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# Influence of drizzle on $Z$ – $M$ relationships in warm clouds

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This paper addresses the sensitivity of the relationships between radar reflectivity ( $Z$ ) and liquid water content ( $M$ ) for liquid water clouds to microphysical drizzle parameters by means of simulated radar observation at a frequency of 3 GHz of modeled cumulus clouds. A power law relationship for non drizzling clouds with water content as high as  $3 \text{ gm}^{-3}$ :  $Z_c = 0.026 M_c^{1.61}$  is numerically derived and agreed with previous empirical relationships relative to cumulus and stratocumulus. This relationship is then used to explore the influence of drizzle on the correlation between radar reflectivity and water content. Due to their large diameters with respect to cloud droplets, drizzle sized drops dominate radar reflectivity but do not carry the cloud water content so that reflectivity and liquid water content are expected to be not correlated in clouds containing drizzle. It is shown that for congestus or extreme congestus cumuli, microphysical conditions for which the  $Z_c$ – $M_c$  relationship can be used with a tolerance of 5 and 10% are provided whereas for humilis or mediocris cumuli, the presence of drizzle breaks down the  $Z_c$ – $M_c$  relationship whatever the situations.

*Keywords:* Cumulus radar reflectivity; Cumulus liquid water content; Drizzle; Radar observation simulations

## 1. Introduction

Clouds – the non precipitating condensed atmosphere water – are of considerable importance in the atmosphere and their studies are of great interest for various reasons. First, clouds are major term in the radiative balance of the atmosphere (e.g., Lenoble, 1993; Sengupta et al., 2003); second, clouds are of prime importance in the dynamics and microphysics of precipitating systems since they initiate and enable

development of precipitation (e. g. Pruppacher and Klett, 1997); third, microscopic cloud droplets have also a non negligible role in radar meteorology and telecommunication since they are responsible of a significant microwave attenuation (e. g., Pujol et al., 2007), and in aeronautics because of aircraft icing. Knowing the spatial and temporal cloud structure is thus of great scientific and operational interest. Nowadays, quantitative observations are made through two complementary methods: in situ measurements and radar soundings, the latter offering the advantage of enabling observations of large clouds volumes with high spatial and temporal resolutions. However, quantitative interpretations of radar observations are contingent on the existence of relationship between radar measurements and cloud physical parameters. Since the work of Diem (1948),

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Boucher (1952), and Atlas (1954), who first attempted to link cloud reflectivity factor ( $Z_c$ ) to the cloud liquid water content ( $M_c$ ), many efforts have been made to propose different meaningful  $Z_c$ - $M_c$  relationship (Sauvageot and Omar, 1987; Fox and Illingworth, 1997; Vivekanandan et al., 1999, Wand and Geerts, 2003 among others). Paluch et al. (1995) used data from the CAPE campaign (Convection And Precipitation/Electrification Experiment) in summer 1991 to study, through a close correlation between  $Z_c$  and  $M_c$ , the development of cloud droplet size distribution in cumulus clouds over Florida under the influence of entrainment and mixing.

However, using radar reflectivity to retrieve physical characteristics of cloud is limited by the presence of drizzle sized drops inside clouds because drizzle destroys the correlation between  $Z_c$  and  $M_c$ . The reason for that is simple: radar reflectivity is proportional to  $D^6$  (the diameter or the equivalent diameter of the back-scattered hydrometeor) so that drizzle sized drops ( $50 < D < 500 \mu\text{m}$ ) dominate the backscattered signal with respect to microscopic sized cloud droplets. On the other hand, these last ones contribute for the most part of the cloud liquid water content which is proportional to  $D^3$ . Drizzle is thus a serious problem in cloud property retrieval from radar reflectivity observations, and lots of studies have been conducted to identify drizzle clouds. Sauvageot and Omar (1987) proposed to characterize drizzle clouds with reflectivity higher than  $-15 \text{ dBZ}$ ; Babb and Albrecht (1995), Frisch et al. (1995), and Fox and Illingworth (1997) suggested to identify drizzle clouds using radar Doppler spectrum since drizzle drops fall with a velocity of about  $1 \text{ m s}^{-1}$  greater than cloud droplets. Using dataset from stratiform clouds in the tropical eastern Pacific Ocean, Matrosov et al. (2004) have proposed an evaluation of the performance of  $Z_c$ - $M_c$  relationships by comparing vertically integrated liquid water content values from radar measurements with estimates from microwave radiometers. They have shown that the reflectivity threshold of Sauvageot and Omar (1987), i.e.  $Z_t = -15 \text{ dBZ}$ , allows accuracies better than a factor 2 and that accuracies are improved if  $Z_t$  is lowered. VanZanten et al. (2005) observed drizzle in nocturnal stratocumulus clouds during the DYCOMS II campaign and showed that it is more prevalent than commonly thought. The importance of drizzle is enhanced by the fact that it is often neglected in the modeling community, especially when dealing with the relation between the cloud macroscopic features and drizzle (VanZanten et al., 2005). In addition, several methods of retrieving microphysical characteristics of warm clouds have been proposed (Frisch et al., 1995; Mace and Sassen, 2000; Leohnert et al., 2001; Dong and

Mace, 2003). These methods are based on the association of radar reflectivity detection with water content determination by passive instruments as radiometers.

The work presented in this paper is exploratory. It is devoted to study the influence of the various parameters characterizing drizzle drops, precisely drizzle water content  $M_d$  and median drizzle diameter  $D_0$ , on the correlation between radar reflectivity and liquid water content of cumulus clouds. In order to test the sensitivity of  $Z_c$ - $M_c$  relationships to the presence of drizzle under different cloud configurations, different types of cloud with various values of the total droplet concentration  $N_t$  are considered. To make the paper consistent, a  $Z_c$ - $M_c$  relationship is proposed from simulations based on realistic data available in the literature. Observed or experimental data have not been used since unavailable. Simulation enables to explore the entire range of variation of the significant parameters and thus provide a framework for further studies. The originality of this study is to test the validity of  $Z$ - $M$  law in presence of droplets or drizzle drops by means of a reasonable and realistic modeling of cumulus clouds and simulation of radar observations.

The model developed to simulated cloud radar observations and the various situations analyzed are presented in Section 2. In Section 3, a  $Z_c$ - $M_c$  power law relationship is proposed and compared to the other ones available in the literature. In Section 4, sensitivity of this relation to the presence of drizzle sized drops is studied in function of drizzle distribution characteristics. Section 5 is the concluding one.

## 2. Cumulus modeling

### 2.1. Cloud water content

Clouds simulated are cumulus ones made only of liquid water droplets. They are characterized by their liquid water content  $M_c$ , assumed to be a spatial two-dimensional function,  $M_c(x,z) = G(x)M_c(z)$ ,  $z$  being the vertical coordinate and  $G$  a weighting function depending on the horizontal coordinate  $x$ . At a given altitude  $z$ ,  $M_c(z)$  is modulated by  $G(x)$ , and thus  $M_c(x)$  is determined. Zaitsev (1950); Warner (1955, 1969, 1970); Squires (1958); Borovikov (1963); Mason (1971); Vulfson et al. (1973) among others, have shown that the global cloud dynamics and  $M_c$  are closely related. Typically,  $M_c(z)$  increases with height above cloud base, reaches a maximum value  $M_c^{\text{max}}$  in the upper half of the cloud and then decreases up to cloud top. In the model, the cumulus base height is fixed to  $z_{\text{CB}} = 1 \text{ km}$  which is a value found during the CAPE experiment for example. In addition, to represent the

well-marked transition between a cumulus and its clear-air environment, it is reasonable to limit  $M_c(z)$  to a value  $M_c^{\text{lim}}$  different from zero at cloud base and cloud top.

