



Dynamics of monopolar vortex structures on the β -plane

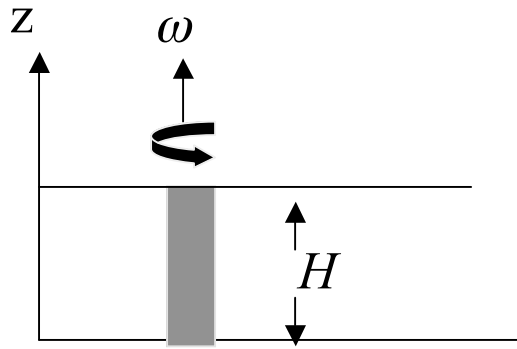
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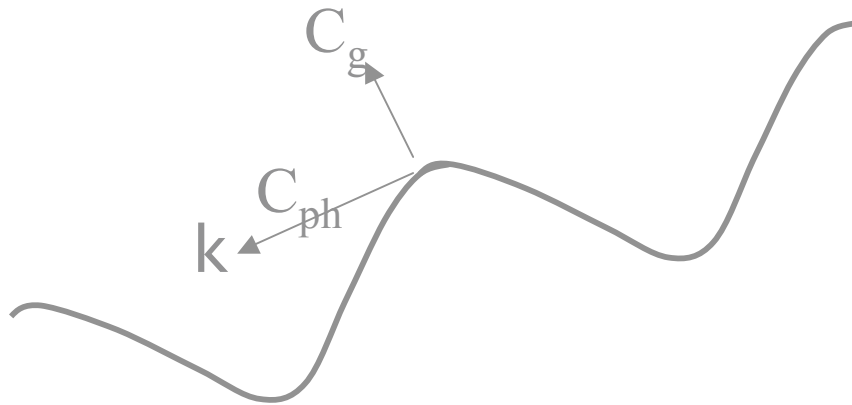
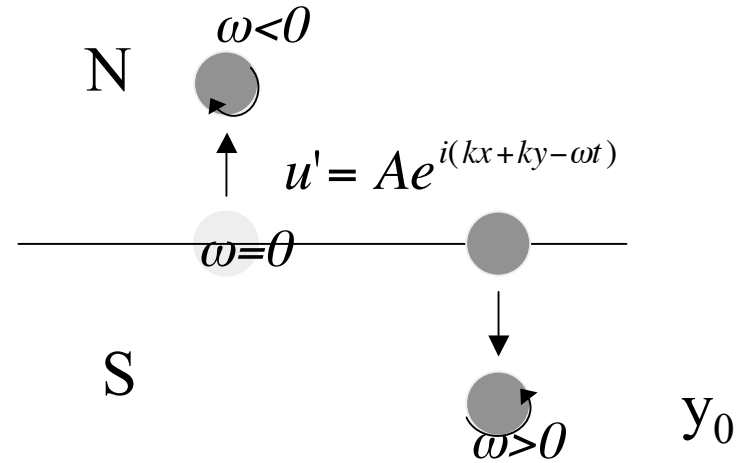
J. Fluid Mech. 2002, vol **456**.

(presentation MIT)

Reminder Rossby Waves



$$\frac{D}{Dt} \left[\frac{f + \omega}{H} \right] = 0$$



$$\omega \cong -\frac{k\beta}{k^2 + l^2}$$

$$\vec{C}_{ph} = -\frac{k\beta}{(k^2 + l^2)^{3/2}} \hat{k} \sim \beta(2R)^2$$

$$\vec{C}_g = \frac{\partial \omega}{\partial k} \hat{i} + \frac{\partial \omega}{\partial l} \hat{j}$$

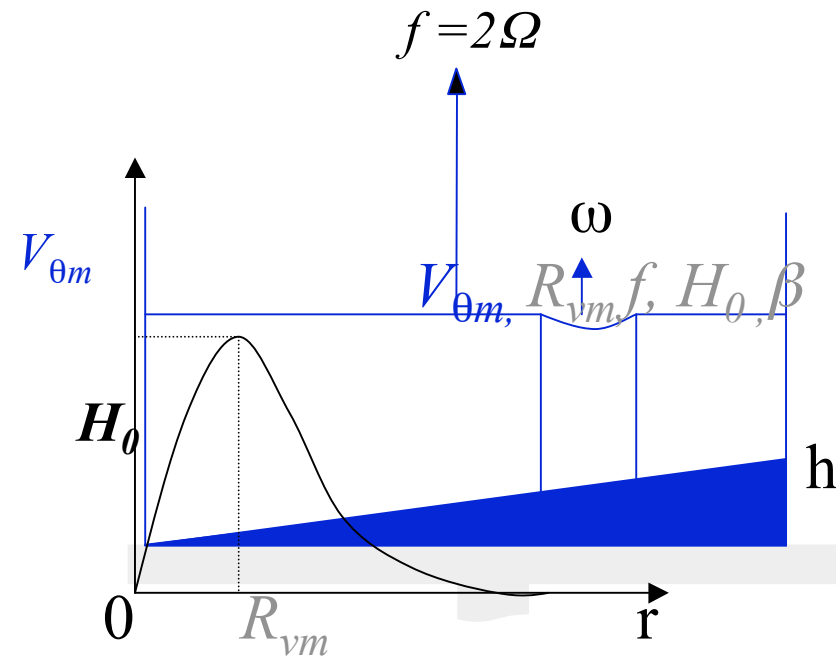
Topographic β -plane:

Using conservation of potential vorticity :

$$\frac{f + \omega}{H_0(y)} = \text{const.}$$

leading order :

$$f = f_0 + \beta y \quad \text{with} \quad \beta = \frac{f_0 dh / dy}{H_0}$$



With numbers $B_u = (R_{vm} / R_d)^2$ $R_d = \sqrt{gH_0 / f}$

the scaled leading order vorticity equation is:

$$\frac{D}{Dt} [\omega - B_u \eta] + \frac{1}{\delta} u_y = 0$$

$$\delta = v_{\theta m} / \beta R_{vm}^2$$

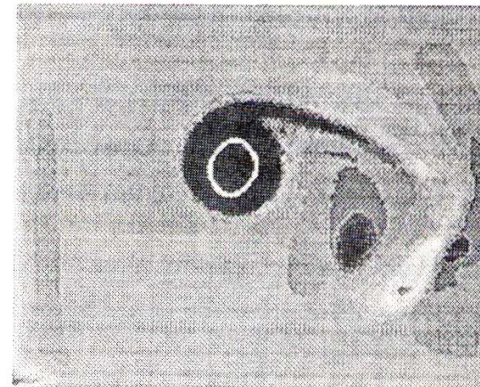
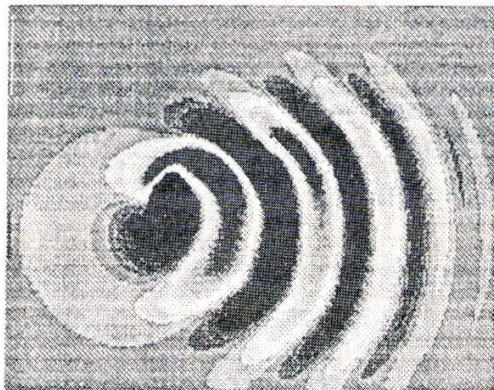
Basic dynamics: Nonlinear effects ($Bu \ll 1$)

$$\frac{\partial \omega}{\partial t} + \underline{u} \cdot \nabla \omega + \frac{1}{\delta} u_y = 0$$

vortex intensity $\delta = v_{\theta m} / \beta R_{vm}^2$

δ small

δ large



Simulation by Walsteijn & Cremers (see van Heijst, 1994)

Dispersion of the vortex into Rossby waves.

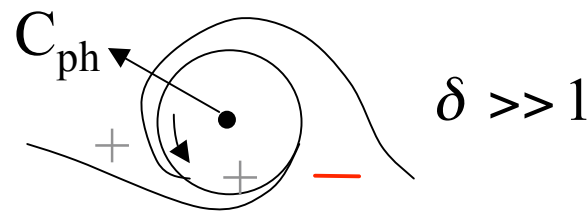
Dominant nonlinear effects.

δ large \implies

- ◆ Advection of ambient vorticity; the vortex is intensified at its west-side
-> displacement of the vortex centre to the West (Flierl et al. 1979):.



- ◆ Intense singular vortices on a β -plane (Reznik, 1992)

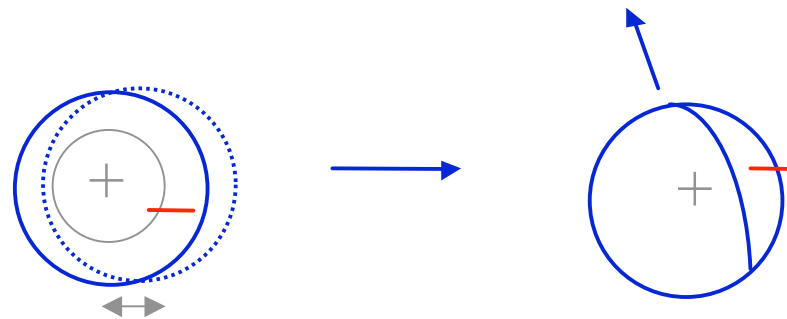


Eventually: wrapping around -> isolated vortex.

For $\delta \geq 1$ strong interactions with Rossby waves:

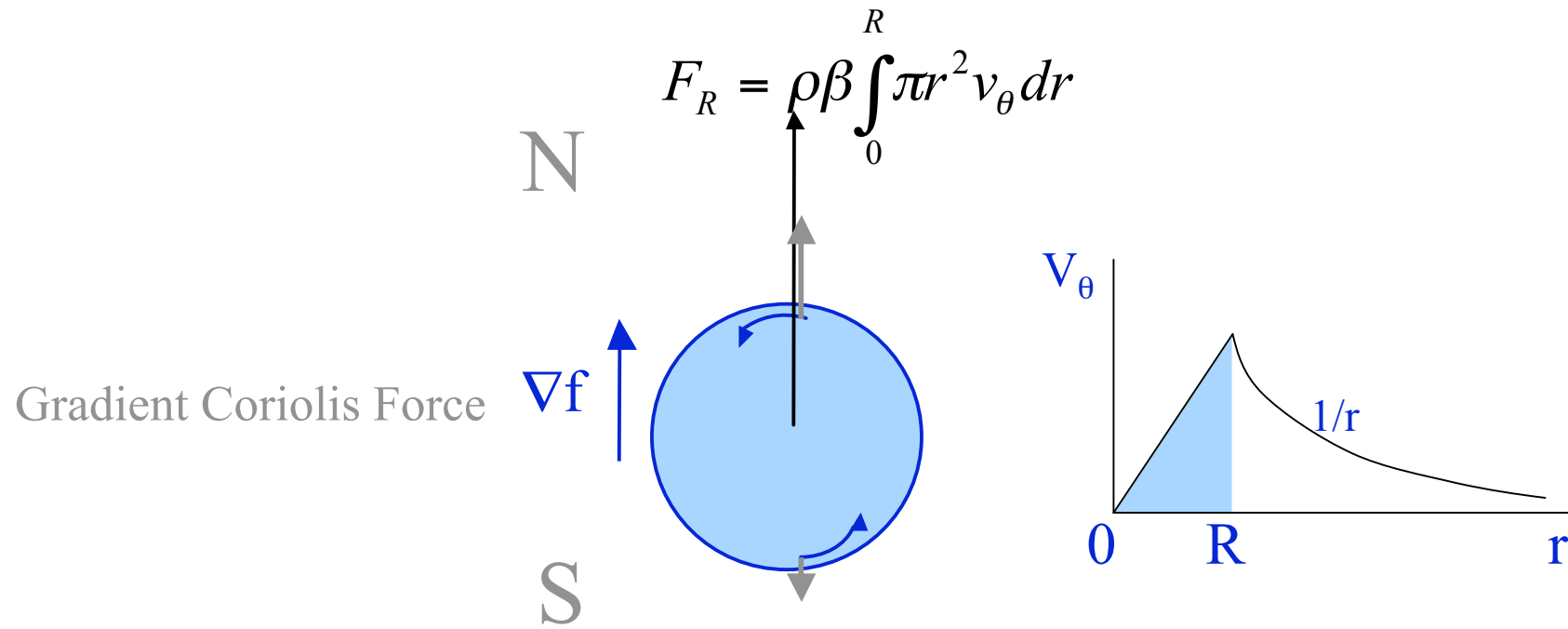
- ◆ Reznik & Dewar, 1994 arbitrary initial velocity field.
Korotaev & Fedotov, 1994:
Llewellyn Smith, 1997: intense isolated vortices
Match ambient Rossby waves ($\delta < 1$) to vortex $\delta > 1$.

- ◆ Dipolar moment due to an Eastward displacement and redistribution of vorticity of a shielded vortex (Stern & Radko, 1998)

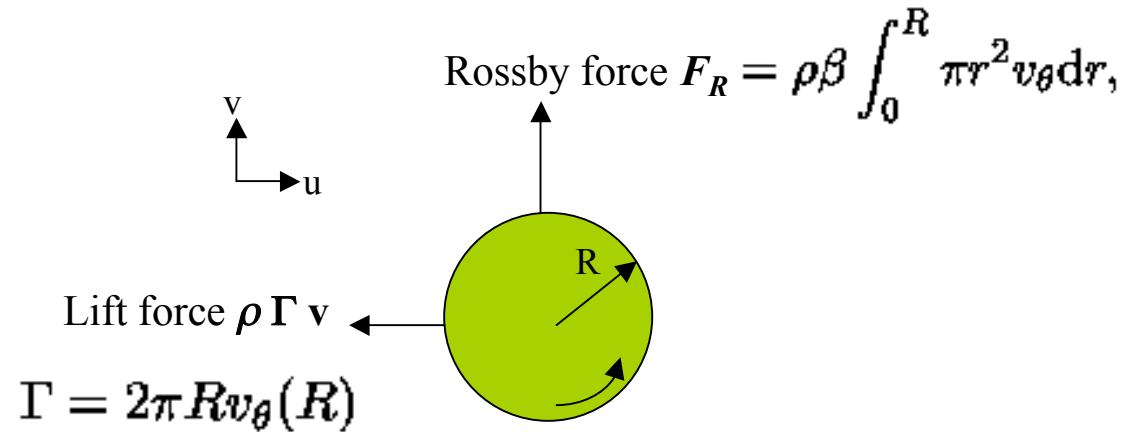


Modelling

- ◆ “Rossby force” (Rossby, 1948)



Model



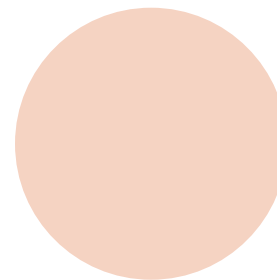
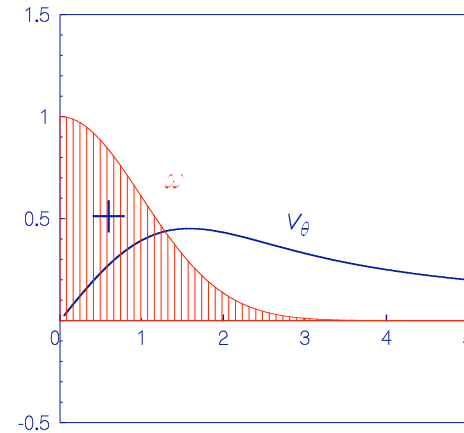
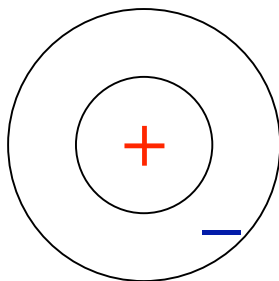
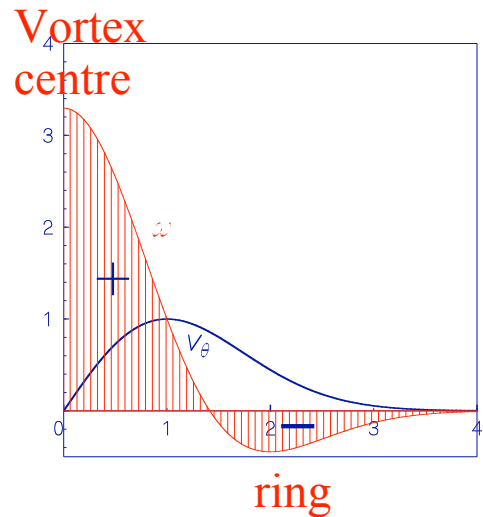
R is the radius of a circular streamline enclosing the fluid containing vorticity. $R \equiv 1\% \Omega_{\max}$.

