



HAL
open science

A 3rd order sliding mode controller based on integral sliding mode for an electropneumatic system

Alexis Girin, Franck Plestan, Xavier Brun, Alain Glumineau

► **To cite this version:**

Alexis Girin, Franck Plestan, Xavier Brun, Alain Glumineau. A 3rd order sliding mode controller based on integral sliding mode for an electropneumatic system. 45th IEEE Conference on Decision and Control, Dec 2006, San Diego, United States. pp.WeIP4.5, 10.1109/CDC.2006.376778. hal-00207686

HAL Id: hal-00207686

<https://hal.science/hal-00207686v1>

Submitted on 2 Apr 2019

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

A 3rd order sliding mode controller based on integral sliding mode for an electropneumatic system

Alexis Girin, Franck Plestan, Xavier Brun, and Alain Glumineau

Abstract—This paper presents the synthesis of a robust 3rd order sliding mode position controller of an electropneumatic system. The controller is a higher order sliding mode one based on integral sliding mode, and its main features are its easy implementation, its finite time convergence and its robustness with respect to uncertainties and parameters variations.

Keywords: Higher order sliding mode, electropneumatic system, integral sliding mode.

NOMENCLATURE

y, v, a	position, velocity and acceleration of the actuator [m][m/s][m/s^2]
y_d, v_d, a_d, j_d	desired position, velocity, acceleration and jerk [m][m/s][m/s^2][m/s^3]
p_X	pressure in the chamber X [Pa]
u_P, u_N	servodistributors voltages [V]
k	polytropic constant
K_r	springs rates [N/m]
V_X	chamber X volume [m^3]
b_v	viscous friction coefficient [$N/m/s$]
F_f	friction force [N]
M	total moving load mass [kg]
T_X	chamber X temperature [K]
r	perfect gaz constant [$J/kg/K$]
S	piston area [m^2]
q_m	mass flow rate provided from the servo-distributor [kg/s]
X	P or N
γ	adiabatique constant
T_r	temperature inside a upstream tank [K]
Q	thermal exchange [J]
λ	thermal exchange coefficient by conduction [$J/K/m^2/s$]
S_{cX}	total area inside X chamber [m^2]
T_{cX}	temperature of the X chamber wall [K]
$q_{mX_{in}}/q_{mX_{out}}$	mass flow rate brought inside/outside of a chamber [kg/s]
t_F	fixed time convergence [s]

I. INTRODUCTION

Control of pneumatic actuators is a challenging problem, viewed their increasing popularity (low maintenance cost, lightweight and good force/weight ratio), in spite of their

traditional drawbacks (friction, variation of the actuators dynamics due to large change of load and piston position along the cylinder stroke, nonlinearities, ...). The development of high-performance closed-loop linear/nonlinear controllers [21], [12], [25], [9], [13], [5], [6], [7] has shown the feasibility of high-level positioning of pneumatic actuator. Due to uncertainties on the model, robust controllers are necessary to ensure position tracking with high precision. In that way, sliding mode controllers have been used for electropneumatic actuators [4], [22], [33], [29]. Their advantages are that they are simply implemented and more robust versus parameters variations and exhibit good dynamic response. However, since the sampling frequency of the controller is limited, chattering will be produced (dangerous high-frequency vibrations of the controlled system). In order to reduce the chattering, the control can be modified to a so-called boundary layer control [28]. However, this type of control implies a deterioration in accuracy and robustness; furthermore, this solution is not enough in pneumatic field [3]: indeed, a good compromise between static position error and chattering cannot be found. So, the spool of the valve is exited which induced noise due to the air going from source to exhaust and an undesirable deterioration of the servodistributor.

Higher order sliding mode control [19], [1], [20], [14], [16], [23] is a recent approach which allows to remove all the standard sliding mode restrictions, while preserving the main sliding-mode features and improving its accuracy. The both first references only concern results on second order sliding mode control, which consist in ensuring in finite time that the sliding variable and its time derivative equal 0. In [20], a general approach (which means for all sliding mode order) has been proposed: the main drawbacks of this approach are that the convergence time is only bounded, and not exactly known in advance, and that the convergence condition is not constructive. The controller proposed in [14], [23] combines standard sliding mode control with linear quadratic one over a finite time interval with a fixed final state. The algorithm needs the relative degree of the system with respect to the sliding variable s and the bounds of uncertainties and has several advantages: the upper bound of the convergence time is known and can be adjusted in advance, the condition on the gain implies that its tuning is constructive, and the structure of the controller is well-adapted to practical implementations, in particular for pneumatic actuators control [15], [17]. However, two drawbacks appear in this approach. It ensures only a *practical* sliding mode establishment (only convergence in

A. Girin, F. Plestan and A. Glumineau are with IR-CCyN, UMR 6597, Ecole Centrale de Nantes, France Alexis.Girin@ircyn.ec-nantes.fr

X. Brun is with LAI, INSA Lyon, Villeurbanne, France Xavier.Brun@insa-lyon.fr

finite time to an arbitrarily small vicinity of the origin is ensured), and the reaching time is bounded but can not be fixed *exactly* and in advance.

The aim of this paper is to apply, to the position control of a pneumatic actuator, an arbitrary-order sliding mode controller for uncertain SISO minimum-phase nonlinear systems developed in [16], [18]. These both references present an alternative to [14], [23] without its drawbacks, and to apply this class of controllers. The main objective of these approaches is to propose a controller for which the implementation is easy, the convergence time is finite and *well-known in advance* and the robustness is ensured during the entire response of the system. The design uses the *integral sliding mode* concept [31]. The control strategy presented in the sequel, whose the basic idea has been introduced in [16], contains two parts: the first part is discontinuous, forces the establishment of a sliding mode on the integral sliding manifold, and ensures the robustness with respect to bounded uncertainties, *throughout the entire response of the system*. The second part, which is obtained through an optimal open-loop control law [16], and through an optimal feedback control over finite time interval with fixed final states [18], is used to stabilize to zero in finite time the r^{th} order input-output dynamics without uncertainties.