Horizontal liquid water content  $M_c(x)$  globally decreases from cloud center, where  $x=0$ , to cloud boundaries, so that a reasonable representation can be made through an exponential function  $G(x)=\exp(-x^2/SD^2)$  where  $SD$  is assumed to define the horizontal cloud extension:  $L=2 \times SD$ . Cloud shape factor can thus be defined as  $s_f=L/e$  with  $e=z_{CT}-z_{CB}$  the thickness of the cumulus. Finally, the multiplication of  $M_c(z)$  by  $G(x)$  for each altitude  $z$  gives the cloud water content  $M_c(x,z)$ , and cumulus edges are chosen such that  $M_c \geq M_c^{\text{lim}}$ ; all other values are put to zero.

## 2.2. Cloud droplets size distribution

Microphysical cumulus modeling requires specifying a cloud droplet size distribution (CDS). A reasonable and convenient way to represent a CDS is to adopt a gamma distribution (Khrghian and Mazin, 1952; Pruppacher and Klett, 1997):

$$N_c(D_c) = N_0 D_c^2 \exp(-AD_c), \quad (1)$$

where  $D_c$  is the cloud droplet spherical diameter,  $N_c$  the volumic concentration per diameter class (i.e. the number of droplets with diameters between  $D_c$  and  $D_c + \Delta D_c$  per unit volume), and  $N_0$  and  $A$  are parameters that can be related to any two moments of the distribution, for instance the total droplet concentration  $N_t$  (zero order moment) and the liquid water content  $M_c$  (proportional to the third order moment). If water density  $\rho_w$  is expressed in  $\text{g cm}^{-3}$ ,  $D_c$  in  $\text{cm}$ ,  $N_t$  in  $\text{cm}^{-3}$ , and  $M_c$  in  $\text{g m}^{-3}$ , Eq. (1) gives  $N_c(D_c)$  in  $\text{cm}^{-3} \mu\text{m}^{-1}$  with:

$$N_0 (\text{cm}^{-6}) \approx 1.27 \times 10^4 \left( \frac{\rho_w N_t^2}{M_c} \right), \quad (2)$$

and

$$A (\text{cm}^{-1}) \approx 3.16 \times 10^2 \left( \frac{\rho_w N_t^2}{M_c} \right)^{1/3}. \quad (3)$$

In the model,  $\Delta D_c$  is  $1 \mu\text{m}$  and a maximum droplet diameter of  $50 \mu\text{m}$  is considered so that 50 diameter classes, centered on diameter values  $(D_{c,i})_{i=1..50} = \{0.5 \mu\text{m}, 0.15 \mu\text{m}, \dots, 48.5 \mu\text{m}, 49.5 \mu\text{m}\}$ , are defined. Knowing  $M_c$  and  $N_t$  thus allows the determination of CDS.  $N_t$  is assumed to be constant throughout the cumulus, and several values of  $N_t$  have been considered, from 100 to  $1000 \text{ cm}^{-3}$ , which are realistic values for cumulus clouds (e.g., Pruppacher and Klett, 1997).

Miles et al. (2000) have shown that  $N_t$  is approximately constant throughout depth maritime clouds while for continental ones,  $N_t$  profiles are highly variable. Assuming  $N_t$  constant is thus realistic for maritime clouds but is a simplification for continental ones. In order to get the model more realistic, many data would be necessary to perform a mean or statistical function for  $N_t(z)$ . However, continental clouds are composed of different part where  $N_t$  is constant, as it was observed by Huan (1963) for continental cumulus in China (see Pruppacher and Klett, 1997, page 22, Figs. 2–19). In this work, computation can be considered to be relative to a given part of real continental cloud.

## 2.3. Cumulus parameters

Four cumulus clouds have been considered in this study. The first cumulus has been modeled from the CAPE observations conducted in Florida and corresponding to cumulus clouds with thickness  $e$  of at least 2 km, base height  $z_{CB}$  of about 1 km, base temperature  $T_{BC}$  around 20–25 °C, and water content reaching  $3 \text{ g m}^{-3}$  (e.g., Paluch et al., 1995). According to Pruppacher and Klett, (1997), such a cloud can be classified as an extreme congestus one. Three other classes of cumulus clouds are considered in the last reference: humilis, mediocris, and congestus, whose maximum liquid water content is  $M_c^{\text{max}} = 0.2, 0.5, \text{ and } 1 \text{ g m}^{-3}$ , respectively. Their modelisation has been done by keeping  $z_{CB}$  at 1 km and choosing  $e$  equal to a multiple half of 2 km, i.e.  $e = 0.25, 0.50, \text{ and } 1 \text{ km}$  for the humilis, the mediocris and the congestus cumulus respectively. In addition,  $N_t$  has been chosen lower than  $200 \text{ cm}^{-3}$  for a humilis cloud, between 200 and  $500 \text{ cm}^{-3}$  for a mediocris, 500 and  $800 \text{ cm}^{-3}$  for a congestus, and 800 and  $1000 \text{ cm}^{-3}$  for an extreme congestus, consistently with the CAPE observations (e.g. Paluch et al., 1995). To justify such realistic choice, one can quote Miles et al. (2000, Table 2) where two kinds of cumulus are listed: the first one with a liquid water content of  $0.7 \text{ g m}^{-3}$ , a thickness  $e = 0.5 \text{ km}$  and  $N_t = 450 \text{ cm}^{-3}$ ; the second one, with a liquid water content of  $0.24 \text{ g m}^{-3}$  (considered here as approaching the humilis type),  $e = 0.1 \text{ km}$  and  $N_t = 220 \text{ cm}^{-3}$ . Finally, cloud base temperature has to be changed. As well known, it is a key factor of the cloud structure: the lower cloud base temperature, the lower maximum liquid water content. Reasonable  $T_{CB}$  values of 5, 10, and 15 °C for humilis, mediocris and congestus cases respectively are used. Temperature is supposed to be uniform throughout the cloud which is an acceptable assumption taken into account the thickness of the cloud.

