Krajewski diagram.

2 Constraints on dimensionless couplings

The spectral action counts the number of eigenvalues of the Dirac operator whose absolute values are less than the energy cut-off Λ. On the input side its continuous parameters are: this cut-off, three positive parameters in the cut-off functions and the parameters of the inner Dirac operator, i.e. fermion masses and mixing angles. On the output side we have: the cosmological constant, Newton's constant, the gauge and the Higgs couplings. Therefore there are constraints, which for the standard model with N generations and three colours read:

$$\frac{5}{3}\, g_1^2 = g_2^2 = g_3^2 = \frac{3}{N}\,\frac{Y_2^2}{H}\,\frac{\lambda}{24} \;=\; \frac{3}{4N}\, Y_2\,. \tag{4}$$

Here Y_2 is the sum of all Yukawa couplings g_f squared, H is the sum of all Yukawa couplings raised to the fourth power. Our normalisations are: $m_f = \sqrt{2}\,(g_f/g_2)\, m_W$, $(1/2)\,(\partial\varphi)^2 + (\lambda/24)\,\varphi^4$. If we define the gauge group by lifted automorphisms rather than unimodular unitairies, then we get an ambiguity parameterized by the central charges. This ambiguity leaves the $U(1)$ coupling g_1 unconstrained and therefore kills the first of the four constraints (4).

Note that the noncommutative constraints (4) are different from Veltman's condition [13], which in our normalisation reads: $\frac{3}{4}g_2^2 + \frac{1}{4}g_1^2 + \frac{1}{3}\lambda - 2g_t^2 = 0$.

Of course the constraints (4) are not stable under the renormalisation group flow and as in grand unified theories we can only interpret them at an extremely high unification energy Λ. But in order to compute the evolution of the couplings between our energies and Λ we must resort to the daring hypothesis of the big desert, popular since grand unification. It says: above presently explored energies and up to Λ no more new particle, no more new forces exist, with the exception of the Higgs, and that all couplings evolve without leaving the perturbative domain. In particular the Higgs self-coupling λ must remain positif. In grand unified theories one believes that new particles exist with masses of the order of Λ, the leptoquarks. They mediate proton decay and stabilize the constraints between the gauge couplings by a bigger group. In the noncommutative approach we believe that at the energy Λ the noncommutative character of space-time ceases to be negligible. The ensuing uncertainty relation in space-time might cure the short distance divergencies and thereby stabilize the constraints. Indeed Grosse & Wulkenhaar have an example of a scalar field theory on a noncommutative space-time whose β-function vanishes to all orders [14].

Let us now use the one-loop β-functions of the standard model with $N = 3$ generations to evolve the constraints (4) from $E = \Lambda$ down to our energies $E = m_Z$. We set: $t := \ln(E/m_Z)$, $\quad \mathrm{d}g/\mathrm{d}t =: \beta_g$, $\quad \kappa := (4\pi)^{-2}$. We will neglect all fermion masses below the top mass and also neglect threshold effects. We admit a Dirac mass m_D for the τ neutrino induced by spontaneous symmetry breaking and take this mass of the order of the top mass. We also admit a Majorana mass m_M for the right-handed τ neutrino. Since this mass is gauge invariant it is natural to take it of the order of Λ. Then we get two physical masses for the τ neutrino: one is tiny, $m_\ell \sim m_D^2/m_M$, the other is huge, $m_r \sim m_M$. This is the popular seesaw mechanism. The renormalisation of these masses is well-known [15]. By the Appelquist-Carazzone decoupling theorem we distinguish two energy domains: $E > m_M$ and $E < m_M$. In the latter, the Yukawa coupling of the τ neutrino drops out of the β-functions and is replaced by an effective coupling

$$k = 2\,\frac{g_\nu^2}{m_M}\,, \quad \text{at } E = m_M. \tag{5}$$

At high energies, $E > m_M$, the β-functions are [16, 17]:

$$\beta_{g_i} \;=\; \kappa b_i g_i^3, \quad b_i = \left(\tfrac{20}{9}N + \tfrac{1}{6}, -\tfrac{22}{3} + \tfrac{4}{3}N + \tfrac{1}{6}, -11 + \tfrac{4}{3}N\right), \tag{6}$$

$$\beta_t \;=\; \kappa\left[-\sum_i c_i^u g_i^2 + Y_2 + \frac{3}{2}\,g_t^2\right] g_t, \quad \beta_\nu \;=\; \kappa\left[-\sum_i c_i^\nu g_i^2 + Y_2 + \frac{3}{2}\,g_\nu^2\right] g_\nu, \tag{7}$$

$$\beta_\lambda \;=\; \kappa\left[\frac{9}{4}\left(g_1^4 + 2g_1^2 g_2^2 + 3g_2^4\right) - \left(3g_1^2 + 9g_2^2\right)\lambda + 4Y_2\lambda - 12H + 4\lambda^2\right], \tag{8}$$

with $c_i^t = \left(\frac{17}{12}, \frac{9}{4}, 8\right)$, $c_i^\nu = \left(\frac{3}{4}, \frac{9}{4}, 0\right)$, $Y_2 = 3g_t^2 + g_\nu^2$, $H = 3g_t^4 + g_\nu^4$. At low energies, $E < m_M$, the β-functions are the same except that $Y_2 = 3g_t^2$, $H = 3g_t^4$ and that β_ν is replaced [15] by:

$$\beta_k = \kappa\left[-3g_2^2 + \frac{3}{2} - \sum_i c_i^\nu g_i^2 + Y_2 + \frac{2}{3}\,\lambda + 2Y_2\right] k. \tag{9}$$

We suppose that all couplings (other than g_ν and k) are continuous at $E = m_M$, no threshold effects. The three gauge couplings decouple from the other equations and have identical evolutions in both energy domains:

$$g_i(t) = g_{i0}/\sqrt{1 - 2\kappa b_i g_{i0}^2 t}. \tag{10}$$

The initial conditions are taken from experiment [18]: $g_{10} = 0.3575$, $g_{20} = 0.6514$, $g_{30} = 1.221$. In a first run we leave g_1 unconstrained. Then the unification scale Λ is the solution of $g_2(\ln(\Lambda/m_Z)) = g_3(\ln(\Lambda/m_Z))$,

$$\Lambda = m_Z \exp\frac{g_{20}^{-2} - g_{30}^{-2}}{2\kappa(b_2 - b_3)} \;=\; 1.1 \times 10^{17}\ \text{GeV}, \tag{11}$$

and is independent of the number of generations.

Then we choose $g_\nu = R g_t$ at $E = \Lambda$ and m_M, and solve numerically the evolution equations for $\lambda,\ g_t,\ g_\nu$ and k with initial conditions at $E = \Lambda$ from the noncommutative constraints (4):

$$g_2^2 = \frac{3+R^2}{3+R^4}\,\frac{\lambda}{24} = \frac{3+R^2}{4}\,g_t^2\,. \tag{12}$$

We note that these constraints imply that all couplings remain perturbative and at our energies we obtain the pole masses of the Higgs, the top and the light neutrino:

$$m_H^2 = \frac{4}{3}\,\frac{\lambda(m_H)}{g_2(m_Z)^2}\,m_W^2, \quad m_t = \sqrt{2}\,\frac{g_t(m_t)}{g_2(m_t)}\,m_W, \quad m_\ell = \frac{k(m_Z)}{g_2(m_Z)^2}\,m_W^2. \tag{13}$$

A few numerical results are collected in table 1.