II. INTEGRAL SLIDING MODE CONTROLLER[16], [18]

Consider the uncertain nonlinear system

$$\begin{aligned} \dot{x} &= f(x) + g(x)u \\ y &= s(x, t) \end{aligned} \quad (1)$$

where $x \in \mathbb{R}^n$ is the state variable, $u \in \mathbb{R}$ is the input control and $s(x, t) \in \mathbb{R}$ is a smooth output function (sliding variable). $f(x)$ and $g(x)$ are uncertain smooth functions. Assume that :

H1. The relative degree r of (1) with respect to s is known and constant, and the associated zero dynamics are stable. ■

The control objective is to fulfill the constraint $s(x, t) = 0$ in finite time and to keep it exactly by some feedback. The r^{th} order sliding mode is defined through the following definition

Definition 1: [20] Consider the nonlinear system (1), and let the system be closed by some possibly-dynamical discontinuous feedback. Then, provided that ¹ $s, \dot{s}, \dots, s^{(r-1)}$ are continuous functions, and the set

$$\mathcal{S}^r = \{x \mid s(x, t) = \dot{s}(x, t) = \dots = s^{(r-1)}(t, x) = 0\},$$

called “ r^{th} order sliding set”, is non-empty and is locally an integral set in the Filippov sense [10], the motion on \mathcal{S}^r is called “ r^{th} order sliding mode” with respect to the sliding variable s .

The r^{th} order sliding mode control approach allows the finite time stabilization to zero of the sliding variable s and its $r -$

¹All over this paper, $s(\cdot)^{(k)}$ ($k \in \mathbb{N}$) denotes the k^{th} time derivative of the function $s(\cdot)$. This notation is also applied for every function.

1 first time derivatives by defining a suitable discontinuous control function [16], [18]. The output s satisfies the equation

$$s^{(r)} = \chi(\cdot) + \Gamma(\cdot)u \quad (2)$$

with $\Gamma = L_g L_f^{r-1} s$ and $\chi = L_f^r s$. Assume that

H2. Functions $\chi(\cdot)$ and $\Gamma(\cdot)$ are bounded uncertain functions, and, without loss of generality, let also the sign of the control gain Γ be constant and strictly positive. Thus, there exist $K_m \in \mathbb{R}^{+*}$, $K_M \in \mathbb{R}^{+*}$, $C_0 \in \mathbb{R}^+$ such that

$$0 < K_m < \Gamma < K_M \quad |\chi| \leq C_0. \quad (3)$$

This assumption is supposed to be fulfilled at least locally. ■

Then, the r^{th} order sliding mode control of (1) with respect to the sliding variable s is equivalent to the finite time stabilization of [16], [18]

$$\begin{aligned} \dot{z}_i &= z_{i+1} \\ \dot{z}_r &= \chi(\cdot) + \Gamma(\cdot)u \end{aligned} \quad (4)$$

with $1 \leq i \leq r - 1$ and $z = [z_1 \ z_2 \ \dots \ z_r]^T := [s \ \dot{s} \ \dots \ s^{(r-1)}]^T$. In the sequel, the control law u is composed by two parts. The first one, named *ideal control* [31], is continuous and stabilizes in finite time (4) at the origin when there is *no uncertainty*. In fact, this control part is used in order to generate trajectories which the system is forced to track. The second part, named *integral sliding mode control*, provides the complete compensation of uncertainty for $t > 0$ and ensures that control objectives are reached.

A. Continuous control part

Consider system (4) which can be trivially rewritten as

$$\begin{aligned} \dot{z}_i &= z_{i+1} \\ \dot{z}_r &= \underbrace{\chi(\cdot) + [\Gamma(\cdot) - 1]u}_{\beta} + u \end{aligned} \quad (5)$$

for $1 \leq i \leq r - 1$. It yields

$$\begin{aligned} \dot{z}_i &= z_{i+1} \\ \dot{z}_r &= \beta(\cdot) + u. \end{aligned} \quad (6)$$

Let define $u = u_0 + u_1$, with u_0 the *ideal control*, and u_1 the *integral sliding mode control*. Consider now the particular case $\beta(\cdot) = 0$. Then, as no control part u_1 is necessary in order to compensate the uncertainties, the control law u reads as $u = u_0$. One gets

$$\dot{z} = Az + Bu_0 \quad (7)$$

where A and B are defined by

$$A = \begin{bmatrix} 0 & 1 & \dots & 0 & \dots \\ \vdots & \ddots & \ddots & \ddots & \ddots \\ 0 & \ddots & \ddots & \dots & 1 \\ 0 & \ddots & \ddots & \ddots & 0 \end{bmatrix}_{r \times r}, B = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}_{r \times 1}.$$

The control objective is to drive the state of (7) to $z = 0$ at the fixed final time $t = t_F$. In [16], this second part is

based on an open-loop control. Its advantage is the easy implementation; however, the open-loop control law depends only on the initial state, is precomputed and is applied for $t \in [0, t_F]$, and displays the classical drawbacks of open-loop solution. In order to improve this point, the solution proposed in [18] is based on an optimal closed-loop controller u_0 ensuring the minimization of the following criterion

$$J = \frac{1}{2} \int_0^{t_F} (z^T Q z + u_0^2) dt, \quad t_F < +\infty \quad (8)$$

with Q a symmetric positive definite matrix under the fixed final state constraint $z(t_F) = 0$.

Theorem 1: [24] Consider the linear system (7) with (A, B) reachable. A control law u_0 minimizing the criteria (8) and driving system (7) to $z(t) = 0$ at $t = t_F$ for an initial condition $z(0)$ is given by (with $0 \leq t \leq t_F$)

$$u_0 = -B^T M z(t) + B^T \delta(t) \quad (9)$$

with M and $\delta(t)$ defined by

$$\begin{aligned} \dot{\delta} &= -(A^T - M B B^T) \delta, \\ 0 &= M A + A^T M - M B B^T M + Q. \end{aligned} \quad (10)$$

Initial condition $\delta(0)$ of $\delta(t)$ is selected in order to satisfy the terminal condition $z(t_F) = 0$. ■

The control law u_0 is defined in order to drive in finite time the system (7) to $z(t) = 0$ at $t = t_F$. The control law u_0 will maintain the required equilibrium state $z = 0$ even after the terminal time, *i.e.* $t > t_F$, provided that the forcing term $\delta(t)$ is removed at the terminal time, *i.e.* $u_0 = -B^T M z$ for $t > t_F$ [24]. Then, in order to reach in a finite time t_F the origin $z = 0$, and to maintain the system (7) at this point for $t > t_F$, a solution is

$$u_0 = \begin{cases} -B^T M z(t) + B^T \delta(t) & \text{for } 0 \leq t \leq t_F \\ -B^T M z(t) & \text{for } t > t_F \end{cases} \quad (11)$$

Let z_0 denote the solution of system (7) controlled by u_0 defined by (11): dynamics of z_0 are described by

$$\dot{z}_0 = A z_0 + B u_0 \quad (12)$$

As $z_0(t_F) = 0$ and given (11), one gets, for $t > t_F$, $z_0 = 0$ and $\dot{z}_0 = 0$: $z_0 = 0$ is a stable equilibrium point.

