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# A 3<sup>rd</sup> order sliding mode controller based on integral sliding mode for an electropneumatic system

Alexis Girin, Franck Plestan, Xavier Brun, and Alain Glumineau

**Abstract**—This paper presents the synthesis of a robust 3<sup>rd</sup> order sliding mode position controller of an electropneumatic system. The controller is a higher order sliding mode one based on integral sliding mode, and its main features are its easy implementation, its finite time convergence and its robustness with respect to uncertainties and parameters variations.

**Keywords:** Higher order sliding mode, electropneumatic system, integral sliding mode.

## NOMENCLATURE

$y, v, a$	position, velocity and acceleration of the actuator [ $m$ ][ $m/s$ ][ $m/s^2$ ]
$y_d, v_d, a_d, j_d$	desired position, velocity, acceleration and jerk [ $m$ ][ $m/s$ ][ $m/s^2$ ][ $m/s^3$ ]
$p_X$	pressure in the chamber X [ $Pa$ ]
$u_P, u_N$	servodistributors voltages [ $V$ ]
$k$	polytropic constant
$K_r$	springs rates [ $N/m$ ]
$V_X$	chamber X volume [ $m^3$ ]
$b_v$	viscous friction coefficient [ $N/m/s$ ]
$F_f$	friction force [ $N$ ]
$M$	total moving load mass [ $kg$ ]
$T_X$	chamber X temperature [ $K$ ]
$r$	perfect gaz constant [ $J/kg/K$ ]
$S$	piston area [ $m^2$ ]
$q_m$	mass flow rate provided from the servo-distributor [ $kg/s$ ]
$X$	$P$ or $N$
$\gamma$	adiabatique constant
$T_r$	temperature inside a upstream tank [ $K$ ]
$Q$	thermal exchange [ $J$ ]
$\lambda$	thermal exchange coefficient by conduction [ $J/K/m^2/s$ ]
$S_{cX}$	total area inside X chamber [ $m^2$ ]
$T_{cX}$	temperature of the X chamber wall [ $K$ ]
$q_{mX_{in}}/q_{mX_{out}}$	mass flow rate brought inside/outside of a chamber [ $kg/s$ ]
$t_F$	fixed time convergence [ $s$ ]

## I. INTRODUCTION

Control of pneumatic actuators is a challenging problem, viewed their increasing popularity (low maintenance cost, lightweight and good force/weight ratio), in spite of their

traditional drawbacks (friction, variation of the actuators dynamics due to large change of load and piston position along the cylinder stroke, nonlinearities, ...). The development of high-performance closed-loop linear/nonlinear controllers [21], [12], [25], [9], [13], [5], [6], [7] has shown the feasibility of high-level positioning of pneumatic actuator. Due to uncertainties on the model, robust controllers are necessary to ensure position tracking with high precision. In that way, sliding mode controllers have been used for electropneumatic actuators [4], [22], [33], [29]. Their advantages are that they are simply implemented and more robust versus parameters variations and exhibit good dynamic response. However, since the sampling frequency of the controller is limited, chattering will be produced (dangerous high-frequency vibrations of the controlled system). In order to reduce the chattering, the control can be modified to a so-called boundary layer control [28]. However, this type of control implies a deterioration in accuracy and robustness; furthermore, this solution is not enough in pneumatic field [3]: indeed, a good compromise between static position error and chattering cannot be found. So, the spool of the valve is exited which induced noise due to the air going from source to exhaust and an undesirable deterioration of the servodistributor.

Higher order sliding mode control [19], [1], [20], [14], [16], [23] is a recent approach which allows to remove all the standard sliding mode restrictions, while preserving the main sliding-mode features and improving its accuracy. The both first references only concern results on second order sliding mode control, which consist in ensuring in finite time that the sliding variable and its time derivative equal 0. In [20], a general approach (which means for all sliding mode order) has been proposed: the main drawbacks of this approach are that the convergence time is only bounded, and not exactly known in advance, and that the convergence condition is not constructive. The controller proposed in [14], [23] combines standard sliding mode control with linear quadratic one over a finite time interval with a fixed final state. The algorithm needs the relative degree of the system with respect to the sliding variable  $s$  and the bounds of uncertainties and has several advantages: the upper bound of the convergence time is known and can be adjusted in advance, the condition on the gain implies that its tuning is constructive, and the structure of the controller is well-adapted to practical implementations, in particular for pneumatic actuators control [15], [17]. However, two drawbacks appear in this approach. It ensures only a *practical* sliding mode establishment (only convergence in

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finite time to an arbitrarily small vicinity of the origin is ensured), and the reaching time is bounded but can not be fixed *exactly* and in advance.

The aim of this paper is to apply, to the position control of a pneumatic actuator, an arbitrary-order sliding mode controller for uncertain SISO minimum-phase nonlinear systems developed in [16], [18]. These both references present an alternative to [14], [23] without its drawbacks, and to apply this class of controllers. The main objective of these approaches is to propose a controller for which the implementation is easy, the convergence time is finite and *well-known in advance* and the robustness is ensured during the entire response of the system. The design uses the *integral sliding mode* concept [31]. The control strategy presented in the sequel, whose the basic idea has been introduced in [16], contains two parts: the first part is discontinuous, forces the establishment of a sliding mode on the integral sliding manifold, and ensures the robustness with respect to bounded uncertainties, *throughout the entire response of the system*. The second part, which is obtained through an optimal open-loop control law [16], and through an optimal feedback control over finite time interval with fixed final states [18], is used to stabilize to zero in finite time the  $r^{th}$  order input-output dynamics without uncertainties.