$g_\nu/g_t\|_\Lambda$	0	1.16	1.16	1.2	1.2	1.3	1.3	1.4	1.4
m_M [GeV]	*	$2\cdot 10^{13}$	10^{14}	$2\cdot 10^{13}$	10^{14}	$3\cdot 10^{13}$	10^{14}	$3\cdot 10^{13}$	10^{14}
m_t [GeV]	186.3	173.3	173.6	172.5	172.8	170.5	170.7	168.4	168.6
m_H [GeV]	188.4	170.5	170.8	169.7	170.0	167.7	168.0	165.8	166.1
m_ℓ [eV]	0	0.29	0.06	0.30	0.06	0.23	0.07	0.25	0.08

Table 1: The top, Higgs and neutrino masses as a function of R and of the Majorana mass for the unification scale $\Lambda = 1.1\times 10^{17}$ GeV

Note that the Higgs mass is not very sensitive to the three input parameters, $\Lambda,\ m_M$, and $R = g_\nu(\Lambda)/g_t(\Lambda)$ as long as they vary in a range reproducing senible masses for the top and the light neutrino, today $m_t = 170.9 \pm 2.6$ GeV and 0.05 eV $<\ m_\ell\ <$ 0.3 eV. Then we have for the Higgs mass

$$m_H = 168.3 \pm 2.5 \text{ GeV}. \tag{14}$$

In a second run we use the constraint on the Abelian coupling $\frac{5}{3}\,g_1^2 = g_2^2$ to compute the unification scale:

$$\Lambda = m_Z \exp\frac{g_{20}^{-2} - (3/5)g_{30}^{-2}}{2\kappa(b_2 - (3/5)b_1)} = 9.8\times 10^{12} \text{ GeV}. \tag{15}$$

Note that the third constraint $\frac{5}{3}\,g_1^2 = g_3^2$ yields an intermediate unification scale, $\Lambda = 2.4\times 10^{14}$ GeV. Again we give a few numerical results, table 2.

Here the upper bound for the light neutrino mass cannot be met strictly with $m_M < \Lambda$. This does not worry us because that bound derives from cosmological hypotheses. Honouring the constraints for all three gauge couplings then yields the combined range for the Higgs mass,

$$m_H = 168.3^{+6.6}_{-2.5} \text{ GeV}. \tag{16}$$

$g_\nu/g_t\|_\Lambda$	0	0.95	1	1.1	1.2
m_t [GeV]	183.4	173.3	172.3	170.3	168.1
m_H [GeV]	188.5	174.9	173.8	171.9	170.2
m_ℓ [eV]	0	0.53	0.57	0.67	0.77

Table 2: The top, Higgs and neutrino masses as a function of R for the Majorana mass and the unification scale $m_M = \Lambda = 9.8 \times 10^{12}$ GeV

3 Constraints on the dimensionful couplings

The spectral action also produces constraints between the quadratic Higgs coupling, the Planck mass, $m_P^2 = 1/G$, and the cosmological constant in terms of the cut-off Λ and of the three moments, f_0, f_2, f_4, of the cut-off function. A step function for example has $2f_0 = f_2 = f_4$. Trading the quadratic Higgs coupling for the W mass, these constraints read:

$$m_W^2 = \frac{45}{4\cdot 96}\frac{(3m_t^2+m_\nu^2)^2}{3m_t^4+m_\nu^4}\left[\frac{f_2}{f_4}\Lambda^2 - \frac{m_\nu^2}{3m_t^2+m_\nu^2}m_M^2\right] \sim \frac{45}{96}\left[\frac{f_2}{f_4}\Lambda^2 - \frac{1}{4}m_M^2\right], \tag{17}$$

$$\begin{aligned} m_P^2 &= \frac{1}{3\pi}\left[\left(96 - 4\frac{(3m_t^2+m_\nu^2)^2}{3m_t^4+m_\nu^4}\right)f_2\Lambda^2 + 2\left(2\frac{m_\nu^2(3m_t^2+m_\nu^2)}{3m_t^4+m_\nu^4} - 1\right)f_4 m_M^2\right] \\ &\sim \frac{1}{3\pi}\left[80 f_2\Lambda^2 + 2f_4 m_M^2\right], \end{aligned} \tag{18}$$

$$\Lambda_c \sim \frac{1}{\pi m_P^2}[(96\cdot 2f_0 - 16 f_2 2/f_4)\Lambda^4 + f_4 m_M^4 + 4f_2\Lambda^2 m_M^2]. \tag{19}$$