Isolated vortex	$v_\theta(R) = 0$	$\Gamma = 0$
Non-isolated vortex	$v_\theta(R) > 0$,	$\Gamma > 0$

Vortex generation

Stirring in a cylinder:
Isolated (shielded) vortex

Suction of fluid:
sink vortex (non-shielded)

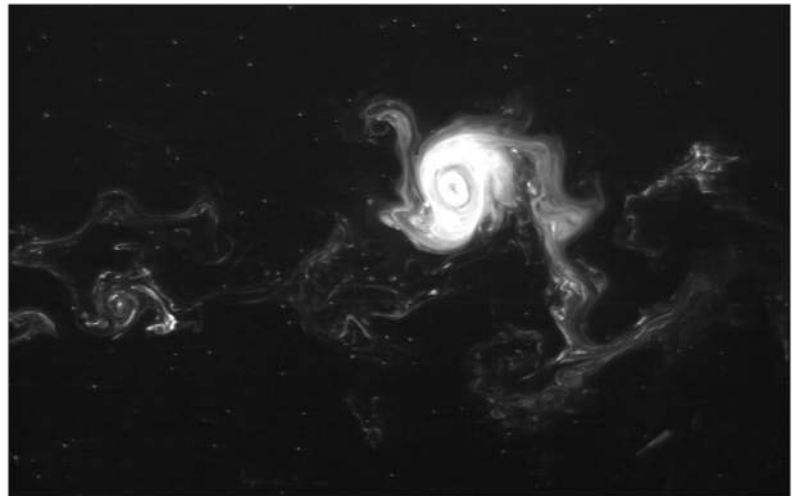
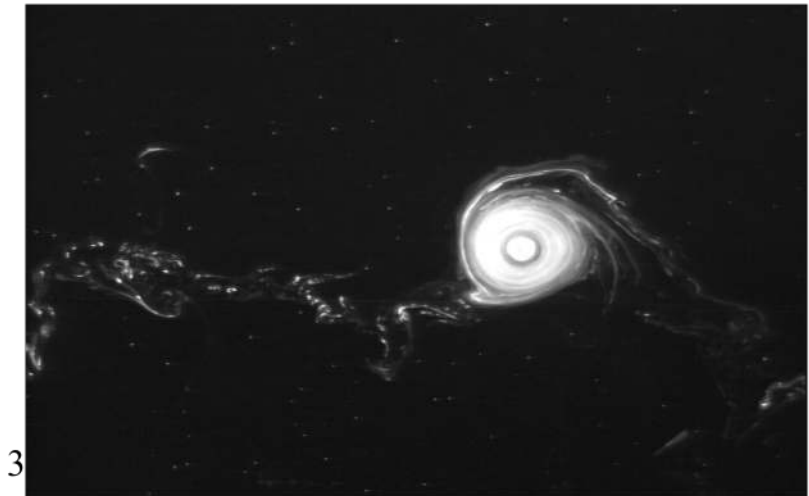
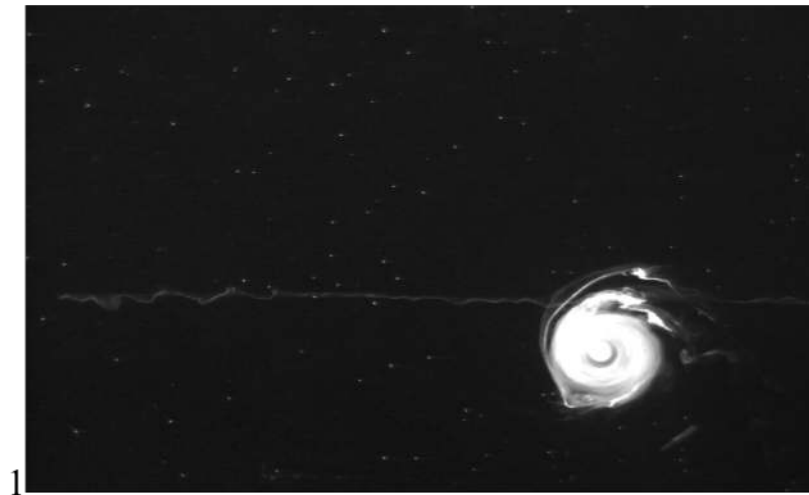