## II. INTEGRAL SLIDING MODE CONTROLLER[16], [18]

Consider the uncertain nonlinear system

$$\begin{aligned}\dot{x} &= f(x) + g(x)u \\ y &= s(x, t)\end{aligned}\quad (1)$$

where  $x \in \mathbb{R}^n$  is the state variable,  $u \in \mathbb{R}$  is the input control and  $s(x, t) \in \mathbb{R}$  is a smooth output function (sliding variable).  $f(x)$  and  $g(x)$  are uncertain smooth functions. Assume that :

**H1.** The relative degree  $r$  of (1) with respect to  $s$  is known and constant, and the associated zero dynamics are stable. ■

*The control objective is to fulfill the constraint  $s(x, t) = 0$  in finite time and to keep it exactly by some feedback.* The  $r^{th}$  order sliding mode is defined through the following definition

**Definition 1:** [20] Consider the nonlinear system (1), and let the system be closed by some possibly-dynamical discontinuous feedback. Then, provided that <sup>1</sup>  $s, \dot{s}, \dots, s^{(r-1)}$  are continuous functions, and the set

$$\mathcal{S}^r = \{x \mid s(x, t) = \dot{s}(x, t) = \dots = s^{(r-1)}(t, x) = 0\},$$

called “ $r^{th}$  order sliding set”, is non-empty and is locally an integral set in the Filippov sense [10], the motion on  $\mathcal{S}^r$  is called “ $r^{th}$  order sliding mode” with respect to the sliding variable  $s$ .

The  $r^{th}$  order sliding mode control approach allows the finite time stabilization to zero of the sliding variable  $s$  and its  $r -$

<sup>1</sup>All over this paper,  $s(\cdot)^{(k)}$  ( $k \in \mathbb{N}$ ) denotes the  $k^{th}$  time derivative of the function  $s(\cdot)$ . This notation is also applied for every function.

1 first time derivatives by defining a suitable discontinuous control function [16], [18]. The output  $s$  satisfies the equation

$$s^{(r)} = \chi(\cdot) + \Gamma(\cdot)u \quad (2)$$

with  $\Gamma = L_g L_f^{r-1} s$  and  $\chi = L_f^r s$ . Assume that

**H2.** Functions  $\chi(\cdot)$  and  $\Gamma(\cdot)$  are bounded uncertain functions, and, without loss of generality, let also the sign of the control gain  $\Gamma$  be constant and strictly positive. Thus, there exist  $K_m \in \mathbb{R}^{+*}$ ,  $K_M \in \mathbb{R}^{+*}$ ,  $C_0 \in \mathbb{R}^+$  such that

$$0 < K_m < \Gamma < K_M \quad |\chi| \leq C_0. \quad (3)$$

This assumption is supposed to be fulfilled at least locally. ■

Then, the  $r^{th}$  order sliding mode control of (1) with respect to the sliding variable  $s$  is equivalent to the finite time stabilization of [16], [18]

$$\begin{aligned}\dot{z}_i &= z_{i+1} \\ \dot{z}_r &= \chi(\cdot) + \Gamma(\cdot)u\end{aligned}\quad (4)$$

with  $1 \leq i \leq r - 1$  and  $z = [z_1 \ z_2 \ \dots \ z_r]^T := [s \ \dot{s} \ \dots \ s^{(r-1)}]^T$ . In the sequel, the control law  $u$  is composed by two parts. The first one, named *ideal control* [31], is continuous and stabilizes in finite time (4) at the origin when there is *no uncertainty*. In fact, this control part is used in order to generate trajectories which the system is forced to track. The second part, named *integral sliding mode control*, provides the complete compensation of uncertainty for  $t > 0$  and ensures that control objectives are reached.

### A. Continuous control part

Consider system (4) which can be trivially rewritten as

$$\begin{aligned}\dot{z}_i &= z_{i+1} \\ \dot{z}_r &= \underbrace{\chi(\cdot) + [\Gamma(\cdot) - 1]u}_{\beta} + u\end{aligned}\quad (5)$$

for  $1 \leq i \leq r - 1$ . It yields

$$\begin{aligned}\dot{z}_i &= z_{i+1} \\ \dot{z}_r &= \beta(\cdot) + u.\end{aligned}\quad (6)$$

Let define  $u = u_0 + u_1$ , with  $u_0$  the *ideal control*, and  $u_1$  the *integral sliding mode control*. Consider now the particular case  $\beta(\cdot) = 0$ . Then, as no control part  $u_1$  is necessary in order to compensate the uncertainties, the control law  $u$  reads as  $u = u_0$ . One gets

$$\dot{z} = Az + Bu_0 \quad (7)$$

where  $A$  and  $B$  are defined by

$$A = \begin{bmatrix} 0 & 1 & \dots & 0 & \dots \\ \vdots & \ddots & \ddots & \ddots & \ddots \\ 0 & \ddots & \ddots & \dots & 1 \\ 0 & \ddots & \ddots & \ddots & 0 \end{bmatrix}_{r \times r}, B = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}_{r \times 1}.$$

The control objective is to drive the state of (7) to  $z = 0$  at the fixed final time  $t = t_F$ . In [16], this second part is

based on an open-loop control. Its advantage is the easy implementation; however, the open-loop control law depends only on the initial state, is precomputed and is applied for  $t \in [0, t_F]$ , and displays the classical drawbacks of open-loop solution. In order to improve this point, the solution proposed in [18] is based on an optimal closed-loop controller  $u_0$  ensuring the minimization of the following criterion

$$J = \frac{1}{2} \int_0^{t_F} (z^T Q z + u_0^2) dt, \quad t_F < +\infty \quad (8)$$

with  $Q$  a symmetric positive definite matrix under the fixed final state constraint  $z(t_F) = 0$ .