We have taken three generations, i.e. a 96-dimensional inner Hilbert space, we only kept the Yukawa couplings of the top quark and of the τ neutrino, and one single Majorana mass in the third generation. The experts still do not agree whether the renormalisation group flow of the quadratic Higgs coupling is logarithmic or quadratic in the energy E. Nobody knows how Newton's and the cosmological constants depend on energy. Therefore we cannot put the above constraints to test. It is however reassuring that, thanks to the seesaw mechanism, a W mass much smaller than the Planck mass is easily obtained. On the other hand it is difficult to produce a small cosmological constant.

4 Is the standard model special?

Despite all constraints, there is still an infinite number of Yang-Mills-Higgs-Dirac-Yukawa models that can be derived from gravity using almost commutative geometry. The exploration of this special class is highly non-trivial and starts with Krajewski diagrams.

At present there are two approaches in this direction. The first by Chamseddine, Connes & Marcolli [5] starts from a left-right symmetric algebra. This algebra admits a privileged bi-module which is identical to the fermionic Hilbert space of the standard

model. The algebra of the standard model is a maximal subalgebra of the left-right symmetric one and the inner Dirac operator is almost the maximal operator satisfying the axioms of a spectral triple. The number of colours and the number of generations remain unexplained in this approach.

The second approach again has nothing to say about the number of colours and generations. It is a more opportunistic approach and copies what grand unified theories did in the frame of Yang-Mills-Higgs-Dirac-Yukawa theories. There, the idea was to cut down on the number of possible models with a 'shopping' list of requirements: one wants the gauge group to be simple, the fermion representation to be irreducible, complex under the gauge group and free of Yang-Mills anomalies, and the model to contain the standard model.

Coming back to Connes' noncommutative model building kit, we remark that the spectral triple of the standard model with one generation of fermions and a massless neutrino is irreducible. It has another remarkable property concerning its built-in spontaneous symmetry breaking: it allows a vacuum giving different masses to the two quarks although they sit in an isospin doublet. Indeed, in the majority of noncommutative models the spontaneous symmetry breaking gives degenerate masses to fermions in irreducible *group* representations. For years we have been looking for viable noncommutative models other than the standard model, without success. We therefore started to scan the Krajewski diagrams with the following shopping list: we want the spectral triple to be irreducible, the fermion representation to be complex under the little group in every irreducible component, and possible massless fermions to transform trivially under the little group. Furthermore we require the fermion representation to be free of Yang-Mills and mixed gravitational Yang-Mills anomalies, and the spectral triple to have no dynamical degeneracy and the colour group of every kinematical degeneracy to remain unbroken.

The first step is to get the list of irreducible Krajewski diagrams [19]. In the case of an inner spectral triple of Euclidean signature, we have no such diagram for a simple algebra, one diagram for an algebra with two simple summands, 30 diagrams for three summands, 22 diagrams for four summands, altogether 53 irreducible diagrams for algebras with up to four simple summands. The situation simplifies when we go to the Lorentzian signature where we remain with only 7 diagrams for up to four summands. These numbers are summarized in table 3.