B. Integral sliding manifold

The basic idea consists in determining a sliding surface such that the state trajectories start on this surface at the initial time $t = 0$, which induces a sliding mode without reaching phase [31]. One has (for $1 \leq i \leq r - 1$)

$$\begin{aligned} \dot{z}_i &= z_{i+1} \\ \dot{z}_r &= \beta(\cdot) + u_0 + u_1 \end{aligned} \quad (13)$$

u_1 is a discontinuous function designed in order to *exactly* reject the perturbation $\beta(\cdot)$ throughout the entire response of the system. In order to reach this objective, the *integral sliding mode control* [31] is used. Let $\sigma \in \mathbb{R}$ define as

$$\sigma = z_r + \zeta_r + \lambda_{r-1}(z_{r-1} + \zeta_{r-1}) + \dots + \lambda_1(z_1 + \zeta_1) \quad (14)$$

with $\zeta_r \in \mathbb{R}$ called *integral sliding term* (which is determined later in this paper as $\zeta_{r-1}, \dots, \zeta_1$). Coefficients $\lambda_{r-1}, \dots, \lambda_1$ are such that $P(p) = p^{r-1} + \lambda_{r-1}p^{r-2} + \dots + \lambda_1$ is Hurwitz. Suppose that a sliding mode is established on the manifold $\mathcal{S}_f = \{x \mid \sigma = 0\}$ from $t = 0$. The time derivative of σ reads as

$$\dot{\sigma} = u_0 + u_1 + \beta(\cdot) + \dot{\zeta}_r + \lambda_{r-1}(\dot{z}_{r-1} + \dot{\zeta}_{r-1}) + \dots + \lambda_1(\dot{z}_1 + \dot{\zeta}_1) \quad (15)$$

As the objective is to ensure that a sliding mode on \mathcal{S}_f is established *early from* $t = 0$, one should have $\dot{\sigma} = 0$ for $t \geq 0$, and the discontinuous control u_1 should be replaced by the equivalent control u_{1eq} : in [16], [18], it is established that, in order to ensure $z = z_0$ for $t \geq 0$ (z_0 describes the *ideal and optimal* trajectory and its dynamics is defined by (12)), u_{1eq} has to satisfy

$$u_{1eq} = -\beta(\cdot), \quad \forall t \geq 0. \quad (16)$$

From (15), condition (16) is satisfied if

$$\dot{\zeta}_r = -u_0, \quad \dot{\zeta}_{r-1} = \zeta_r, \quad \dot{\zeta}_{r-2} = \zeta_{r-1}, \quad \dots, \quad \dot{\zeta}_1 = \zeta_2,$$

with $\zeta_r(0) = -z_r(0)$, $\zeta_{r-1}(0) = -z_{r-1}(0)$, \dots , $\zeta_1(0) = -z_1(0)$. Then, at $t = 0$, one has $\sigma(0) = 0$ and the sliding mode is stated on the *integral sliding manifold* \mathcal{S}_f for $t \geq 0$.

C. Discontinuous control part

The control law u_1 is designed in order to ensure that the sliding motion on $\mathcal{S}_f = \{x \mid \sigma = 0\}$ is guaranteed for $t \geq 0$ in spite of uncertainties, and reads as

$$u_1 = -\alpha \text{sign}(\sigma), \quad (17)$$

where the gain α is a positive constant tuned such that the η -attractivity condition is satisfied

$$\sigma \dot{\sigma} \leq -\eta |\sigma|, \quad \eta > 0. \quad (18)$$

From (15) and given that $\beta = \chi + (\Gamma - 1)u$ with $u = u_0 + u_1$, one has

$$\begin{aligned} \dot{\sigma} &= [\chi + (\Gamma - 1)(u_0 + u_1)] + u_0 + u_1 - u_0 \\ &\quad + \underbrace{\lambda_{r-1}(\dot{z}_{r-1} + \dot{\zeta}_{r-1}) + \dots + \lambda_1(\dot{z}_1 + \dot{\zeta}_1)}_{:= \delta} \\ &= \chi + (\Gamma - 1)u_0 + \Gamma u_1 + \delta. \end{aligned} \quad (19)$$

Define u_{0M} as the bound of u_0 , and $|\delta| < \Delta$. The convergence condition (18) is ensured if

$$\alpha > \frac{C_0 + |K_M - 1|u_{0M} + \Delta + \eta}{K_m}. \quad (20)$$

Under control law (17) with condition (20) fulfilled, system (13) evolves on the sliding manifold \mathcal{S}_f , its dynamics reads as (12). Its trajectories converge to zero in a finite time $t = t_F$ and are maintained in this equilibrium stable point.