**Theorem 1:** [24] Consider the linear system (7) with  $(A, B)$  reachable. A control law  $u_0$  minimizing the criteria (8) and driving system (7) to  $z(t) = 0$  at  $t = t_F$  for an initial condition  $z(0)$  is given by (with  $0 \leq t \leq t_F$ )

$$u_0 = -B^T M z(t) + B^T \delta(t) \quad (9)$$

with  $M$  and  $\delta(t)$  defined by

$$\begin{aligned} \dot{\delta} &= -(A^T - M B B^T) \delta, \\ 0 &= M A + A^T M - M B B^T M + Q. \end{aligned} \quad (10)$$

Initial condition  $\delta(0)$  of  $\delta(t)$  is selected in order to satisfy the terminal condition  $z(t_F) = 0$ . ■

The control law  $u_0$  is defined in order to drive in finite time the system (7) to  $z(t) = 0$  at  $t = t_F$ . The control law  $u_0$  will maintain the required equilibrium state  $z = 0$  even after the terminal time, *i.e.*  $t > t_F$ , provided that the forcing term  $\delta(t)$  is removed at the terminal time, *i.e.*  $u_0 = -B^T M z$  for  $t > t_F$  [24]. Then, in order to reach in a finite time  $t_F$  the origin  $z = 0$ , and to maintain the system (7) at this point for  $t > t_F$ , a solution is

$$u_0 = \begin{cases} -B^T M z(t) + B^T \delta(t) & \text{for } 0 \leq t \leq t_F \\ -B^T M z(t) & \text{for } t > t_F \end{cases} \quad (11)$$

Let  $z_0$  denote the solution of system (7) controlled by  $u_0$  defined by (11): dynamics of  $z_0$  are described by

$$\dot{z}_0 = A z_0 + B u_0 \quad (12)$$

As  $z_0(t_F) = 0$  and given (11), one gets, for  $t > t_F$ ,  $z_0 = 0$  and  $\dot{z}_0 = 0$ :  $z_0 = 0$  is a stable equilibrium point.

### B. Integral sliding manifold

The basic idea consists in determining a sliding surface such that the state trajectories start on this surface at the initial time  $t = 0$ , which induces a sliding mode without reaching phase [31]. One has (for  $1 \leq i \leq r - 1$ )

$$\begin{aligned} \dot{z}_i &= z_{i+1} \\ \dot{z}_r &= \beta(\cdot) + u_0 + u_1 \end{aligned} \quad (13)$$

$u_1$  is a discontinuous function designed in order to *exactly* reject the perturbation  $\beta(\cdot)$  throughout the entire response of the system. In order to reach this objective, the *integral sliding mode control* [31] is used. Let  $\sigma \in \mathbb{R}$  define as

$$\sigma = z_r + \zeta_r + \lambda_{r-1}(z_{r-1} + \zeta_{r-1}) + \dots + \lambda_1(z_1 + \zeta_1) \quad (14)$$

with  $\zeta_r \in \mathbb{R}$  called *integral sliding term* (which is determined later in this paper as  $\zeta_{r-1}, \dots, \zeta_1$ ). Coefficients  $\lambda_{r-1}, \dots, \lambda_1$  are such that  $P(p) = p^{r-1} + \lambda_{r-1}p^{r-2} + \dots + \lambda_1$  is Hurwitz. Suppose that a sliding mode is established on the manifold  $\mathcal{S}_f = \{x \mid \sigma = 0\}$  from  $t = 0$ . The time derivative of  $\sigma$  reads as

$$\begin{aligned} \dot{\sigma} &= u_0 + u_1 + \beta(\cdot) + \dot{\zeta}_r + \lambda_{r-1}(\dot{z}_{r-1} + \dot{\zeta}_{r-1}) \\ &\quad + \dots + \lambda_1(\dot{z}_1 + \dot{\zeta}_1) \end{aligned} \quad (15)$$

As the objective is to ensure that a sliding mode on  $\mathcal{S}_f$  is established *early from*  $t = 0$ , one should have  $\dot{\sigma} = 0$  for  $t \geq 0$ , and the discontinuous control  $u_1$  should be replaced by the equivalent control  $u_{1eq}$ : in [16], [18], it is established that, in order to ensure  $z = z_0$  for  $t \geq 0$  ( $z_0$  describes the *ideal and optimal* trajectory and its dynamics is defined by (12)),  $u_{1eq}$  has to satisfy

$$u_{1eq} = -\beta(\cdot), \quad \forall t \geq 0. \quad (16)$$

From (15), condition (16) is satisfied if

$$\dot{\zeta}_r = -u_0, \quad \dot{\zeta}_{r-1} = \zeta_r, \quad \dot{\zeta}_{r-2} = \zeta_{r-1}, \quad \dots, \quad \dot{\zeta}_1 = \zeta_2,$$

with  $\zeta_r(0) = -z_r(0)$ ,  $\zeta_{r-1}(0) = -z_{r-1}(0)$ ,  $\dots$ ,  $\zeta_1(0) = -z_1(0)$ . Then, at  $t = 0$ , one has  $\sigma(0) = 0$  and the sliding mode is stated on the *integral sliding manifold*  $\mathcal{S}_f$  for  $t \geq 0$ .