#(summands)	Eulidean	Lorentzian
1	0	0
2	1	0
3	30	0
4	22	7
≤ 4	53	7

Table 3: The number of irreducible Krajewski diagrams for algebras with up to 4 simple summands and inner spaces with Euclidean and Lorentzian signatures

The second step is to scan all models derived from the irreducible Krajewski diagrams with respect to our shopping list. In both signatures we remain with the following models: The standard model with one generation of fermions, an arbitrary number of colours $C \geq 2$ and a massless neutrino: $SU(2) \times U(1) \times SU(C) \to U(1) \times SU(C)$. We also have three possible submodels with identical fermion content, but with $SU(2)$ replaced by $SO(2)$, no W-bosons, or with $SU(C)$ replaced by $SO(C)$ or $USp(C/2)$, C even, less gluons. There is one more possible model, the elctro-strong model: $U(1) \times SU(C) \to U(1) \times SU(C)$. The fermionic content is $\underline{C} \oplus \underline{1}$, one quark and one charged lepton. The two electric charges are arbitrary but vectorlike. The model has no scalar and no symmetry breaking.

For those of you who think that our shopping list is unreasonably restrictive already in the frame of Yang-Mills-Higgs-Dirac-Yukawa models, here is a large class of such models satisfying our shopping list: Take any group that has complex representations (like E_6) and take any irreducible, complex, unitary representation of this group. Put the left- and right-handed fermions in two copies of this representation, choose the Hilbert space for scalars 0-dimensional and a gauge-invariant mass for all fermions.

5 Beyond the standard model

For many years we have been trying to construct models from noncommutative geometry that go beyond the standard model [20] and we failed to come up with anything physical if it was not to add more generations and right-handed neutrinos to the standard model.

The noncommutative constraints on the continuous parameters of the standard model with $N = 4$ generations fail to be compatible with the hypothesis of the big desert [21].

Since a computer program [22] was written to list the irreducible Krajewski diagrams for algebras with more than three summands we do have a genuine extension of the standard model satisfying all physical desirata. It comes from an algebra with six summands [23] and is identical to the standard model with two additional leptons A^{--} and C^{++} whose electric charge is two in units of the electron charge. These new leptons couple neither to the charged gauge bosons, nor to the Higgs scalar. Their hypercharges are vector-like, so that they do not contribute to the electroweak gauge anomalies. Their masses are gauge-invariant and they constitute viable candidates for cold dark matter [24].

Also, by trial and error, two more models could be found recently [25, 26]. The first model is based on an algebra with six summands and adds to the standard model a lepton-like, weakly charged, left-handed doublet and two right-handed hypercharge singlets. These four particles are each colour-doublets under a new $SU(2)_c$ colour group. They participate in the Higgs mechanism and the noncommutative constraints require their masses to be around 75 GeV. Since they have a non-Abelian colour group one expects a macroscopic confinement [27] with a confinement radius of $\sim 10^{-7}$ m. Although these particles have electro-magnetic charge after symmetry breaking it is not yet clear whether they could have been detected in existing experiments.

The second model adds to the standard model three generations of vectorlike doublets with weak and hypercharge. After symmetry breaking one particle of each doublet becomes electrically neutral while its partner acquires a positive or a negative electro-

magnetic charge (depending on the choice of the hypercharge). These particles, like the AC leptons, do not couple to the Higgs boson and should have masses of the order of $\sim 10^3$ TeV. Due to differing self-interaction terms with the photon and the Z-boson the neutral particle will be slightly lighter than the charged particle and is therefore the stable state [28]. Together with its neutrino-like cross section, the neutral particle constitutes an interesting dark matter candidate [29].

6 Conclusions

There are two clear-cut achievements of noncommutative geometry in particle physics:

- Connes' derivation of the Yang-Mills-Higgs-Dirac-Yukawa ansatz from the Einstein-Hilbert action,
- the fact that this unification of all fundamental forces allows you to compute correctly the representation content of the Higgs scalar (i.e. one weak isospin doublet, colour singlet with hyper-charge one half) from the experimentally measured representation content of the fermions.

The other clear achievements are restrictions on the gauge groups, severe restrictions on the fermion representations and their compatibility with experiment.

Finally there are constraints on the top and Higgs masses. They do rely on the hypothesis of the big desert. Nevertheless we look forward to the Tevatron and LHC verdict.

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