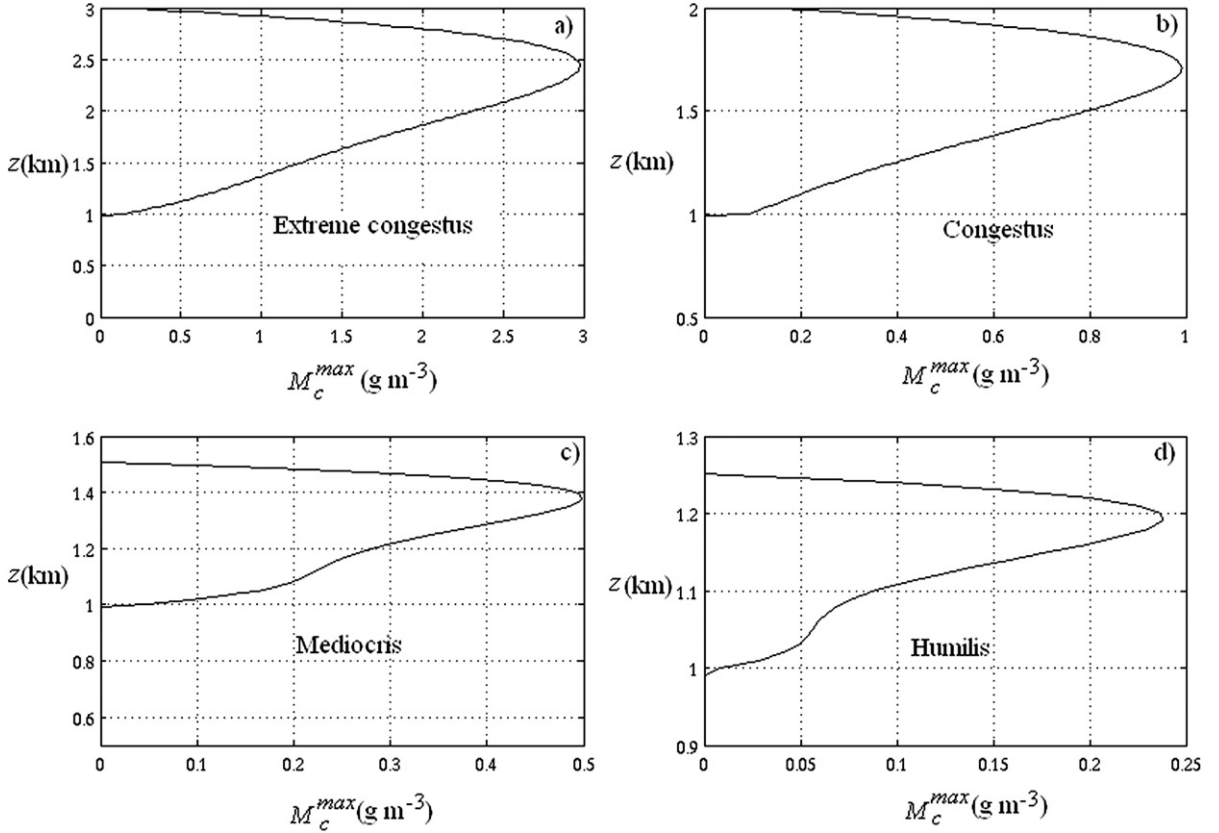


Fig. 1. Liquid water content profile of the different cumuli constructed: a) extreme congestus, b) congestus, c) mediocris, and d) humilis.

The authors are conscious that these assumptions made to simplify the model and to limit the number of parameters could be questionable, but it can be note that, to our knowledge,  $T_{CB}-M_c^{max}$  relationships do not really exist. Only it can be find some relations between  $T$  and mean  $M_c$  profiles (Mazin, 1995; Gulpepe and Isaac, 1997). The above values have been chosen in order to account for the decreasing of  $M_c^{max}$  with  $T_{CB}$ ; keeping a  $T$  value identical for all kinds of constructed cumuli would be unrealistic. Further, for the purpose of this

paper, temperature should not play a crucial role, so that these above approximations are acceptable.

Fig. 1 displays the liquid water content vertical profile  $M_c(z)$  of the four modeled cumuli. These profiles have been constructed by fitting in situ data of  $M_c$  available on the literature. Fig. 2 sketches the cumuli considered and Table 1 summarizes their characteristics:  $M_c^{max}$ ,  $N_b$ ,  $z_{CB}$ ,  $z_{CT}$ ,  $L$ , and  $T_{CB}$ . Then Fig. 3 displays the CDSC associated to the different cumuli modeled,  $(M_c^{max}, N_b) = (0.2, 200)$ ,  $(0.5, 500)$ ,  $(1, 800)$ ,  $(3, 1000)$ ;

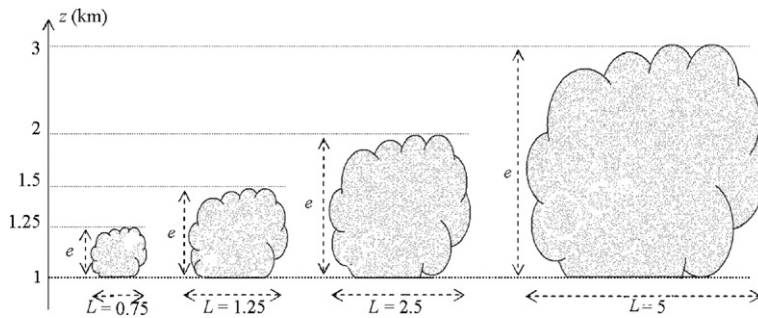


Fig. 2. Schematic representation of the different cumuli constructed with their geometrical characteristics in km (thickness  $e$ , horizontal extension  $L$ ). The first is a cumulus humuli, the second a mediocris one, the third a congestus one and the last an extreme congestus one.

Table 1  
Characteristics of the modeled cumulus cloud

	Humilis	Mediocris	Congestus	Extreme congestus
$z_{CB}$ (km)	1	1	1	1
$z_{CT}$ (km)	1.25	1.50	2	3
$L$ (km)	0.625	1.25	2.5	5
$M_c^{\max}$ ( $\text{g m}^{-3}$ )	0.2	0.5	1	3
$N_t$ ( $\text{cm}^{-3}$ )	100–200	200–400	400–800	800–1000
$T_{CB}$ ( $^{\circ}\text{C}$ )	5	10	15	20

Parameters  $z_{CB}$  and  $z_{CT}$  are cloud base and top heights respectively,  $L$  is the horizontal cloud extension,  $M_c^{\max}$  is the maximum water content,  $N_t$  is the total droplet concentration and  $T_{CB}$  the cloud base temperature.

smaller droplets are in higher concentration than larger ones. These CDS are consistent with those found in the literature: for example, Paluch et al. (1995) have observed CDS in cumulus clouds characterized by droplet diameters lower or equal than  $50 \mu\text{m}$  and concentration reaching  $10^2 \text{ cm}^{-3} \mu\text{m}^{-1}$ , which corresponds to droplet water content as large as  $3 \text{ g m}^{-3}$ .

#### 2.4. Backscattering cross section and equivalent radar reflectivity factor

For spherical scatterers as cloud droplets, the backscattering cross section  $\sigma$  is given by the Mie formulas (Mie, 1908), whose coefficients are computed with the Deirmendjian (1969) algorithm. They depend on the radar frequency, the diameter, and the complex index of refrac-

tion of the scatterer. The latter is computed using the Ray (1972) model, supposing a cloud water temperature uniform throughout the cloud and equal to the cloud base temperature  $T_{CB}$ . Thus,  $\sigma$  is computed for all scatterers, that is  $(\sigma_i)_{i=1\dots 50}$  for homogeneous cumulus droplets of diameter  $(D_{c,i})_{i=1\dots 50}$ .

Cloud droplets are small enough to satisfy the Rayleigh approximation whatever may be radar wavelengths  $\lambda$ , so that Mie formulas directly give the Rayleigh backscattering cross section. The average power backscattered by a population of droplets homogeneously distributed in an elementary volume  $V$  is proportional to the radar reflectivity ( $\eta$ ), defined as the sum of the backscattering cross sections ( $\sigma_i$ ) of individual droplets:

$$\eta = \frac{1}{V} \sum_i \sigma_i \quad (4)$$

where  $\eta$  is commonly expressed in  $\text{cm}^{-1}$ . The cloud radar reflectivity factor  $Z_c$  is by definition:

$$Z_c (\text{mm}^6 \text{m}^{-3}) = \frac{\eta}{2.84 \times 10^{-10} \lambda^{-4}}, \quad (5)$$

where  $\lambda$  is expressed in cm. Usually,  $Z_c$  is expressed in logarithmic unit:

$$Z_c (\text{dBZ}) = 10 \log [Z_c (\text{mm}^6 \text{m}^{-3})] \quad (6)$$

Under Rayleigh's approximation, the reflectivity is independent from the wave frequency.