### C. Discontinuous control part

The control law  $u_1$  is designed in order to ensure that the sliding motion on  $\mathcal{S}_f = \{x \mid \sigma = 0\}$  is guaranteed for  $t \geq 0$  in spite of uncertainties, and reads as

$$u_1 = -\alpha \text{sign}(\sigma), \quad (17)$$

where the gain  $\alpha$  is a positive constant tuned such that the  $\eta$ -attractivity condition is satisfied

$$\sigma \dot{\sigma} \leq -\eta |\sigma|, \quad \eta > 0. \quad (18)$$

From (15) and given that  $\beta = \chi + (\Gamma - 1)u$  with  $u = u_0 + u_1$ , one has

$$\begin{aligned} \dot{\sigma} &= [\chi + (\Gamma - 1)(u_0 + u_1)] + u_0 + u_1 - u_0 \\ &\quad + \underbrace{\lambda_{r-1}(\dot{z}_{r-1} + \dot{\zeta}_{r-1}) + \dots + \lambda_1(\dot{z}_1 + \dot{\zeta}_1)}_{:= \delta} \\ &= \chi + (\Gamma - 1)u_0 + \Gamma u_1 + \delta. \end{aligned} \quad (19)$$

Define  $u_{0M}$  as the bound of  $u_0$ , and  $|\delta| < \Delta$ . The convergence condition (18) is ensured if

$$\alpha > \frac{C_0 + |K_M - 1|u_{0M} + \Delta + \eta}{K_m}. \quad (20)$$

Under control law (17) with condition (20) fulfilled, system (13) evolves on the sliding manifold  $\mathcal{S}_f$ , its dynamics reads as (12). Its trajectories converge to zero in a finite time  $t = t_F$  and are maintained in this equilibrium stable point.

**Theorem 2 ([16], [18]):** Consider the nonlinear system (1) with a relative degree  $r$  with respect to the sliding variable  $s(x, t)$ . Suppose the hypotheses H1-H2 fulfilled and  $s(0), \dot{s}(0), \dots, s^{(r-1)}(0)$  bounded. Then, the control law  $u$

$$u = u_0 + u_1$$

with  $u_0$  defined by (11) and

$$\begin{aligned} u_1 &= -\alpha \cdot \text{sign} [s^{(r-1)} + \zeta_r + \lambda_{r-1}(s^{(r-2)} + \\ &\quad \zeta_{r-1}) + \dots + \lambda_1(s + \zeta_1)], \\ \dot{\zeta}_r &= -u_0 \text{ with } \zeta_r(0) = -s^{(r-1)}(0), \\ \zeta_{r-1}(0) &= -s^{(r-2)}(0), \dots, \zeta_1(0) = -s(0), \\ \alpha &> \frac{C_0 + |K_M - 1|u_{0M} + \Delta + \eta}{K_m}, \end{aligned} \quad (21)$$

allows the establishment of a  $r^{\text{th}}$  order sliding mode with respect to  $s$  with a finite time convergence  $t = t_F < \infty$ . ■

### III. ELECTROPNEUMATIC SYSTEM

#### A. Description

The electropneumatic system under interest is a double acting actuator controlled by two servodistributors (see Figure 1) and composed by two chambers denoted  $P$  and  $N$ . Piston diameter is  $63 \text{ mm}$  and rod diameter  $16 \text{ mm}$ . With a source pressure equal to  $7 \text{ bar}$ , the maximum force developed by the actuator is  $1750 \text{ N}$ . The air mass flow rates  $q_m$  entering in the chambers are modulated by two three-way servodistributors Servotronic (Asco-Joucomatic) controlled by a micro-controller. The pneumatic jack horizontally moves a load carriage of mass  $M$ . This carriage is coupled to 4 springs (which restrain the displacement of the carriage and restore the initial position in the middle of the total stroke equal to  $50 \text{ mm}$  - see Figure 1) for a total of  $75000 \text{ N/m}$  rate. Additional dry friction is controlled by two skates, with a maximum value equal to  $40 \text{ N}$ . As the maximal displacement of carriage is  $16 \text{ mm}$ , the maximal spring force is  $1200 \text{ N}$ . The electropneumatic plant model is obtained from three

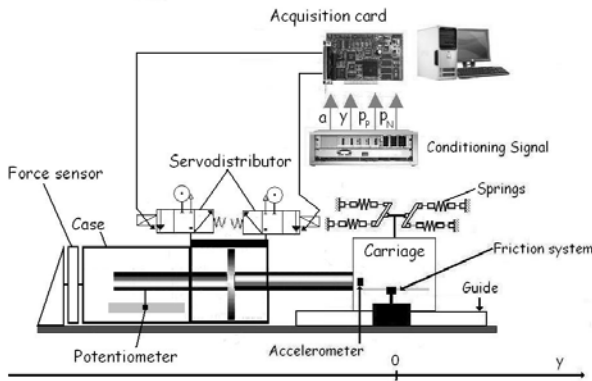


Fig. 1. Electropneumatic system

physical laws: the mass flow rate through a restriction, the pressure behaviour in a chamber with variable volume and the fundamental mechanical equation. The experimental set-up is simulated with a fluid power systems dedicated software AMESim, and the control law is developed under Matlab/Simulink, which implies a cosimulation program. In the sequel, two models are displayed

- the first one, which takes into account physical phenomena as temperature variations, practical values of mass flow rate, dynamics of servodistributors..., is developed under Amesim.
- The second one, which is simpler than the previous, is used in order to design the controller under Simulink.

#### B. Simulation model

**Servodistributor model.** The servodistributor model is composed in two parts, a dynamic part and a static one :

- Dynamic part is modeled by a second order transfer function

$$F(s) = \frac{\omega_{ns}^2}{s^2 + 2 \cdot \zeta_s \cdot \omega_{ns} s + \omega_{ns}^2} \quad (22)$$

with  $\omega_{ns} = 246 \text{ rad} \cdot \text{s}^{-1}$  and  $\zeta_s = 0.707$ .

- Static part is modeled by an experimental table where mass flow rate is given in function of ratio pressure (upstream/downstream) and control voltage [26].

