

Fluid properties probed by vibrating microcantilevers

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ABSTRACT

The aim of this paper is to demonstrate that vibrating microcantilevers can be used to quantify fluid properties as both density and viscosity. For this, an analytical model is exploited and the preliminary measurements, allowing the determination of the viscosity as a function of frequency, are encouraging for the development of a useful microrheometer on a silicium chip for microfluidic applications.

1. Introduction

Contrary to classical rheological measurements using microcantilever sensors^{1,2}, the development of the proposed microrheometer is based on the measurement of fluid properties at each vibration frequency, without necessarily being bound to resonance phenomenon. Simultaneous measurements of both phase and amplitude lead to the fast determination of both unknown fluid parameters (viscosity and density).

2. Beam Vibrations in Viscous Fluid

Viscosity, commonly perceived as the resistance to flow is an intrinsic constant which characterizes newtonian fluid. In Fourier space, a viscous fluid exerts on the beam surface the drag force^{3,4}.

$$F_{fluid} = -g_1 \omega j w(\omega) + g_2 \omega^2 w(\omega) \quad (1)$$

with g_1 and g_2 terms due respectively to the dissipative and inertial parts, w the deflection at the end of the beam, ω the angular frequency.

In Fig.1, the analytical resolution of the equation governing the deflection of a microcantilever in a viscous fluid demonstrates that the evolution of both, the amplitude and phase, as a function of the frequency depends on the fluid properties.

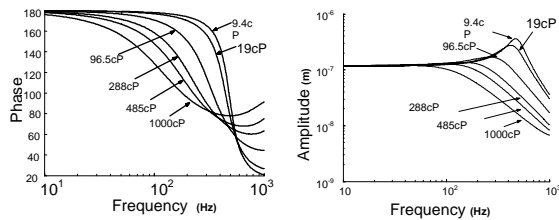


Fig. 1: Theoretical phase and amplitude of a silicon cantilever (23 μ m, 600 μ m, 4mm) oscillating in fluids with different viscosities.

An analogy between beam dynamic in viscous fluid and second order low-pass band allows the simplification of the mechanical transfer function. Then, by identification, knowing the amplitude and phase we can retrieve the drag force components and see their variations according to the frequency:

$$g_1 = -2 \pi m_L \left| \frac{H_0}{H} \right| \frac{f_{0,vac}^2}{f} \sin \phi \quad (2)$$

$$g_2 = m_L \left[\left(1 - \left| \frac{H_0}{H} \right| \cos \phi \right) \left(\frac{f_{0,vac}}{f} \right)^2 - 1 \right] \quad (3)$$

with m_L the cantilever mass per unit length, $f_{0,vac}$ the resonant frequency in vacuum, ϕ the phase, f the frequency, H the transfer function, H_0 its static value.

Thus, this determination of the two parts of the drag force allows to extract analytically at each frequency viscosity and density in order to see the rheological behaviour of the studied fluid.

3. Measurements and Discussion

Using microcantilevers of various geometrical properties, measurements were made in silicon oils covering a wide viscosity range (Fig.2). The measurements confirm that, even without resonant phenomenon, fluid properties govern the vibrational characteristics of the beam oscillation.

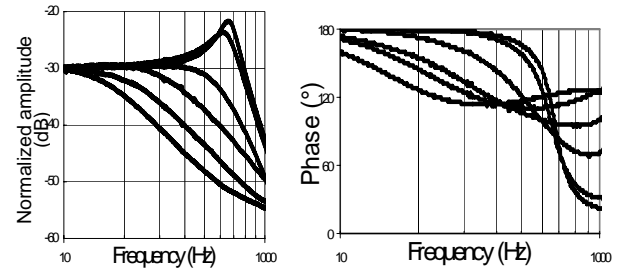


Fig. 2: Measurements corresponding to the simulation presented in Fig.1 for a silicon cantilever (23 μ m x 600 μ m x 4000 μ m) immersed in various silicon oils: 9.4cP, 19cP, 96.5cP, 288cP, 485cP and 1000cP.

Viscosity, density and the rheological behaviour can be deduced after drag force components extraction from beam spectrum. First measurements (Fig.3) are in good agreement with the Newtonian behaviour of the silicon oils for this frequency range.

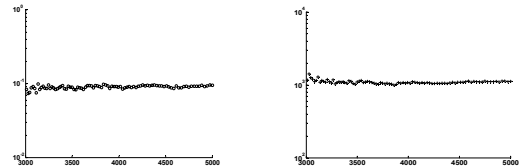


Fig. 3: Determination of viscosity and density with measurements of a silicon cantilever (70 μ m x 200 μ m x 3000 μ m) immersed in silicon oil whose viscosity and density are respectively equal to 96cP and 1000kg.m⁻³.

3. CONCLUSION

The results are very encouraging as they demonstrate the potential for beam-based microstructures to serve as useful tools for rheological measurements at fixed frequency over a wide frequency-range.

4. ACKNOWLEDGMENTS

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5. REFERENCES

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