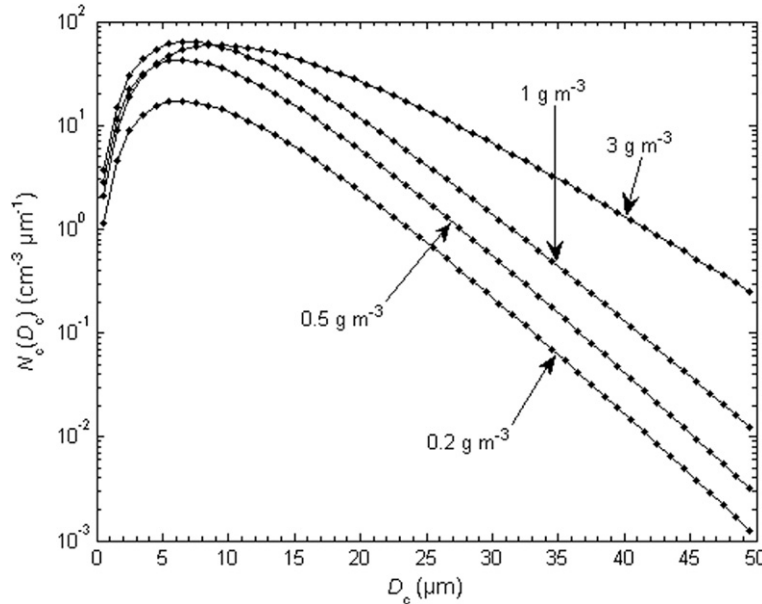


Fig. 3. Cloud droplet size distribution (CDS) represented by (1).  $D_c$  is the droplet diameter and  $N_c$  the volumic concentration per class of diameter. CDS is displayed for four  $(M_c^{\max}, N_t)$  couples corresponding to the different cumuli modeled: (0.2, 200), (0.5, 500), (1, 800), and (3, 1000) [ $M_c^{\max}$  is in  $\text{g m}^{-3}$  and  $N_t$  in  $\text{cm}^{-3}$ ].

## 2.5. Radar beam modeling

In the model, observations are simulated for a ground based radar pointing upward located at the origin of the coordinate below the cumulus and operating at a frequency  $f$  of 3 GHz. This frequency is the least attenuated by hydrometeors compared to other common frequencies (as 10, 35, and 94 GHz). The modeled radar beam is approximated to a 3-dB beamwidth ( $\theta_{3\text{dB}}$ ) equal to  $1.8^\circ$ , and a gate spacing  $\Delta r$  of 150 m. The cumulus is digitized through a two-dimensional meshing with a grid resolution of 100 m and 10 m along the horizontal and vertical direction, respectively. Each grid point is characterized by specific values of  $M_c$  and, consequently, by specific CSD. The computational procedure consists first, for a given radar resolution volume  $V$ , in identifying the grid points included in  $V$  with non zero values of  $M_c$ . For each of these grid points, the backscattering cross sections of the whole droplets ( $i$ ) are ( $\sigma_i = \sum_i \sigma_i$ ) computed. Then,  $\eta$  associated to  $V$  is deduced from Eq. (4) and, finally,  $Z_c$  is obtained using (5) and (6). The simulation is fully completed when the successive radar beams separated by  $\Delta\beta=1^\circ$  in azimuth have covered the whole extent of the target. This procedure differs from previous studies based on the use of in situ microphysical data to derive reflectivity factor and water content from retrieved hydrometeor size distributions. Since reflectivity corresponds to the backscattered signal from a collection of hydrometeors inside a radar resolution volume  $V$ , simulating radar observations is particularly interesting. That is why an average over each volume  $V$  is performed. Of importance to note is that the position of the radar is not a crucial parameter in the present work and do not fundamentally modify the results: this parameter only concerns volumes  $V$  considered.

The power received by the radar is generally due to several backscatterers: 1) precipitating hydrometeors 2) cloudy hydrometeors although their reflectivity is often beyond the sensitivity of commonly used radars (Pujol et al., 2007) 3) index-of-refraction heterogeneity in the atmosphere due to turbulence; this contribution, the Bragg scattering, increases with the wavelength and at 10 cm (3 GHz) is a non negligible contribution for non or few precipitating cumulus (Knight and Miller, 1993), 4) birds and insects, 5) building, mountains or other contaminating targets. Clearly, these contributions will have to be investigated and compared with each of them in a complete study of cumulus reflectivity. In this work, we focus only on reflectivity of cloud droplets and drizzle drops and the influence of these last ones on  $Z_c$ - $M_c$  relationships, so that the other terms of reflectivity are outside the scope of this paper.

It is worth nothing that using simulation of radar observations is an original way to study influence of drizzle size drops on  $Z_c$ - $M_c$  relationships in warm clouds which differs from previous studies based only on in situ measurements or microwave observations.

## 3. Cloud reflectivity $Z_c$ vs. cloud liquid content $M_c$

Some  $Z_c$  ( $\text{mm}^6 \text{m}^{-3}$ )- $M_c$  ( $\text{g m}^{-3}$ ) relationships can be found in the literature; Table 2 lists the available relations derived in various context and for different types of warm clouds (stratus, stratocumulus, cumulus). For ground observed unknown types of clouds, Atlas (1954) first proposed  $Z_c=0.048 M_c^2$  and suggested that better relations could be obtained by considering cloud type. For more than thirty years, this relation was the only reference. Sauvageot and Omar (1987) investigated relations between  $Z_c$  and cloud parameters in warm non precipitating cumulus and stratocumulus clouds at mid-latitude from in situ aircraft measurements. They showed that meaningful  $Z_c$ - $M_c$  relationships seemed to exist only if clouds contained droplets of diameter smaller than 100  $\mu\text{m}$ , corresponding to a reflectivity lower than  $-15$  dBZ. Their observations indicated power law relations between  $a$  and  $b$  coefficients ranging respectively from 0.014 to 0.069, and from 1.34 to 1.92. Moreover, the correlation coefficient,  $\rho$ , ranges from 0.70 to 0.84. They proposed  $-15$  dBZ as an upper limit for the reflectivity factor for the non precipitating (non drizzling) warm clouds. For diameter lower than 100  $\mu\text{m}$ , Sauvageot and Omar (1987, Fig. 4) found the mean relation:  $Z_c=0.030 M_c^{1.31}$  with  $\rho=0.77$ . From in situ measurements in marine stratocumulus, Fox and Illingworth (1997) found  $Z_c=0.031 M_c^{1.56}$ . From computations of non precipitating clouds, based on droplet parameters from the WISP (Winter Icing and Storm Project) experiment, Vivekanandan et al. (1999) obtained the relation:  $Z_c=0.34 M_c^{1.42}$ . Recently, Wang and Geerts (2003) analyzed and compared airborne cloud radar and cloud

Table 2  
Coefficient ( $a$ ,  $b$ ) of some  $Z_c=aM_c^b$  relationships proposed by various authors