-> different vortex velocity profiles, velocity steepness α

$\alpha=3$ isolated, $\alpha=1$ non isolated vortex Carnevale et al, 1991

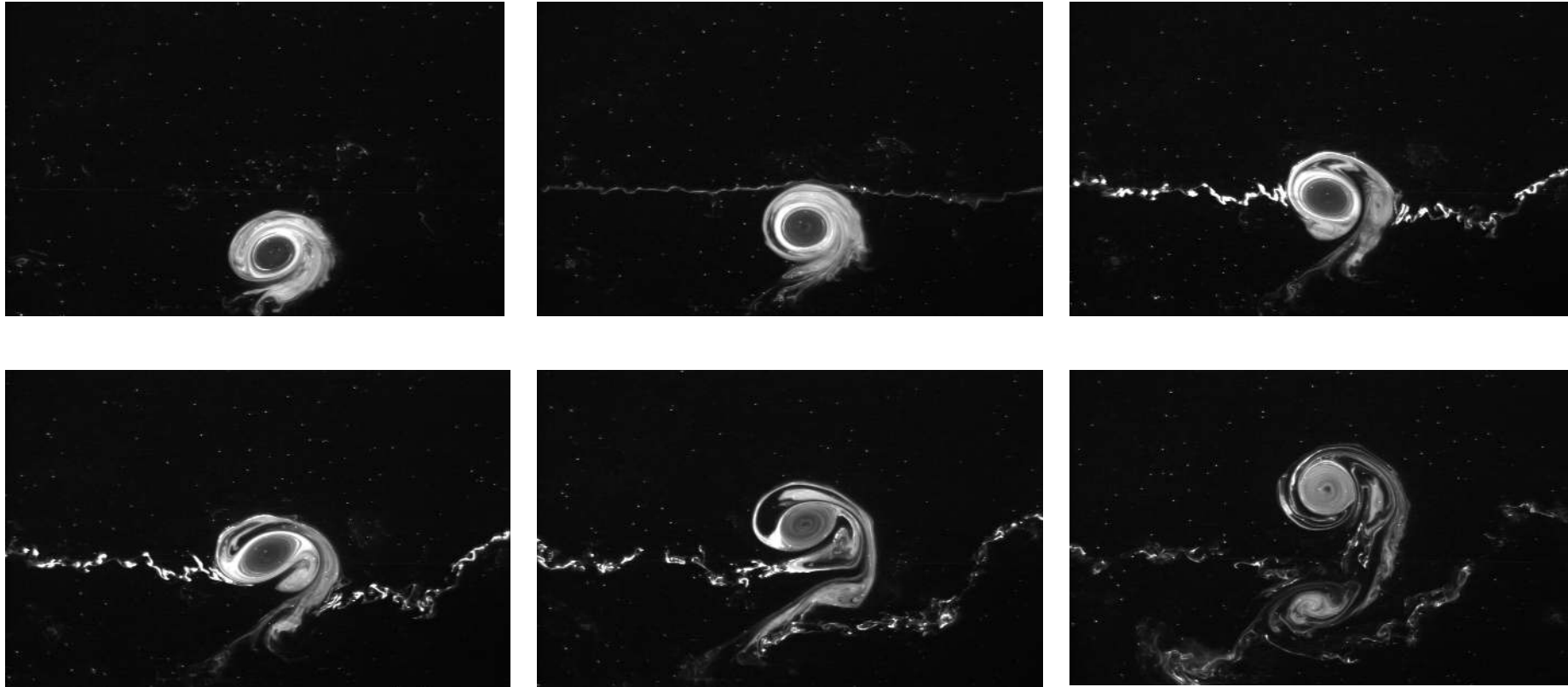
==> **Varie α**

Suction vortex ('non-isolated')