Theorem 2 ([16], [18]): Consider the nonlinear system (1) with a relative degree r with respect to the sliding variable $s(x, t)$. Suppose the hypotheses H1-H2 fulfilled and $s(0), \dot{s}(0), \dots, s^{(r-1)}(0)$ bounded. Then, the control law u

$$u = u_0 + u_1$$

with u_0 defined by (11) and

$$\begin{aligned} u_1 &= -\alpha \cdot \text{sign} [s^{(r-1)} + \zeta_r + \lambda_{r-1}(s^{(r-2)} + \\ &\quad \zeta_{r-1}) + \dots + \lambda_1(s + \zeta_1)], \\ \dot{\zeta}_r &= -u_0 \text{ with } \zeta_r(0) = -s^{(r-1)}(0), \\ \zeta_{r-1}(0) &= -s^{(r-2)}(0), \dots, \zeta_1(0) = -s(0), \\ \alpha &> \frac{C_0 + |K_M - 1|u_{0M} + \Delta + \eta}{K_m}, \end{aligned} \quad (21)$$

allows the establishment of a r^{th} order sliding mode with respect to s with a finite time convergence $t = t_F < \infty$. ■

III. ELECTROPNEUMATIC SYSTEM

A. Description

The electropneumatic system under interest is a double acting actuator controlled by two servodistributors (see Figure 1) and composed by two chambers denoted P and N . Piston diameter is 63 mm and rod diameter 16 mm . With a source pressure equal to 7 bar , the maximum force developed by the actuator is 1750 N . The air mass flow rates q_m entering in the chambers are modulated by two three-way servodistributors Servotronic (Asco-Joucomatic) controlled by a micro-controller. The pneumatic jack horizontally moves a load carriage of mass M . This carriage is coupled to 4 springs (which restrain the displacement of the carriage and restore the initial position in the middle of the total stroke equal to 50 mm - see Figure 1) for a total of 75000 N/m rate. Additional dry friction is controlled by two skates, with a maximum value equal to 40 N . As the maximal displacement of carriage is 16 mm , the maximal spring force is 1200 N . The electropneumatic plant model is obtained from three

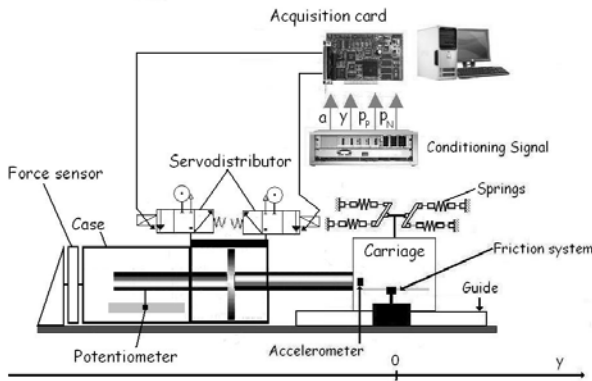


Fig. 1. Electropneumatic system

physical laws: the mass flow rate through a restriction, the pressure behaviour in a chamber with variable volume and the fundamental mechanical equation. The experimental set-up is simulated with a fluid power systems dedicated software AMESim, and the control law is developed under Matlab/Simulink, which implies a cosimulation program. In the sequel, two models are displayed

- the first one, which takes into account physical phenomena as temperature variations, practical values of mass flow rate, dynamics of servodistributors..., is developed under Amesim.
- The second one, which is simpler than the previous, is used in order to design the controller under Simulink.

B. Simulation model

Servodistributor model. The servodistributor model is composed in two parts, a dynamic part and a static one :

- Dynamic part is modeled by a second order transfer function

$$F(s) = \frac{\omega_{ns}^2}{s^2 + 2 \cdot \zeta_s \cdot \omega_{ns} s + \omega_{ns}^2} \quad (22)$$

with $\omega_{ns} = 246 \text{ rad} \cdot \text{s}^{-1}$ and $\zeta_s = 0.707$.

- Static part is modeled by an experimental table where mass flow rate is given in function of ratio pressure (upstream/downstream) and control voltage [26].

Pneumatic chamber variable volume model. Each chamber of the pneumatic actuator is considered as a variable volume, in which the air mass evolves with time. State the following assumptions :

- A1.** Air is a perfect gas and its kinetic is inconsequential. ■
- A2.** The pressure and the temperature are homogeneous in each chamber. ■
- A3.** The mass flow is pseudo-stationary. ■

The first dynamic principle applied to the air mass and the thermodynamic evolution of air in each chamber read as (with $X = P$ or N) [27]

$$\begin{aligned} \frac{dp_X}{dt} &= -\gamma \frac{p_X}{V_X} \frac{dV_X}{dt} + \frac{\gamma r T_r}{V_X} q_{mX_{in}} - \frac{\gamma r T_X}{V_X} q_{mX_{out}} \\ &\quad + \frac{(\gamma - 1) dQ_X}{V_X dt} \\ \frac{dT_X}{dt} &= -(\gamma - 1) \frac{T_X}{V_X} \frac{dV_X}{dt} + \frac{r T_X}{p_X V_X} (\gamma T_r - T_X) q_{mX_{in}} \\ &\quad - \frac{r T_X^2}{p_X V_X} (\gamma - 1) q_{mX_{out}} + (\gamma - 1) \frac{T_X}{p_X V_X} \frac{dQ_X}{dt} \end{aligned} \quad (23)$$

with γ the adiabatic constant, T_r the temperature inside the upstream tank, $q_{mX_{in}}$ the mass flow rate brought inside the X chamber, and $q_{mX_{out}}$ the mass flow rate brought outside the X chamber. Q_X , the thermal exchange with the X chamber wall, is described by assumption A4.

- A4.** The thermal exchange is due only by conduction described by

$$\frac{dQ_X}{dt} = \lambda S_{cX} (T_{cX} - T_X) \quad (24)$$

with λ the thermal exchange coefficient by conduction, S_{cX} the total area inside a X chamber, and T_{cX} the temperature of the X chamber wall. ■

Mechanical model. The second Newton law gives

$$\begin{aligned}\frac{dv}{dt} &= \frac{1}{M} [S(p_P - p_N) - F_f - b_v v - K_r y] \\ \frac{dy}{dt} &= v\end{aligned}\quad (25)$$

With friction force F_f included stiction, Coulomb and Stribeck phenomena.

Samplers and saturation. Samplers are added in AMESim's model in order to take into account samplers of acquisition card; sample time is 1 *ms*. Saturation signal control are added, *i.e.* $|u_{sat}| = 10$ V.

C. Control model

This model is developed in order to design the control law and a simplest version of the simulation one. The following hypotheses are added

A5. The process is polytropic and characterized by coefficient k (with $1 < k < \gamma$). In this case, Q dynamics are [27]

$$\begin{aligned}\frac{dQ_X}{dt} &= \frac{1}{\gamma - 1} [-r q_{mX_{in}} (\gamma T_r - k T_X) \\ &\quad + r T_X q_{mX_{out}} (\gamma - k) + p_X (\gamma - k) \frac{dV_X}{dt}]\end{aligned}\quad (26)$$

A6. Furthermore, the temperature variations in each chamber are inconsequential with respect to the supply temperature, *i.e.* $T_P = T_N = T$.