Author	Cloud type	$a$	$b$
Atlas (1954)	Unknown	0.048	2
Sauvageot and Omar (1987)	Stratocumulus-Cumulus	0.030	1.31
Fox and Illingworth (1997)	Marine stratocumulus	0.031	1.56
Wang and Geerts (2003)	Marine stratus	0.044	1.34
Vivekanandan et al. (1999)	Non precipitating liquid clouds	0.34	1.42
This study	Warm clouds	0.026	1.61

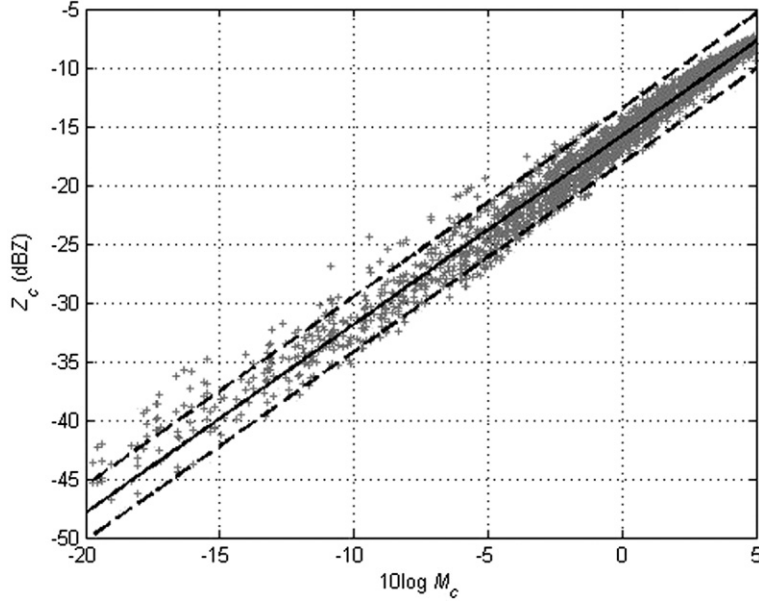


Fig. 4. Scatter plot of cumulus reflectivity  $Z_c$  vs. cloud liquid water content  $M_c$  resulting from various simulation of radar observations of four cumulus clouds (Figs. 1 and 2), with total droplet concentration between 100 and 1000  $\text{cm}^{-3}$ . Stars represent simulated data while the black solid curve is the  $Z_c$ - $M_c$  expression obtained from a least-square fitting on the simulated data. Dotted lines represent the 95% confidence interval.

microphysical in situ data to obtain, for non drizzling marine stratus\*:  $Z_c = 0.044 M_c^{1.34}$ .

The  $Z_c$ - $M_c$  relationship obtained in the present study is necessary for the consistency of the whole paper and has not to be considered as a new one more but as a way to test the reliability of our modeling by comparison with the previously mentioned one.

### 3.1. Simulation-derived $Z_c$ - $M_c$ relationships

Each cumulus considered for this study is defined by  $M_c$ -field, characterized by its maximum value  $M_c^{\max}$  (0.2, 0.5, 1, and 3  $\text{g m}^{-3}$ ), an interval of values of the total droplet concentration  $N_t$ , and consequently by an associated computed  $Z_c$ -field. Note that each  $M_c$  and  $Z_c$  values have to be considered respectively as the cloud water content and the associated reflectivity over a given radar resolution volume. Since  $N_t$  affects the  $a$  coefficient of the  $Z_c$ - $M_c$  relationship (Sassen and Laio, 1996), computation of  $Z_c$  for a given type of cumulus is made for several values of  $N_t$ , namely,  $N_t = 800, 900$  and  $1000 \text{ cm}^{-3}$  for the extreme cumulus,  $N_t = 600$  and  $700 \text{ cm}^{-3}$  for the congestus ( $M_c^{\max} = 1 \text{ g m}^{-3}$ ),  $N_t = 400$  and  $500 \text{ cm}^{-3}$  for the mediocris and  $N_t = 100, 200,$  and  $300 \text{ cm}^{-3}$  for the humilis. All these computations provide a set of  $M_c$

values, ranging from 0 to 3  $\text{g m}^{-3}$ , which is associated with the computed  $Z_c$  values for cumulus clouds; a scatter plot of  $Z_c$  vs.  $M_c$  is displayed in Fig. 4. A least-square fitting to the simulated data leads to the expression:

$$Z_c = 0.026 M_c^{1.61}, \quad (7)$$

where  $Z_c$  is in  $\text{mm}^6 \text{ m}^{-3}$  and  $M_c$  in  $\text{g m}^{-3}$ . Standard deviation in  $a$  and  $b$  are  $\pm 0.002$  and  $\pm 0.01$ , respectively. Correlation coefficient and root mean square are 0.97 and 1.2 dBZ, respectively. In addition, confidence bounds at 95% have been represented (dashed lines in Fig. 4). The inferior bound corresponds to  $a = 0.015$  while the superior corresponds to  $a = 0.045$ ; values of  $b$  are identical since curves are parallel. These bounds indicate precisely that, for a given  $M_c$ , the associated computed  $Z_c$  is included with a 95% of certainty in the domain between the two dashed lines which is centered on the  $Z_c$  value calculated from (7).

### 3.2. Comparison with other relations

A comparison between (7) and earlier published power law  $Z_c$ - $M_c$  relations (Table 2) is performed in Fig. 5. It is noteworthy to recall that the relations in Table 2 correspond to situation characterized by  $M_c$  values lower or equal than about 1  $\text{g m}^{-3}$ , whereas (7) holds for  $M_c$  values up to 3  $\text{g m}^{-3}$ . Such values are not exceptional since during the CaPE experiment, cumuli

\* In the paper by Wang and Geerts (2003), the correct relation can be found in Table 5, last line. There is a typographical error in equations numbered (10) and (11).