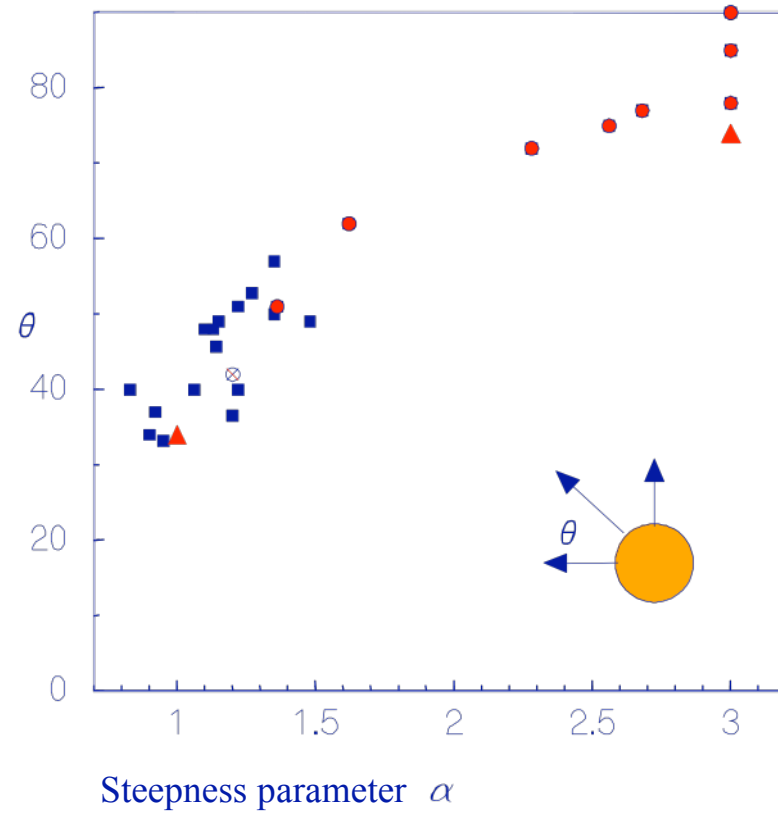
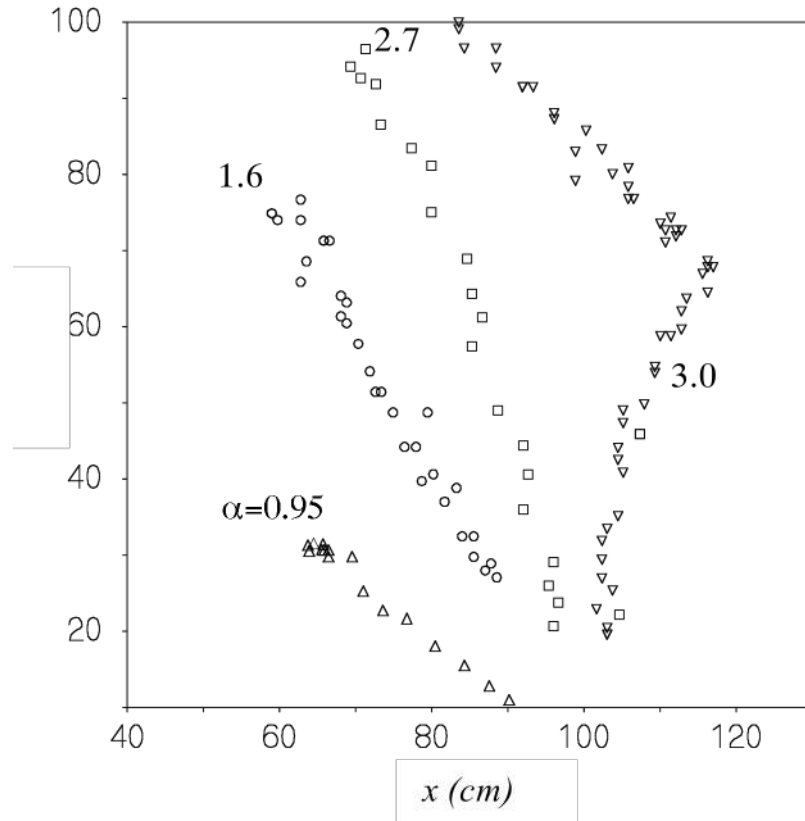
1T, 2.7T, 6.2T, 14.7T

Stir vortex (isolated)



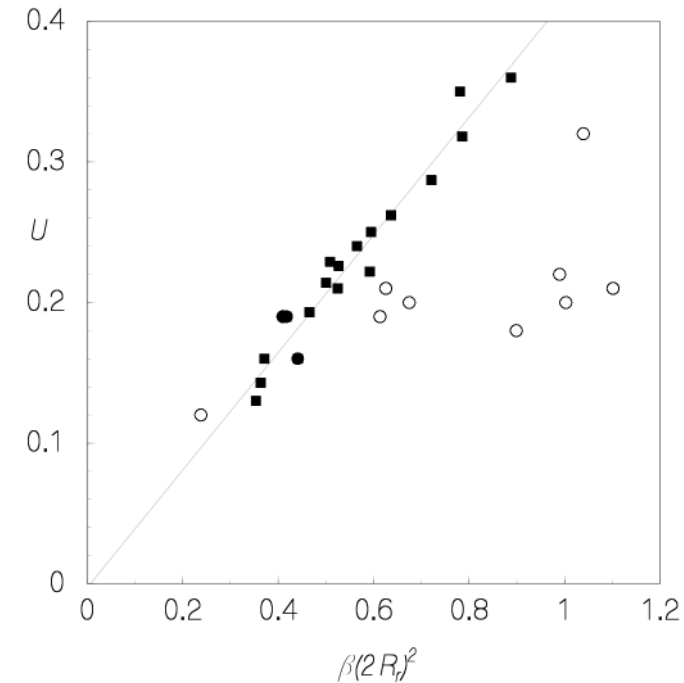
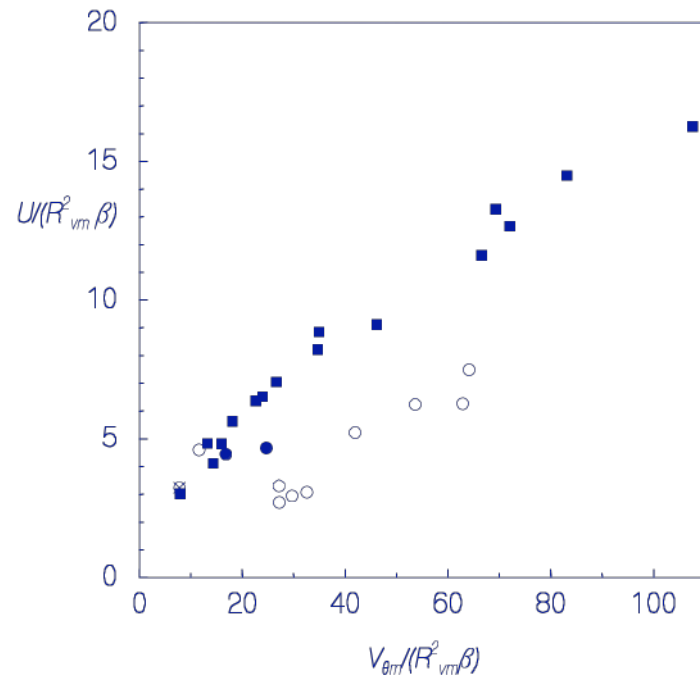
1.5T, 2.5T, 3.7T, 4.2T, 6.0T, 9.7T