**Pneumatic chamber variable volume model.** Each chamber of the pneumatic actuator is considered as a variable volume, in which the air mass evolves with time. State the following assumptions :

- A1.** Air is a perfect gas and its kinetic is inconsequential. ■
- A2.** The pressure and the temperature are homogeneous in each chamber. ■
- A3.** The mass flow is pseudo-stationary. ■

The first dynamic principle applied to the air mass and the thermodynamic evolution of air in each chamber read as (with  $X = P$  or  $N$ ) [27]

$$\begin{aligned} \frac{dp_X}{dt} &= -\gamma \frac{p_X}{V_X} \frac{dV_X}{dt} + \frac{\gamma r T_r}{V_X} q_{mX_{in}} - \frac{\gamma r T_X}{V_X} q_{mX_{out}} \\ &\quad + \frac{(\gamma - 1) dQ_X}{V_X dt} \\ \frac{dT_X}{dt} &= -(\gamma - 1) \frac{T_X}{V_X} \frac{dV_X}{dt} + \frac{r T_X}{p_X V_X} (\gamma T_r - T_X) q_{mX_{in}} \\ &\quad - \frac{r T_X^2}{p_X V_X} (\gamma - 1) q_{mX_{out}} + (\gamma - 1) \frac{T_X}{p_X V_X} \frac{dQ_X}{dt} \end{aligned} \quad (23)$$

with  $\gamma$  the adiabatic constant,  $T_r$  the temperature inside the upstream tank,  $q_{mX_{in}}$  the mass flow rate brought inside the  $X$  chamber, and  $q_{mX_{out}}$  the mass flow rate brought outside the  $X$  chamber.  $Q_X$ , the thermal exchange with the  $X$  chamber wall, is described by assumption A4.

- A4.** The thermal exchange is due only by conduction described by

$$\frac{dQ_X}{dt} = \lambda S_{cX} (T_{cX} - T_X) \quad (24)$$

with  $\lambda$  the thermal exchange coefficient by conduction,  $S_{cX}$  the total area inside a  $X$  chamber, and  $T_{cX}$  the temperature of the  $X$  chamber wall. ■

**Mechanical model.** The second Newton law gives

$$\begin{aligned}\frac{dv}{dt} &= \frac{1}{M} [S(p_P - p_N) - F_f - b_v v - K_r y] \\ \frac{dy}{dt} &= v\end{aligned}\quad (25)$$

With friction force  $F_f$  included stiction, Coulomb and Stribeck phenomena.

**Samplers and saturation.** Samplers are added in AMESim's model in order to take into account samplers of acquisition card; sample time is 1 *ms*. Saturation signal control are added, *i.e.*  $|u_{sat}| = 10$  V.

### C. Control model

This model is developed in order to design the control law and a simplest version of the simulation one. The following hypotheses are added

**A5.** The process is polytropic and characterized by coefficient  $k$  (with  $1 < k < \gamma$ ). In this case,  $Q$  dynamics are [27]

$$\begin{aligned}\frac{dQ_X}{dt} &= \frac{1}{\gamma - 1} [-r q_{mX_{in}} (\gamma T_r - k T_X) \\ &\quad + r T_X q_{mX_{out}} (\gamma - k) + p_X (\gamma - k) \frac{dV_X}{dt}]\end{aligned}\quad (26)$$

**A6.** Furthermore, the temperature variations in each chamber are inconsequential with respect to the supply temperature, *i.e.*  $T_P = T_N = T$ .

Then, pressures dynamics read as

$$\begin{aligned}\frac{dp_X}{dt} &= -k \frac{p_X}{V_X} \frac{dV_X}{dt} \\ &\quad + \frac{krT}{V_X} (q_{mX_{in}} - q_{mX_{out}})\end{aligned}\quad (27)$$

**A7.** The leakage between the two chamber and between servodistributor and jack are negligible.

By defining  $q_m(u_X, p_X) := q_{mX_{in}} - q_{mX_{out}}$ , one gets

$$\begin{aligned}\frac{dp_P}{dt} &= -k \frac{p_P}{V_P(y)} \frac{dV_P(y)}{dt} + \frac{krT}{V_P} q_m(u_P, p_P) \\ \frac{dp_N}{dt} &= -k \frac{p_N}{V_N(y)} \frac{dV_N(y)}{dt} + \frac{krT}{V_N} q_m(u_N, p_N)\end{aligned}\quad (28)$$

**A8.** All dry frictions forces are neglected.

**A9.** There is no control signal saturation.

**A10.** Dynamic part of servodistributor is neglected, and static part reads as

$$q_m(u_X, p_X) = \varphi(p_X) + \psi(p_X, \text{sign}(u_X)) u_X$$

with  $\varphi$  and  $\psi$  defined as 5<sup>th</sup>-order polynomials with respect to  $p_X$  [2].

**A11.** Only the position of the actuator is controlled, which means that the problem is a single input-single output (SISO). It implies that  $u_P = -u_N = u$ . ■

Then, with  $V_P(y) = V_0 + S \cdot y$  and  $V_N(y) = V_0 - S \cdot y$  ( $V_0$  being equal to the half of the cylinder volume), the model used for the design of controller is a nonlinear system reads as

$$\begin{aligned}\dot{p}_P &= \frac{krT}{V_P(y)} [\varphi(p_P) + \psi(p_P, \text{sign}(u)) u - \frac{S}{rT} p_P v] \\ \dot{p}_N &= \frac{krT}{V_N(y)} [\varphi(p_N) - \psi(p_N, \text{sign}(-u)) u + \frac{S}{rT} p_N v] \\ \dot{v} &= \frac{1}{M} [S p_P - S p_N - b_v v - K_r y] \\ \dot{y} &= v\end{aligned}\quad (29)$$