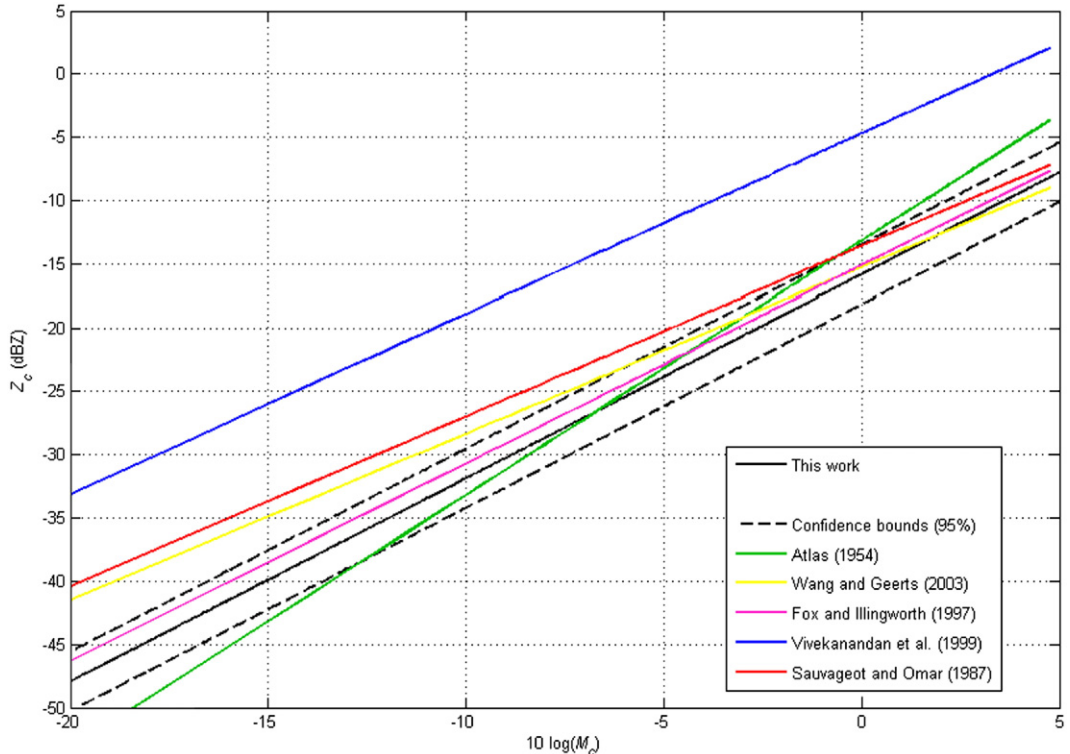


Fig. 5. Comparison between the various  $Z_c$ - $M_c$  relations. Dashed lines are the 95% confidence interval.

with liquid water content greater than  $1 \text{ g m}^{-3}$  was often observed (Paluch et al., 1995) and in addition, Poellot and Pflaum (1989) have even reported values of  $5 \text{ g m}^{-3}$  in particular cumuli with very high updraughts.

Whatever the considered value of  $M_c$ , the relation proposed by Vivekanandan et al. (1999) overestimates cumulus reflectivity with reflectivity values ranging from  $-30$  to  $2 \text{ dBZ}$ . That can result from the low total droplet concentrations considered in their study. On the contrary, the other equations are more in agreement with (7) especially that of Fox and Illingworth (1997), with reflectivity values ranging from  $-50$  to  $-5 \text{ dBZ}$ . The equations proposed by Atlas (1954); Sauvageot and Omar (1987); Wang and Geerts (2003) are included in the 95% confidence bound interval, if not, close to it. Agreement between all these relations is better for high  $M_c$  (about  $0.3 \text{ g m}^{-3}$ , or  $10 \log(M_c) \approx -5$ ). Note that a significant disagreement occurs for small  $M_c$  values ( $0.01 \text{ g m}^{-3}$ ,  $10 \log M_c = -10$ ) for which measurements are generally more uncertain. But, on the overall, all these relations are consistent which supports the reliability of our modeling. It is worth noting that the relation proposed in this paper enables to complete the empirical relationships by extrapolation towards high

water content. In addition, it appears that they do not depend on the kind of cloud considered since they have been established for different systems (cumulus, stratocumulus, stratus). This can be explained by different similar dynamic and microphysical characteristics of these clouds. All of them are liquid clouds characterized by droplets of diameter lower than  $100 \mu\text{m}$  and similar liquid water content vertical profiles.

To conclude this section, we want to insist that (7) should not be seen as a new relation although it is deduced in a different way than previous ones, i.e., from simulations of radar observations, and although it is an extrapolation from high cloud water content values (till  $3 \text{ g m}^{-3}$ ). The goal of this section was to make the whole paper consistent with respect to the following section.

#### 4. Influence of drizzle sized drops on the $Z_c$ - $M_c$ relationship

It is well known that drizzle sized drops destroy the correlation between  $Z_c$  and  $M_c$ : Sauvageot and Omar (1987) found that radar reflectivity  $Z$  and liquid water content  $M$  are correlated for droplets with diameters lower than  $100 \mu\text{m}$  and attributed non correlation for

higher diameters to the presence of drizzle; later, Fox and Illingworth (1997) investigated the radar retrieval of stratocumulus properties and showed that drizzle disables the existence of  $Z-M$  relationships because it dominates the backscattering signal (due to the relatively high diameter of drizzle drops), although its contribution to the liquid water content is negligible (due to its low numerical concentration). Drizzle appears thus as a serious problem in radar observation of warm clouds and its study is important for cloud microphysics and radar meteorology. Paluch et al. (1995) investigated entrainment and mixing in cumulus cloud on the base of  $Z-M$  relation during the CAPE experiment and said that they were not able to say with certainty which clouds they observed were free of drizzle; they noted also that the poor correlation they obtained in some situations could be due to the presence of drizzle.

Many studies have been conducted in order to investigate drizzle (e.g. VanZanten et al., 2005 and the DYCOMS II campaign) and to discriminate drizzle cloud from drizzle free cloud and to determine the conditions of application of the  $Z_c-M_c$  relationships. As stated in the introduction, a reflectivity of  $-15$  dBZ has been proposed by Sauvageot and Omar (1987) as a threshold under which clouds are not expected to contain drizzle sized drops; Babb and Albrecht (1995); Frisch et al. (1995); Fox and Illingworth (1997) found that drizzle drops fall with a velocity of about  $1 \text{ m s}^{-1}$

greater than cloud droplets and suggested to identify drizzle in clouds using radar Doppler spectrum; Wang and Geerts (2003) found that drizzle appear in marine stratocumulus as the reflectivity values exceed a threshold  $Z_t$  depending on the normalized altitude  $\phi = (z-z_{CB})/(z_{CT}-z_{CB})$  within clouds:

$$Z_t = 0.046 \phi^{1.413}, \quad (8)$$

where  $z_{CB}$  and  $z_{CT}$  are cloud base and top altitudes respectively. They also indicated that reflectivity threshold is less uncertain in the lower half part of the cloud. The threshold of  $-15$  dBZ identified by Sauvageot and Omar (1987) corresponds to a normalized altitude of 0.77, i.e. to the upper two-third of the cloud where water content is maximum. The concept of reflectivity threshold has also been investigated by Matrosov et al. (2004) by comparing water content retrievals from radar measurements and microwave radiometers.

All these studies investigate the limit of validity of the  $Z_c-M_c$  relationship in function of measured radar parameters (reflectivity, Doppler velocity). On the other hand, Gerber (1996) studied the dependence of drizzle water content on cloud observable during the ASTEX campaign (Atlantic Stratocumulus Transition Experiment). He found that for droplet with radius greater than  $16 \text{ }\mu\text{m}$ , stratocumulus should be classified as heavy

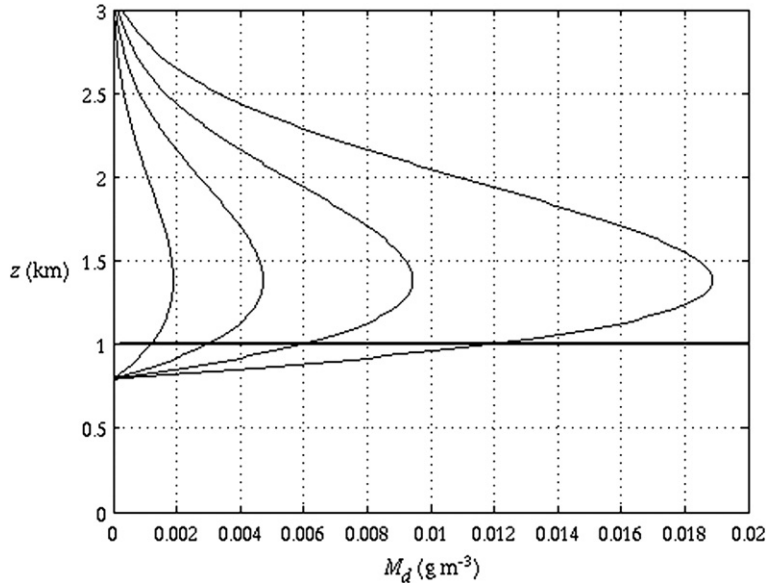


Fig. 6. Drizzle water content profile  $M_d(z)$  considered in this work with maxima  $M_d^{\max} \approx 0.002, 0.005, 0.01$  and  $0.02 \text{ g m}^{-3}$ . These profiles are associated to an extreme cumulus congestus with cloud top at 3 km. Profiles are similar for the other cumuli except that cloud tops are 2, 1.5, and 1.25 km for the congestus, mediocris, and humilis cumulus respectively. The horizontal black line represents cloud base.