Vortex trajectories



- ▲ Carnevale, Kloosterziel & van Heijst 1991
- Coriolis platform Isolated & suction vortices
- Small platform suction vortices

Translation velocity U



○: strong velocity gradient $\alpha > 1.4$

■: weak velocity gradient $\alpha < 1.4 \implies$

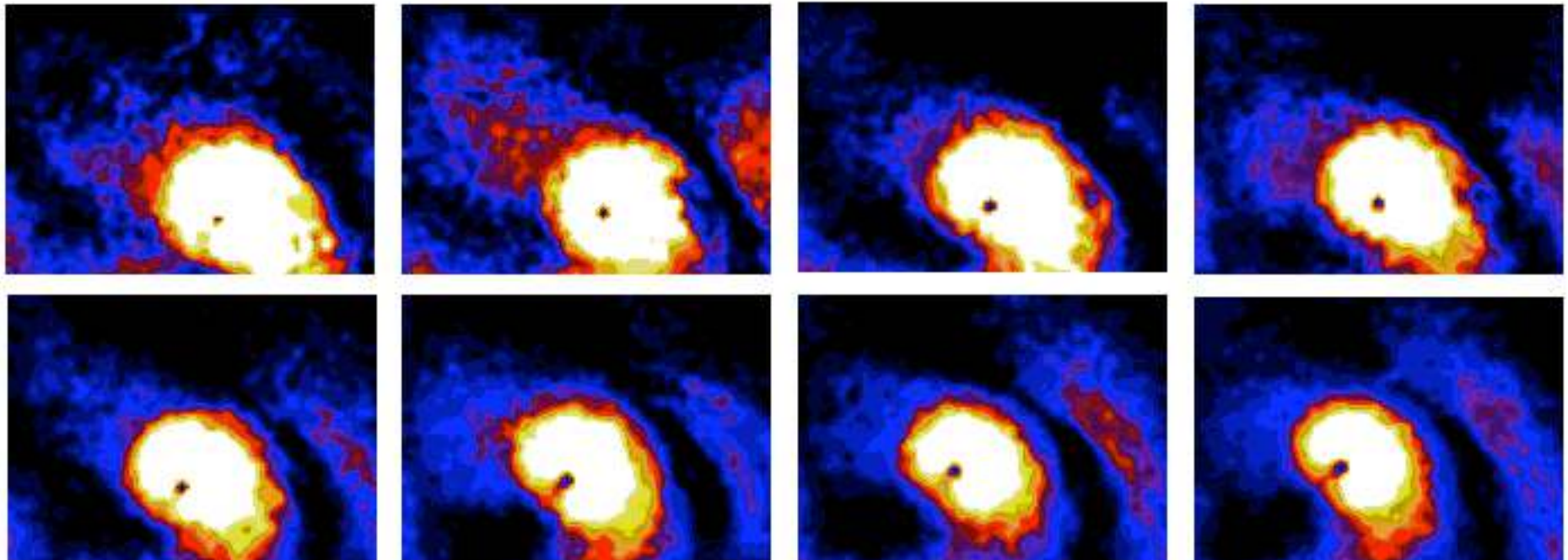
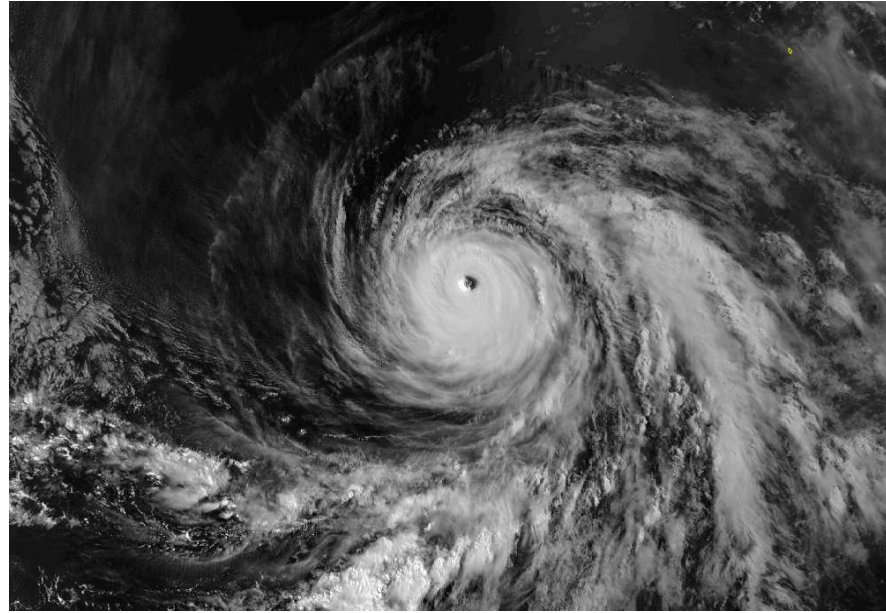
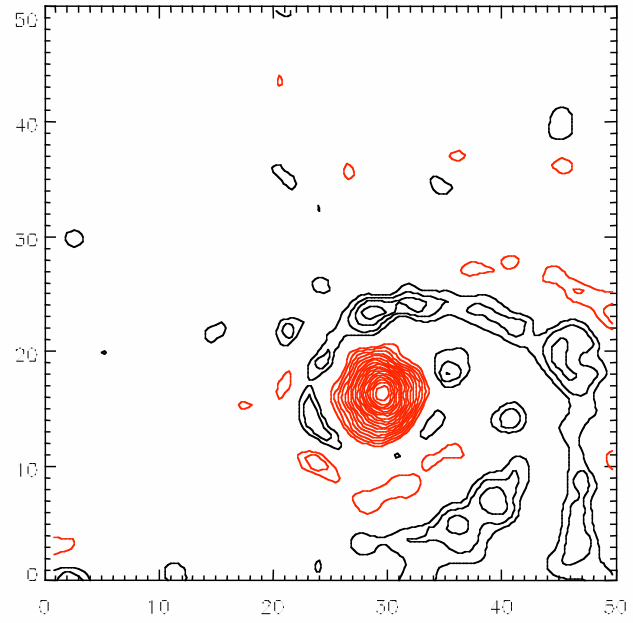
Rossby waves are in phase with V_θ