### D. Conclusion

The use of different models for the simulation and for the control design implies that, if high accuracy is required, it is necessary to design robust control law with respect to uncertainties generated by the difference between these two models. These uncertainties are taken into account, in the controller design, through two models

- The so-called “nominal” model (29),
- The so-called “real” model which takes into account the uncertainties on several parameters or functions (Temperature  $T$ , functions  $\varphi$  and  $\psi$ , mass  $M$ )

$$\begin{aligned}\dot{p}_P &= \frac{kr(T + \Delta T)}{V_P(y)} [\varphi + \Delta\varphi - \frac{S}{r(T + \Delta T)} p_P v] \\ &\quad + \frac{kr(T + \Delta T)}{V_P(y)} (\psi + \Delta\psi) u \\ \dot{p}_N &= \frac{kr(T + \Delta T)}{V_N(y)} [\varphi + \Delta\varphi - \frac{S}{r(T + \Delta T)} p_N v] \\ &\quad - \frac{kr(T + \Delta T)}{V_N(y)} (\psi + \Delta\psi) u \\ \dot{v} &= \frac{1}{M + \Delta M} [S p_P - S p_N - b_v v - K_r y] \\ \dot{y} &= v\end{aligned}\quad (30)$$

with  $|\Delta T| < T_M$ ,  $|\Delta\varphi| < \varphi_M$ ,  $|\Delta\psi| < \psi_M$  and  $|\Delta M| < M_M$  all bounded values.

### E. Desired trajectory

The desired position trajectory, named  $y_d(t)$ , is displayed in Figure 2 and has been designed such that associated velocity and acceleration are continuous functions. The maximum velocity (resp. acceleration) is  $0.8$  *m·s<sup>-1</sup>* (resp.  $33$  *m·s<sup>-2</sup>*). This displacement corresponds to 64% of the total stroke around the central position. The initial conditions are such that  $[y(0) \ v(0) \ \dot{v}(0)]^T = [0 \ 0 \ 0]^T$ .

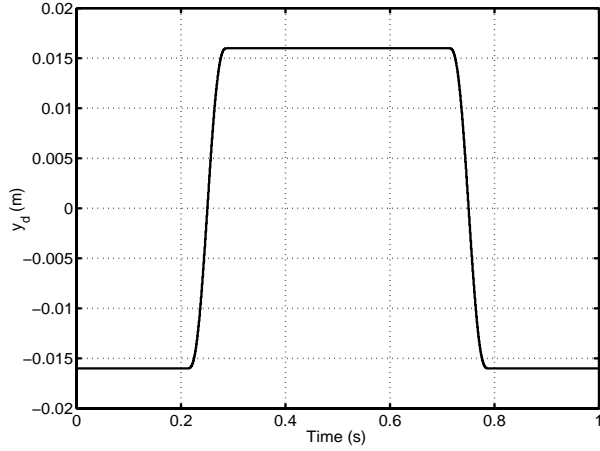


Fig. 2. Desired position (m) versus time (sec)

#### IV. APPLICATION OF INTEGRAL SLIDING MODE CONTROLLER

##### A. Controller design

The objective consists in designing a robust (with respect to uncertainties/disturbances) position controller. Define  $s$  the sliding variable as  $s = y - y_d(t)$ : from (30), its relative degree with respect to  $u$  equals 3, which implies that a 3<sup>rd</sup> order sliding mode controller is designed. The validity of the control law depends on the stability of the unobservable subsystem of dimension one, which is supposed. One has

$$s^{(3)} = \chi(\cdot) + \Gamma(\cdot)u \quad (31)$$

with  $\bar{T} = T + \Delta T$ ,  $\bar{\varphi} = \varphi + \Delta\varphi$ ,  $\bar{\psi} = \psi + \Delta\psi$ ,  $\bar{M} = M + \Delta M^2$ , and

$$\begin{aligned} \chi(\cdot) &= \frac{kr\bar{T}S}{M} \left( \frac{\bar{\varphi}(p_P)}{V_P(y)} - \frac{\bar{\varphi}(p_N)}{V_N(y)} \right) - \frac{kS^2v}{M} \left( \frac{p_P}{V_P(y)} - \frac{p_N}{V_N(y)} \right) \\ &\quad - \frac{b_v}{M^2} (S(p_P - p_N) - b_v v - K_r y) - \frac{K_r v}{M} - y_d^{(3)} \\ \Gamma(\cdot) &= \frac{kr\bar{T}S}{M} \left( \frac{\bar{\psi}(p_P, \text{sign}(u))}{V_P(y)} + \frac{\bar{\psi}(p_N, \text{sign}(-u))}{V_N(y)} \right) \end{aligned} \quad (32)$$

The control law is defined as  $u = \Gamma_{Nom}^{-1} \cdot [-\chi_{Nom} + v]$  with  $\Gamma_{Nom}$  (resp.  $\chi_{Nom}$ ) the nominal value of  $\Gamma$  (resp.  $\chi$ ), i.e. derived from (32) with no uncertainties. It is important to note that  $\Gamma_{nom}$  is always strictly positive. So  $u$  has the same sign as  $(-\chi_{nom} + v)$ . By definition,  $\chi_{nom}$  and  $v$  are independent of  $u$ . Consequently, the control law is not implicit. This explains the choice of the function  $\varphi$ , independent of  $u$ , in the mass flow rate expression of assumption A10. Then, one gets