drizzle cloud with drizzle water content greater than  $0.01 \text{ g m}^{-3}$ . Below this value, stratocumulus should be considered as light-drizzle clouds. The threshold proposed by Gerber (1996) is a microphysical indication of the  $Z$ - $M$  relationship validity for light drizzle stratocumulus. The present work focuses more particularly on cloud microphysical parameters, precisely cloud water content  $M_c$ , total droplet concentration  $N_t$ , drizzle water content  $M_d$ , and drizzle drop median volume diameter  $D_0$ . The present simulation is used to test the sensitivity of  $Z$ - $M$  relationships in function of the microphysical characteristics.

#### 4.1. Drizzle water content and drizzle drop size distribution

The procedure for representing drizzle is identical as that for clouds. It is based on a two dimensional drizzle water content field and a drizzle drop size distribution (DDSD). The drizzle water content field is written  $M_d(x, z) = M_d(z)G_d(x)$  where  $M_d(z)$  is the drizzle water content vertical profile ( $z$  is the vertical coordinate) and  $G_d(x)$  a weighting function which gives, at a fixed  $z$ , the horizontal dependence ( $x$  is the horizontal coordinate). The vertical profile  $M_d(z)$  is zero at cloud top, increases from cloud top to a maximum value  $M_d^{\text{max}}$  above cloud base as a consequence of drizzle drop weight and decreases towards zero under cloud base because of evapo-

ration in unsaturated air conditions. Fig. 6 displays different considered  $M_d(z)$  profiles with  $M_d^{\text{max}} = 0.002, 0.005, 0.01$  and  $0.02 \text{ g m}^{-3}$ . Horizontal variation of drizzle water content is expressed by function  $G_d(x) = \exp(-x^2/SD)$  where  $SD$  is assumed to represent the horizontal extension of the drizzle field with  $L_d = 2 \times SD$ . Kebe et al. (2005) shown that horizontal extension of tropical rain is lower than the horizontal generating cloud by a factor 1.68. In this work, we have supposed that this factor also applies to drizzle cumulus so that  $L_d = L_c/1.68$ . As proposed by Ulbrich (1983) for rain, the drizzle drop size distribution (DDSD) is assumed to be a gamma modified distribution of the form:

$$N_d(D_d) = N_0 D_d^\mu \exp(-AD_d), \quad (9)$$

where  $D_d$  is drizzle drop diameter,  $N_d$  the volumic concentration per class of diameter, and  $N_0$ ,  $\mu$ , and  $A$  are parameters. The latter is related to the median volume diameter  $D_0$  (mm) by:  $AD_0 = 3.67 + \mu$  (Ulbrich, 1983). If  $D_d$  is in mm and  $M_d$  in  $\text{g m}^{-3}$ , then

$$N_0 (\text{m}^{-3} \text{mm}^{-1}) = \left( \frac{6}{\pi \Gamma(\mu + 4)} \right) \left( \frac{M_d}{10^{-3} \rho_w} \right) \left( \frac{3.67 + \mu}{D_0} \right)^{\mu+4} \quad (10)$$

( $\Gamma$  being the gamma function), so that  $N_d(D_d)$  is expressed in  $\text{m}^{-3} \text{mm}^{-1}$ . In the present work, drizzle drop diameters range from a minimum value of  $50 \mu\text{m}$  to a maximum

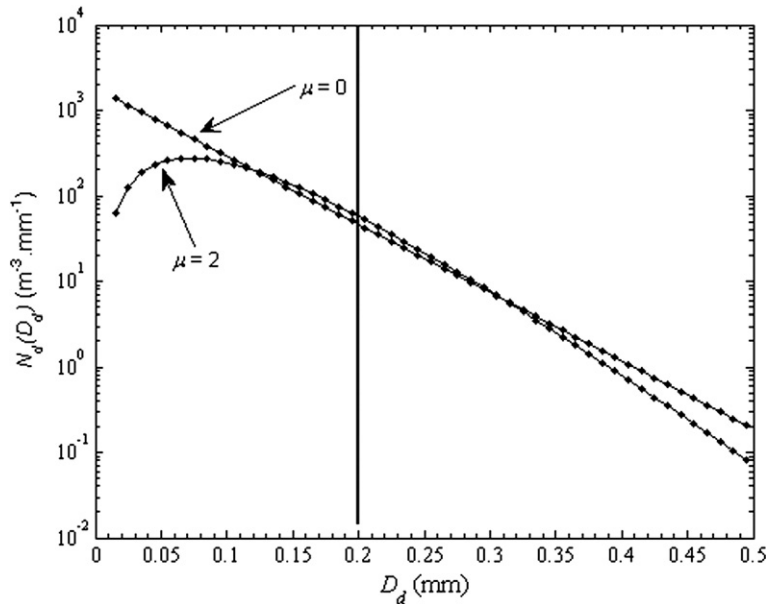


Fig. 7. Drizzle drop size distribution (DDSD) represented by (9) for  $\mu=0$  and 2, a median volume drop diameter  $D_0$  of 0.2 mm (vertical black line), and a maximum drizzle water content  $M_d^{\text{max}} = 0.005 \text{ g m}^{-3}$ .  $D_d$  is the drop diameter.

value of 0.5 mm, with a size interval  $\Delta D_d=0.01$  mm so that 46 diameter classes  $(D_{d,i})_{i=1, \dots, 46}$  are defined. Knowing  $M_d$ ,  $D_0$ , and  $\mu$  thus enables the determination of the DDS. Such a DDS accounts well for the rapid evaporation of the smallest drizzle drops outside the cloud where the atmosphere is drier. Their concentration falls down significantly in a relatively very short distance out of the cloud. The smaller the drizzle drop, the faster it evaporates, and the lower the concentration is. For rain, Ulbrich (1983) indicates that various physical processes in unsaturated air, as evaporation, seem to transform a size distribution defined by  $\mu=0$  inside a cloud to a gamma distribution with  $\mu>0$  outside a cloud. Assuming that these processes also apply for the drizzle size distribution,  $\mu$  is taken equal to zero inside the cloud and to 2 outside so that small drizzle drops are in higher concentration in the cloud. In addition, several values of  $D_0$  have been chosen in order to look at the influence of this parameter on the

breaking down of  $Z-M$  correlation:  $D_0=0.07, 0.1, 0.15, 0.2, 0.25,$  and  $0.3$  mm. Fig. 7 displays an example of DDS for a drizzle water content of  $0.005 \text{ g m}^{-3}$  and  $D_0=0.2$  mm for the two cases  $\mu=0$  and 2. Concentrations can reach  $10^2-10^3 \text{ m}^3 \text{ mm}^{-1}$  for high drizzle contents. They are very low compared to those of cumulus droplets (of the order of  $10 \text{ cm}^{-3} \mu\text{m}^{-1}$ ) such that drizzles does not bring significant water concentration in clouds.