Then, pressures dynamics read as

$$\begin{aligned}\frac{dp_X}{dt} &= -k \frac{p_X}{V_X} \frac{dV_X}{dt} \\ &\quad + \frac{krT}{V_X} (q_{mX_{in}} - q_{mX_{out}})\end{aligned}\quad (27)$$

A7. The leakage between the two chamber and between servodistributor and jack are negligible.

By defining $q_m(u_X, p_X) := q_{mX_{in}} - q_{mX_{out}}$, one gets

$$\begin{aligned}\frac{dp_P}{dt} &= -k \frac{p_P}{V_P(y)} \frac{dV_P(y)}{dt} + \frac{krT}{V_P} q_m(u_P, p_P) \\ \frac{dp_N}{dt} &= -k \frac{p_N}{V_N(y)} \frac{dV_N(y)}{dt} + \frac{krT}{V_N} q_m(u_N, p_N)\end{aligned}\quad (28)$$

A8. All dry frictions forces are neglected.

A9. There is no control signal saturation.

A10. Dynamic part of servodistributor is neglected, and static part reads as

$$q_m(u_X, p_X) = \varphi(p_X) + \psi(p_X, \text{sign}(u_X)) u_X$$

with φ and ψ defined as 5th-order polynomials with respect to p_X [2].

A11. Only the position of the actuator is controlled, which means that the problem is a single input-single output (SISO). It implies that $u_P = -u_N = u$. ■

Then, with $V_P(y) = V_0 + S \cdot y$ and $V_N(y) = V_0 - S \cdot y$ (V_0 being equal to the half of the cylinder volume), the model used for the design of controller is a nonlinear system reads as

$$\begin{aligned}\dot{p}_P &= \frac{krT}{V_P(y)} [\varphi(p_P) + \psi(p_P, \text{sign}(u)) u - \frac{S}{rT} p_P v] \\ \dot{p}_N &= \frac{krT}{V_N(y)} [\varphi(p_N) - \psi(p_N, \text{sign}(-u)) u + \frac{S}{rT} p_N v] \\ \dot{v} &= \frac{1}{M} [S p_P - S p_N - b_v v - K_r y] \\ \dot{y} &= v\end{aligned}\quad (29)$$

D. Conclusion

The use of different models for the simulation and for the control design implies that, if high accuracy is required, it is necessary to design robust control law with respect to uncertainties generated by the difference between these two models. These uncertainties are taken into account, in the controller design, through two models

- The so-called “nominal” model (29),
- The so-called “real” model which takes into account the uncertainties on several parameters or functions (Temperature T , functions φ and ψ , mass M)

$$\begin{aligned}\dot{p}_P &= \frac{kr(T + \Delta T)}{V_P(y)} [\varphi + \Delta\varphi - \frac{S}{r(T + \Delta T)} p_P v] \\ &\quad + \frac{kr(T + \Delta T)}{V_P(y)} (\psi + \Delta\psi) u \\ \dot{p}_N &= \frac{kr(T + \Delta T)}{V_N(y)} [\varphi + \Delta\varphi - \frac{S}{r(T + \Delta T)} p_N v] \\ &\quad - \frac{kr(T + \Delta T)}{V_N(y)} (\psi + \Delta\psi) u \\ \dot{v} &= \frac{1}{M + \Delta M} [S p_P - S p_N - b_v v - K_r y] \\ \dot{y} &= v\end{aligned}\quad (30)$$

with $|\Delta T| < T_M$, $|\Delta\varphi| < \varphi_M$, $|\Delta\psi| < \psi_M$ and $|\Delta M| < M_M$ all bounded values.

E. Desired trajectory

The desired position trajectory, named $y_d(t)$, is displayed in Figure 2 and has been designed such that associated velocity and acceleration are continuous functions. The maximum velocity (resp. acceleration) is 0.8 *m*·*s*⁻¹ (resp. 33 *m*·*s*⁻²). This displacement corresponds to 64% of the total stroke around the central position. The initial conditions are such that $[y(0) \ v(0) \ \dot{v}(0)]^T = [0 \ 0 \ 0]^T$.

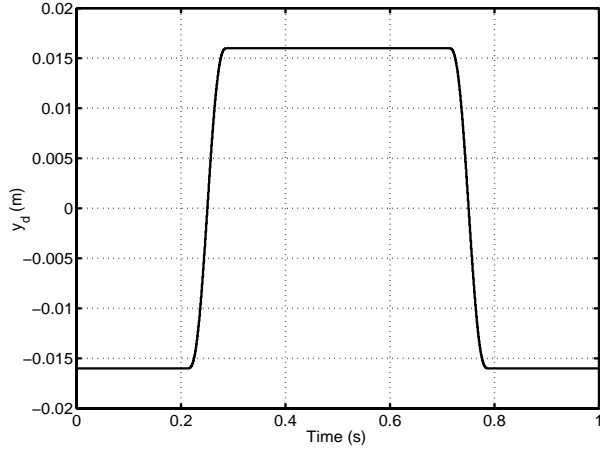


Fig. 2. Desired position (m) versus time (sec)

IV. APPLICATION OF INTEGRAL SLIDING MODE CONTROLLER

A. Controller design

The objective consists in designing a robust (with respect to uncertainties/disturbances) position controller. Define s the sliding variable as $s = y - y_d(t)$: from (30), its relative degree with respect to u equals 3, which implies that a 3rd order sliding mode controller is designed. The validity of the control law depends on the stability of the unobservable subsystem of dimension one, which is supposed. One has

$$s^{(3)} = \chi(\cdot) + \Gamma(\cdot)u \quad (31)$$

with $\bar{T} = T + \Delta T$, $\bar{\varphi} = \varphi + \Delta\varphi$, $\bar{\psi} = \psi + \Delta\psi$, $\bar{M} = M + \Delta M^2$, and

$$\begin{aligned} \chi(\cdot) &= \frac{kr\bar{T}S}{M} \left(\frac{\bar{\varphi}(p_P)}{V_P(y)} - \frac{\bar{\varphi}(p_N)}{V_N(y)} \right) - \frac{kS^2v}{M} \left(\frac{p_P}{V_P(y)} - \frac{p_N}{V_N(y)} \right) \\ &\quad - \frac{b_v}{M^2} (S(p_P - p_N) - b_v v - K_r y) - \frac{K_r v}{M} - y_d^{(3)} \\ \Gamma(\cdot) &= \frac{kr\bar{T}S}{M} \left(\frac{\bar{\psi}(p_P, \text{sign}(u))}{V_P(y)} + \frac{\bar{\psi}(p_N, \text{sign}(-u))}{V_N(y)} \right) \end{aligned} \quad (32)$$