$$s^{(3)} = \bar{\chi}(\cdot) + \bar{\Gamma}(\cdot)v \quad (33)$$

with  $\bar{\chi} = \chi - \Gamma\Gamma_{Nom}^{-1}\chi_{Nom}$  and  $\bar{\Gamma} = \Gamma\Gamma_{Nom}^{-1}$ . Let  $z = [s \ \dot{s} \ \ddot{s}]^T$ . Then, as shown in Section II, the 3<sup>rd</sup> order sliding

mode control is equivalent to the finite time stabilization of

$$\dot{z} = \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}}_A z + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}_B v + \begin{bmatrix} 0 \\ 0 \\ \beta(\cdot) \end{bmatrix} \quad (34)$$

where  $\beta(\cdot)$  is defined as  $\beta = \bar{\chi} + (\bar{\Gamma} - 1)v$ . As mentioned in Section II, the integral sliding mode controller  $v$  reads as

$$v = v_0 + v_1.$$

The first part  $v_0$  is a continuous one which ensures that  $s$ ,  $\dot{s}$ , and  $\ddot{s}$  converge to 0 at a fixed time  $t_F$ ; the second one  $v_1$  is a discontinuous one which ensures the previous convergence in spite of uncertainties. As mentioned in Section II, the design follows several steps.

**Continuous part  $v_0$ .** Matrix  $Q$  is stated as

$$Q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Then, from (10), one gets

$$M = \begin{bmatrix} 2.4142 & 2.4142 & 1.0000 \\ 2.4142 & 4.8284 & 2.4142 \\ 1.0000 & 2.4142 & 2.4142 \end{bmatrix}$$

From (10), one gets  $\delta(t) = e^{A_m t} \delta(0)$  with  $A_m = -[A^T - MBB^T]$ . Then, given  $\delta(t)$ , and from (7)-(9), one gets  $\dot{z} = -A_m^T z + BB^T e^{A_m t} \delta(0)$ . By multiplying both side of previous equation by  $e^{A_m^T t}$ , and integrating from  $t = 0$  to  $t = t_F$ , with  $z(t_F) = 0$ , one gets

$$z(0) = - \underbrace{\left[ \int_0^{t_F} e^{A_m^T t} BB^T e^{A_m t} dt \right]}_H \cdot \delta(0)$$

Matrix  $H$  is the partial reachability gramian of linear system  $\dot{x} = A_m x + Bv$  and is, viewed the form of  $A_m$ ,  $B$  and  $t_F < \infty$ , invertible. Then, the initial condition  $\delta(0)$  ensuring that  $z(t_F) = 0$  can be derived from  $\delta(0) = -H^{-1}z(0)$ . The matrix  $H$  is evaluated using an algorithm from [32] which yields  $\delta_1(0) = -3.6034e4$ ,  $\delta_2(0) = -3.6034e3$  and  $\delta_3(0) = -1.2002e2$  for  $z(0) = [0.016 \ 0 \ 0]^T$  and a convergence time fixed to  $t_F = 0.2$  sec. Then,  $v_0$  reads as (with  $|v_0| < v_{0M} = 120$ )

$$\begin{aligned} v_0 &= \begin{cases} -B^T M z(t) + B^T \delta(t) & \text{for } 0 \leq t \leq t_F \\ -B^T M z(t) & \text{for } t > t_F \end{cases} \\ \dot{\delta} &= -(A^T - MBB^T) \delta \end{aligned} \quad (35)$$

**Discontinuous part  $v_1$ .** The switching variable  $\sigma$  reads as  $\sigma = \ddot{s} + \zeta_3 + 2\xi\omega_n(\dot{s} + \zeta_2) + \omega_n^2(s + \zeta_1)$  with  $\zeta_3(0) = -\ddot{s}(0)$ ,  $\zeta_2(0) = -\dot{s}(0)$  and  $\zeta_1(0) = -s(0)$ , and  $\dot{\zeta}_1 = \zeta_2$ ,  $\dot{\zeta}_2 = \zeta_3$ ,  $\zeta_3 = v_0$ . Then, one gets  $v_1 = -\alpha \cdot \text{sign}(\sigma)$  with  $\xi = 0.7$ ,  $\omega_n = 190 \text{ rad} \cdot \text{s}^{-1}$  and  $\alpha = 10^5$  (in order to satisfy (18)-(20) by taking into account the uncertainties due to the difference of simulation and control models, and the variations of mass (+125%)).

<sup>2</sup>It is supposed that  $\Delta\dot{M} = 0$ .

### B. Simulation results

Simulations have been made in cosimulation with Amesim-Simulink : the “complete” model of the experimental set-up displayed in Section III-B is simulated by Amesim, and the previous controller based on the “simplified” model displayed in Section III-C is implemented with Simulink. Two kind of simulations have been made. The first simulations, named “Nominal case”, consist in considering that the moving mass is  $0.8kg$  (i.e. the control law has been designed with this hypothesis). The second simulations consist in increasing the moving mass to  $1.8kg$ , without changing the structure of the controller and its gains values. In order to evaluate the performances of integral sliding mode controller, a classical state feedback control, defined as

$$u = K_y(y - y_d) + K_v v + K_a a \quad (36)$$

has been designed by using a linear model around actuator position  $y = 0$  [5], and by computing gains  $K_y$ ,  $K_v$  and  $K_a$  by Ackerman’s approach such that poles placement authorized 4.6% overshoot [11], which gives  $K_y = 3119 \text{ V/m}$ ,  $K_v = 2.618 \text{ V/m/s}$  and  $K_a = 0.03125 \text{ V/m/s}^2$ .