Locks on Rossby wave speed C_{ph} at radius when $\delta=1$

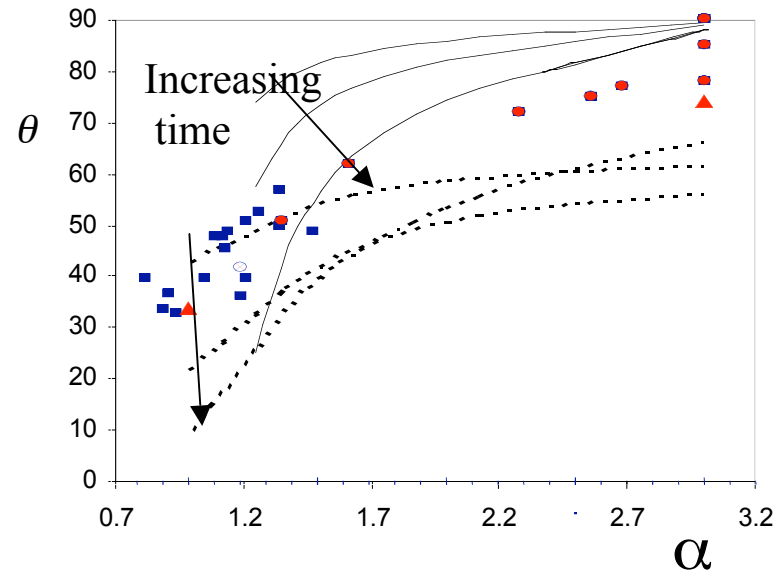
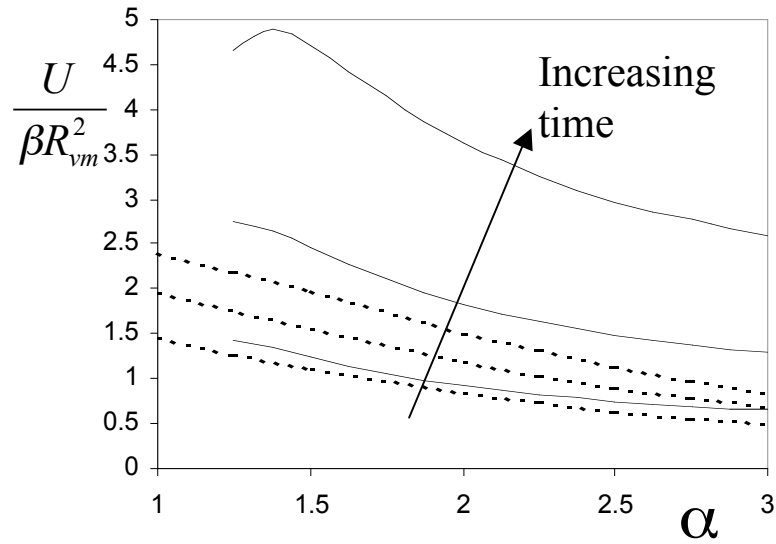
$$v_\theta(R_r) = \beta(2R_r)^2 \rightarrow R_r$$

Vortices with $\alpha < 1.4$: optimal correlation for $U = 0.6 \beta(2R_r)^2$

propagation velocity \sim Rossby-wave groupe velocity



Comparison



----- *Reznik & Dewar, 1994*

———— *potential flow model*

Conclusions

Direction of propagation depends on steepness parameter α .

- Two regimes:
 - $\alpha < 1.4$ strong Rossby waves
in phase with the vortex
determine the propagation speed.
 - $\alpha > 1.4$ weak Rossby waves
vortex dynamics dominate and
propagation speed is slow.
- Model: Potential flow model (initial value model)
 - conservation of the vortex coherence
 - ignores advection and Rossby waves
 - no leaking of vorticitynevertheless promising results.