The control law is defined as $u = \Gamma_{Nom}^{-1} \cdot [-\chi_{Nom} + v]$ with Γ_{Nom} (resp. χ_{Nom}) the nominal value of Γ (resp. χ), i.e. derived from (32) with no uncertainties. It is important to note that Γ_{nom} is always strictly positive. So u has the same sign as $(-\chi_{nom} + v)$. By definition, χ_{nom} and v are independent of u . Consequently, the control law is not implicit. This explains the choice of the function φ , independent of u , in the mass flow rate expression of assumption A10. Then, one gets

$$s^{(3)} = \bar{\chi}(\cdot) + \bar{\Gamma}(\cdot)v \quad (33)$$

with $\bar{\chi} = \chi - \Gamma\Gamma_{Nom}^{-1}\chi_{Nom}$ and $\bar{\Gamma} = \Gamma\Gamma_{Nom}^{-1}$. Let $z = [s \ \dot{s} \ \ddot{s}]^T$. Then, as shown in Section II, the 3rd order sliding

mode control is equivalent to the finite time stabilization of

$$\dot{z} = \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}}_A z + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}_B v + \begin{bmatrix} 0 \\ 0 \\ \beta(\cdot) \end{bmatrix} \quad (34)$$

where $\beta(\cdot)$ is defined as $\beta = \bar{\chi} + (\bar{\Gamma} - 1)v$. As mentioned in Section II, the integral sliding mode controller v reads as

$$v = v_0 + v_1.$$

The first part v_0 is a continuous one which ensures that s , \dot{s} , and \ddot{s} converge to 0 at a fixed time t_F ; the second one v_1 is a discontinuous one which ensures the previous convergence in spite of uncertainties. As mentioned in Section II, the design follows several steps.

Continuous part v_0 . Matrix Q is stated as

$$Q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Then, from (10), one gets

$$M = \begin{bmatrix} 2.4142 & 2.4142 & 1.0000 \\ 2.4142 & 4.8284 & 2.4142 \\ 1.0000 & 2.4142 & 2.4142 \end{bmatrix}$$

From (10), one gets $\delta(t) = e^{A_m t} \delta(0)$ with $A_m = -[A^T - MBB^T]$. Then, given $\delta(t)$, and from (7)-(9), one gets $\dot{z} = -A_m^T z + BB^T e^{A_m t} \delta(0)$. By multiplying both side of previous equation by $e^{A_m^T t}$, and integrating from $t = 0$ to $t = t_F$, with $z(t_F) = 0$, one gets

$$z(0) = - \underbrace{\left[\int_0^{t_F} e^{A_m^T t} BB^T e^{A_m t} dt \right]}_H \cdot \delta(0)$$

Matrix H is the partial reachability gramian of linear system $\dot{x} = A_m x + Bv$ and is, viewed the form of A_m , B and $t_F < \infty$, invertible. Then, the initial condition $\delta(0)$ ensuring that $z(t_F) = 0$ can be derived from $\delta(0) = -H^{-1}z(0)$. The matrix H is evaluated using an algorithm from [32] which yields $\delta_1(0) = -3.6034e4$, $\delta_2(0) = -3.6034e3$ and $\delta_3(0) = -1.2002e2$ for $z(0) = [0.016 \ 0 \ 0]^T$ and a convergence time fixed to $t_F = 0.2$ sec. Then, v_0 reads as (with $|v_0| < v_{0M} = 120$)

$$\begin{aligned} v_0 &= \begin{cases} -B^T M z(t) + B^T \delta(t) & \text{for } 0 \leq t \leq t_F \\ -B^T M z(t) & \text{for } t > t_F \end{cases} \\ \dot{\delta} &= -(A^T - MBB^T) \delta \end{aligned} \quad (35)$$

Discontinuous part v_1 . The switching variable σ reads as $\sigma = \ddot{s} + \zeta_3 + 2\xi\omega_n(\dot{s} + \zeta_2) + \omega_n^2(s + \zeta_1)$ with $\zeta_3(0) = -\ddot{s}(0)$, $\zeta_2(0) = -\dot{s}(0)$ and $\zeta_1(0) = -s(0)$, and $\dot{\zeta}_1 = \zeta_2$, $\dot{\zeta}_2 = \zeta_3$, $\zeta_3 = v_0$. Then, one gets $v_1 = -\alpha \cdot \text{sign}(\sigma)$ with $\xi = 0.7$, $\omega_n = 190 \text{ rad} \cdot \text{s}^{-1}$ and $\alpha = 10^5$ (in order to satisfy (18)-(20) by taking into account the uncertainties due to the difference of simulation and control models, and the variations of mass (+125%).

²It is supposed that $\Delta\dot{M} = 0$.

B. Simulation results

Simulations have been made in cosimulation with Amesim-Simulink : the “complete” model of the experimental set-up displayed in Section III-B is simulated by Amesim, and the previous controller based on the “simplified” model displayed in Section III-C is implemented with Simulink. Two kind of simulations have been made. The first simulations, named “Nominal case”, consist in considering that the moving mass is $0.8kg$ (i.e. the control law has been designed with this hypothesis). The second simulations consist in increasing the moving mass to $1.8kg$, without changing the structure of the controller and its gains values. In order to evaluate the performances of integral sliding mode controller, a classical state feedback control, defined as

$$u = K_y(y - y_d) + K_v v + K_a a \quad (36)$$

has been designed by using a linear model around actuator position $y = 0$ [5], and by computing gains K_y , K_v and K_a by Ackerman’s approach such that poles placement authorized 4.6% overshoot [11], which gives $K_y = 3119 \text{ V/m}$, $K_v = 2.618 \text{ V/m/s}$ and $K_a = 0.03125 \text{ V/m/s}^2$.