**Nominal case.** The actuator position (Figure 3) converges to the desired trajectory in  $0.2s$  (which is the stated convergence time  $t_F$ ) for ISM<sup>3</sup> controller without overshoot. With CF controller, the convergence is ensured in  $0.15s$ , with a 10% overshoot. The maximum error position in steady state is  $0.18mm$  for ISM controller and  $0.16mm$  for CF controller. During all trajectory tracking with ISM controller, there is no pressure saturation (Figure 4-Top) and control input is realistic (Figure 5). Note that, in steady state, the force developed by the actuator,  $F = S \cdot (p_P - p_N)$ , allows to compensate springs force, as shown in Figure 4-Bottom.

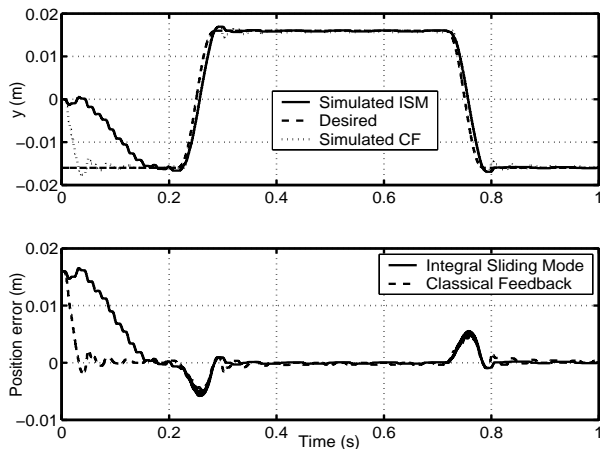


Fig. 3. **Top.** Desired and current positions ( $m$ ) versus time ( $sec$ ). **Bottom.** Positions errors ( $m$ ) versus time ( $sec$ ).

**Robustness evaluation.** The ISM controller still ensures

<sup>3</sup>In the sequel, “ISM” denotes Integral Sliding Mode controller, and “CF” denote Classical State Feedback controller

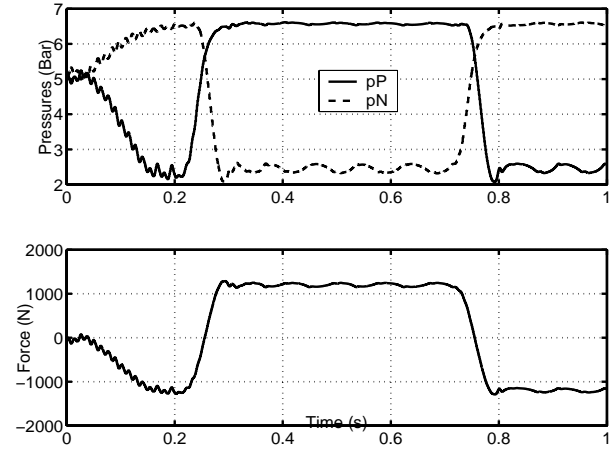


Fig. 4. **Top.** pressure ( $p_P$  and  $p_N$ ) (bar) versus time ( $sec$ ). **Bottom.** Force ( $N$ ) versus time ( $sec$ ).

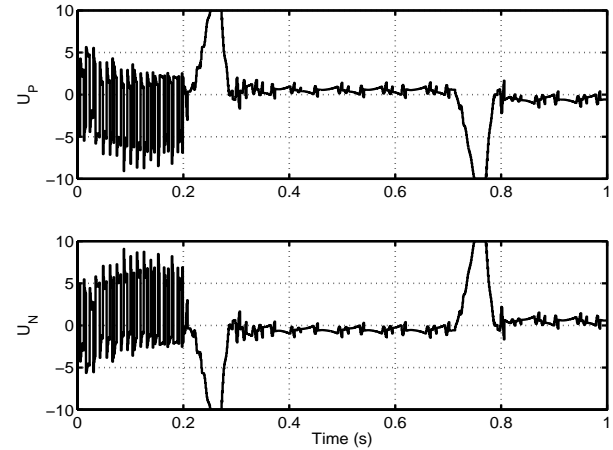


Fig. 5. **Top.**  $u_P$  (V) versus time ( $sec$ ). **Bottom.**  $u_N$  (V) versus time ( $sec$ ).

convergence in  $0.2s$  without overshoot (Figure 6). With CF controller, overshoot increased at 19%. The maximum position error in steady state equals  $0.3mm$  for ISM controller and  $0.65mm$  for CF controller. This robustness evaluation with respect to mass modification confirms the best efficiency of ISM controller versus CF one.

## V. CONCLUSIONS

This paper has proposed the application of a higher order sliding mode controller based on integral sliding mode [16], [18] to the position control of a pneumatic actuator. Design and simulations have shown the efficiency and applicability of the control approach to the pneumatic area. The further works concern the practical application of the control solution, and the design of 4<sup>th</sup> order sliding mode controller in order to improve the trajectories tracking accuracy.

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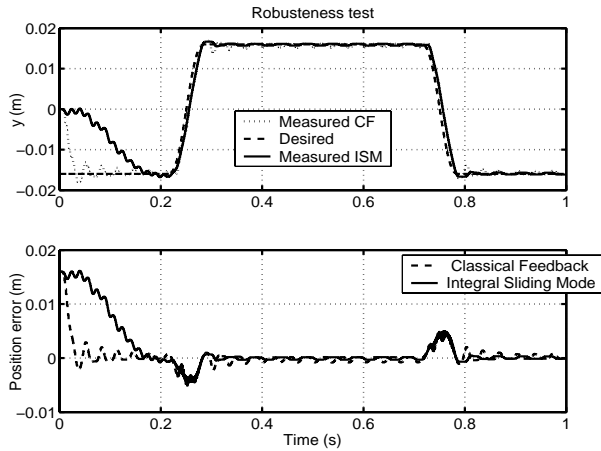


Fig. 6. **Top.** Desired and current positions (m) versus time (sec). **Bottom.** Positions errors (m) versus time (sec).

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