Nominal case. The actuator position (Figure 3) converges to the desired trajectory in $0.2s$ (which is the stated convergence time t_F) for ISM³ controller without overshoot. With CF controller, the convergence is ensured in $0.15s$, with a 10% overshoot. The maximum error position in steady state is $0.18mm$ for ISM controller and $0.16mm$ for CF controller. During all trajectory tracking with ISM controller, there is no pressure saturation (Figure 4-Top) and control input is realistic (Figure 5). Note that, in steady state, the force developed by the actuator, $F = S \cdot (p_P - p_N)$, allows to compensate springs force, as shown in Figure 4-Bottom.

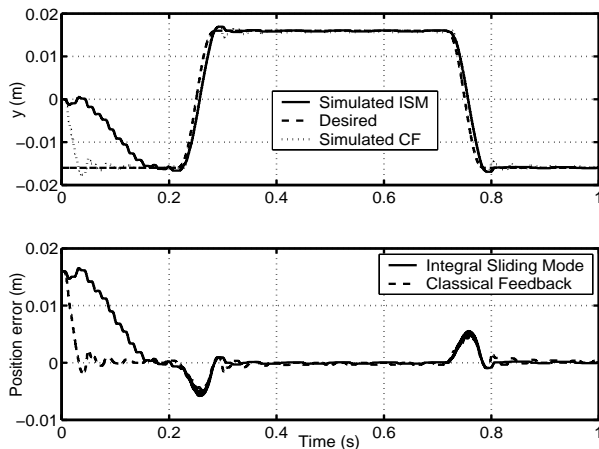


Fig. 3. **Top.** Desired and current positions (m) versus time (sec). **Bottom.** Positions errors (m) versus time (sec).

Robustness evaluation. The ISM controller still ensures

³In the sequel, “ISM” denotes Integral Sliding Mode controller, and “CF” denote Classical State Feedback controller

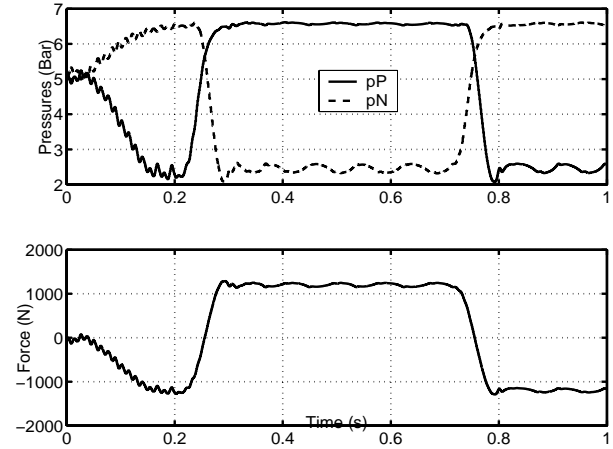


Fig. 4. **Top.** pressure (p_P and p_N) (bar) versus time (sec). **Bottom.** Force (N) versus time (sec).

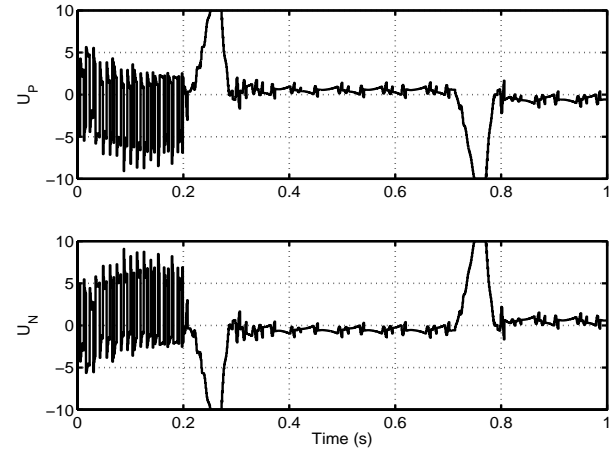


Fig. 5. **Top.** u_P (V) versus time (sec). **Bottom.** u_N (V) versus time (sec).

convergence in $0.2s$ without overshoot (Figure 6). With CF controller, overshoot increased at 19%. The maximum position error in steady state equals $0.3mm$ for ISM controller and $0.65mm$ for CF controller. This robustness evaluation with respect to mass modification confirms the best efficiency of ISM controller versus CF one.

V. CONCLUSIONS

This paper has proposed the application of a higher order sliding mode controller based on integral sliding mode [16], [18] to the position control of a pneumatic actuator. Design and simulations have shown the efficiency and applicability of the control approach to the pneumatic area. The further works concern the practical application of the control solution, and the design of 4th order sliding mode controller in order to improve the trajectories tracking accuracy.

ACKNOWLEDGEMENTS

This work is supported by Direction Générale de l’Armement (DGA-French Defense Ministry) through the Alexis GIRIN’s Ph.D. grant.

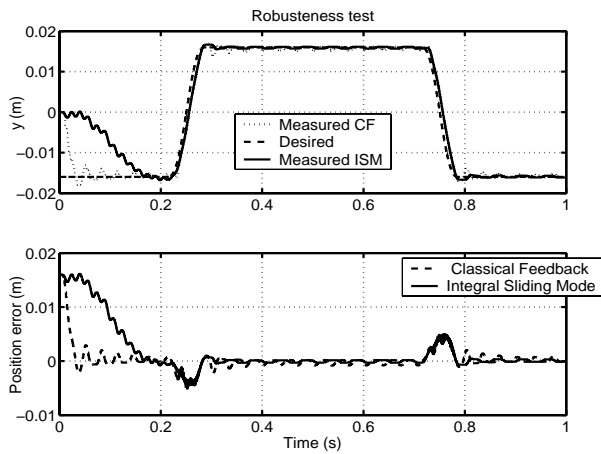


Fig. 6. **Top.** Desired and current positions (m) versus time (sec). **Bottom.** Positions errors (m) versus time (sec).

REFERENCES

- [1] G. Bartolini, A. Ferrara, and E. Usai, "Chattering avoidance by second-order sliding mode control", *IEEE Trans. Automat. Control*, vol.43, no.2, 1998, pp.241-246.
- [2] M. Belgharbi, D. Thomasset, S. Scavarda, and S. Sesmat, "Analytical model of the flow stage of a pneumatic servo-distributor for simulation and nonlinear control", in *Scandinavian International Conference on Fluid Power SICFP'99*, Tampere, Finland, 1999, pp.847-860.
- [3] M. Bouri, D. Thomasset, and S. Scavarda, "Integral sliding mode controller of a rotational servodrive", in *JHPS International Symposium on Fluid Power*, Tokyo, Japan, 1996, pp.145-150.
- [4] M. Bouri, and D. Thomasset, "Sliding control of an electropneumatic actuator using an integral switching surface", *IEEE Trans. Control Syst. Technology*, vol.2, no.2, 2001, pp.368-375.
- [5] X. Brun, S. Sesmat, D. Thomasset, and S. Scavarda, "A comparative study between two control laws of an electropneumatic actuator", in *European Control Conference ECC'99*, Karlsruhe, Germany, 1999.
- [6] X. Brun, and D. Thomasset, "Choice of control law in electropneumatics. Expertise using an industrial benchmark and some new trends", in *Conference on Decision and Control CDC'00*, Sydney, Australia, 2000.
- [7] X. Brun, D. Thomasset, and E. Bideaux, "Influence of the process design on the control strategy: application in electropneumatic field", *Control Engineering Practice*, vol.10, no.7, 2002, pp.727-735.
- [8] P. de Larminat, "Contrôle d'état standart" [in French], Hermès, Paris, France, 2000.
- [9] K.A. Edge, "The control of fluid power systems - responding to the challenge", *Journal of Systems and Control Engineering*, vol.211, no.12, 1997, pp.91-110.
- [10] A.F. Filippov, *Differential Equations with Discontinuous Right-Hand Side*, Kluwer, Dordrecht, The Netherlands, 1988.
- [11] G.F. Franklin, J.D. Powell, and A. Emani-Naeini, *Feedback Control of Dynamic Systems*, Addison-Wesley, Reading, Massachusetts, 1987.
- [12] K. Hamiti, A. Voda-Besançon, and H. Roux-Buisson, "Position control of a pneumatic actuator under the influence of stiction", *Control Engineering Practice*, vol.4, no.8, 1996, pp.1079-1088.
- [13] T. Kimura, S. Hara, T. Fujita, and T. Kagawa, "Feedback linearization for pneumatic actuator systems with static friction", *Control Engineering Practice*, vol.5, no.10, 1997, pp.1385-1394.
- [14] S. Laghrouche, F. Plestan, and A. Glumineau, "Higher order sliding mode control based on optimal linear quadratic control", in *European Control Conference ECC'03*, Cambridge, England, 2003.
- [15] S. Laghrouche, M. Smaoui, X. Brun, and F. Plestan, "Second order sliding mode controllers for pneumatic actuators", in *American Control Conference ACC'04*, Boston, Massachusetts, 2004.
- [16] S. Laghrouche, F. Plestan, and A. Glumineau, "Higher order sliding mode control based on optimal LQ control and integral sliding mode", in *IFAC Symposium on Nonlinear Control Systems NOLCOS'04*, Stuttgart, Germany, 2004.

- [17] S. Laghrouche, M. Smaoui, and F. Plestan, "Third-order sliding mode controller for electropneumatic actuators", in *IEEE Conference on Decision and Control CDC'04*, Paradise Island, The Bahamas, 2004.
- [18] S. Laghrouche, F. Plestan, and A. Glumineau, "Higher order sliding mode control based on integral sliding mode", submitted to *Automatica* (second lecture), 2006.
- [19] A. Levant, "Sliding order and sliding accuracy in sliding mode control", *International Journal of Control*, vol.58, no.6, 1993, pp.1247-1263.
- [20] A. Levant, "Universal SISO sliding-mode controllers with finite-time convergence", *IEEE Trans. Automat. Control*, vol. 49, no.9, 2001, pp.1447-1451.
- [21] S. Ming-Chang, and T. Shy-I, "Identification and position control of a servo pneumatic cylinder", *Control Engineering Practice*, vol.3, no.9, 1995, pp.1285-1290.
- [22] A.K. Paul, J.K. Mishra, and M.G. Radke, "Reduced order sliding mode control for pneumatic actuator", *IEEE Trans. Control Syst. Technology*, vol.2, no.3, 1994, pp.271-276.
- [23] F. Plestan, S. Laghrouche, and A. Glumineau, "Multivariable practical higher order sliding mode control", in *IEEE Conference on Decision and Control - European Control Conference CDC-ECC'05*, Sevilla, Spain, 2005.
- [24] Z.V. Rekasius, "An alternate approach to the fixed terminal point regulator problem", *IEEE Trans. Autom. Control*, vol.9, 1964, pp.290-292.
- [25] E. Richard, and S. Scavarda, "Comparison between linear and nonlinear control of an electropneumatic servodrive", *J. of Dynamic Systems, Measurement, and Control*, vol.118, 1996, pp.245-252.
- [26] S. Sesmat, and S. Scavarda, "Static characteristics of a three way servovalve", in *Conference on Fluid Power Technology*, Aachen, Germany, 1996, pp.643-652.
- [27] J.L. Shearer, "Study of pneumatic processes in the continuous control of motion with compressed air", *Trans. Am. Soc. Mech. Eng.*, vol.78, 1956, pp.233-249.
- [28] J.J. Slotine, "Sliding mode controller design for non-linear systems", *International Journal of Control*, vol.40, no.2, 1984, pp.421-434.
- [29] M.Smaoui, X.Brun, D. Thomasset, "A combined first and second order sliding mode approach for position and pressure control of an electropneumatic system", in *American Control Conference ACC'05*, Portland, Oregon, 2005.
- [30] V.I. Utkin, *Sliding Mode in Control and optimization*, Springer-Verlag, Berlin, Germany, 1992.
- [31] V.I. Utkin, and J. Shi, "Integral sliding mode in systems operating under uncertainty", in *IEEE Conference on Decision and Control CDC'96*, Kobe, Japan, 1996.
- [32] C.F. VanLoan, "Computing integrals involving the matrix exponential", *IEEE Trans. Automat. Control*, vol.23, no.3, 1978, pp.395-404.
- [33] L. Yang, and J.H. Lilly, "Sliding mode tracking for pneumatic muscle actuators in bicep/tricep pair configuration", in *American Control Conference ACC'03*, Denver, Colorado, 2003.