#### 4.2. Influence of microphysical parameters on the $Z-M$ correlation

New simulations of radar observations of cumulus clouds are performed in the presence of drizzle. By the next, reflectivity ( $Z$ ) and liquid water content ( $M$ ) concern a radar resolution volume. Fig. 8 displays  $Z-M$  scatter plots (blue circles) of an extreme congestus cloud ( $M_c^{\max}=3 \text{ g m}^{-3}$ ,  $N_t=800-1000 \text{ cm}^{-3}$ ) containing

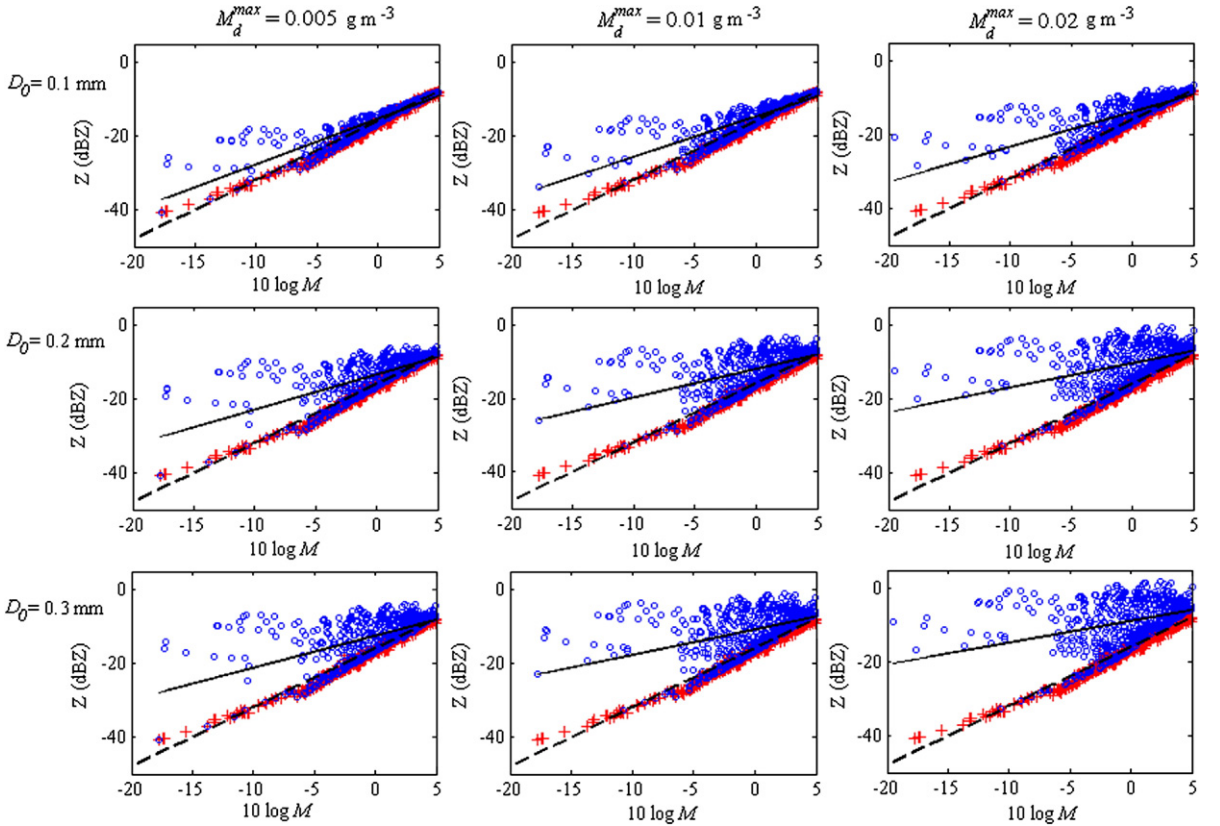


Fig. 8. Scatter plots (logarithmic scales) of the radar reflectivity  $Z$  (dBZ) vs. liquid water content  $M$  ( $\text{g m}^{-3}$ ) for an extreme congestus cumulus ( $M_c^{\max}=3 \text{ g m}^{-3}$ ) with a total droplet concentration  $N_t=1000 \text{ cm}^{-3}$  for different microphysical drizzle parameter. From top to bottom,  $D_0=0.1, 0.2,$  and  $0.3$  mm, and, from left to right,  $M_d^{\max}=0.005, 0.01,$  and  $0.02 \text{ g m}^{-3}$ . In each subplot, red crosses are only relative to cloud droplets and the reference curve  $Z_c=0.026 M_c^{1.61}$  is displayed in dashed line, blue circles correspond to  $(Z, M)$  couples of cloud droplets and drizzle drops and solid black line is the fitted curve on these circles.

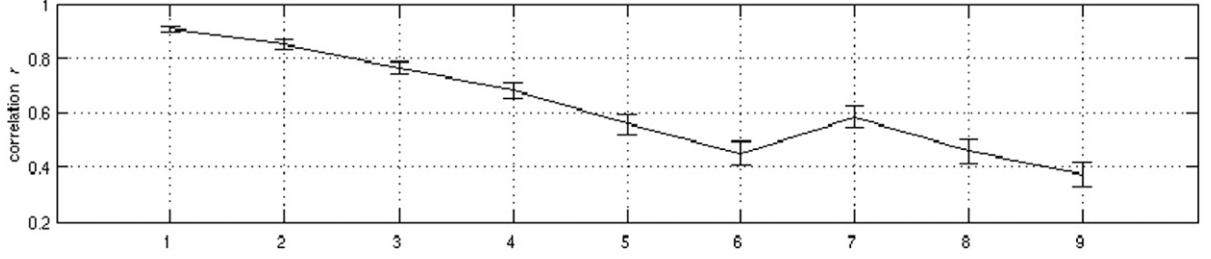


Fig. 9. Correlation coefficient  $r$  of the fitted curve on the  $(Z, M)$  couples (blue circles) of Fig. 8. The numbers on the abscissa correspond to the number of the associated subplot of Fig. 8 with a progression from the left panels to the right ones and from the top panels to the bottom ones. Thus, the number 1 refers to the upper left subplot of Fig. 8 while the number 9 corresponds to the lower right one. Vertical error bars represent 95% confidence bounds on the calculus of  $r$ .

drizzle drops. For the sake of clarity, only the scatter plots such that  $D_0=0.1, 0.2,$  and  $0.3$  mm (from top to bottom) and  $M_d^{\max}=0.005, 0.01,$  and  $0.02$  g m<sup>-3</sup> (from left to right) are displayed. This results from the superimposition of the cloud and drizzle characteristics, so that  $Z$  is the backscattered signal from cloud droplets and drizzle drops and  $M$  is the total water content

( $M=M_c+M_d$ ). A fit of these simulated data is performed and represented by the black solid line. Scatter plots shows that the great majority of blue circles is located at the top right of the scatter plot, corresponding to high  $M$  values and high reflectivity  $Z$ , where discrepancy between the fitted curve and the reference is minimum. Not surprisingly, for a fixed  $M_d^{\max}$  value, the greater  $D_0$

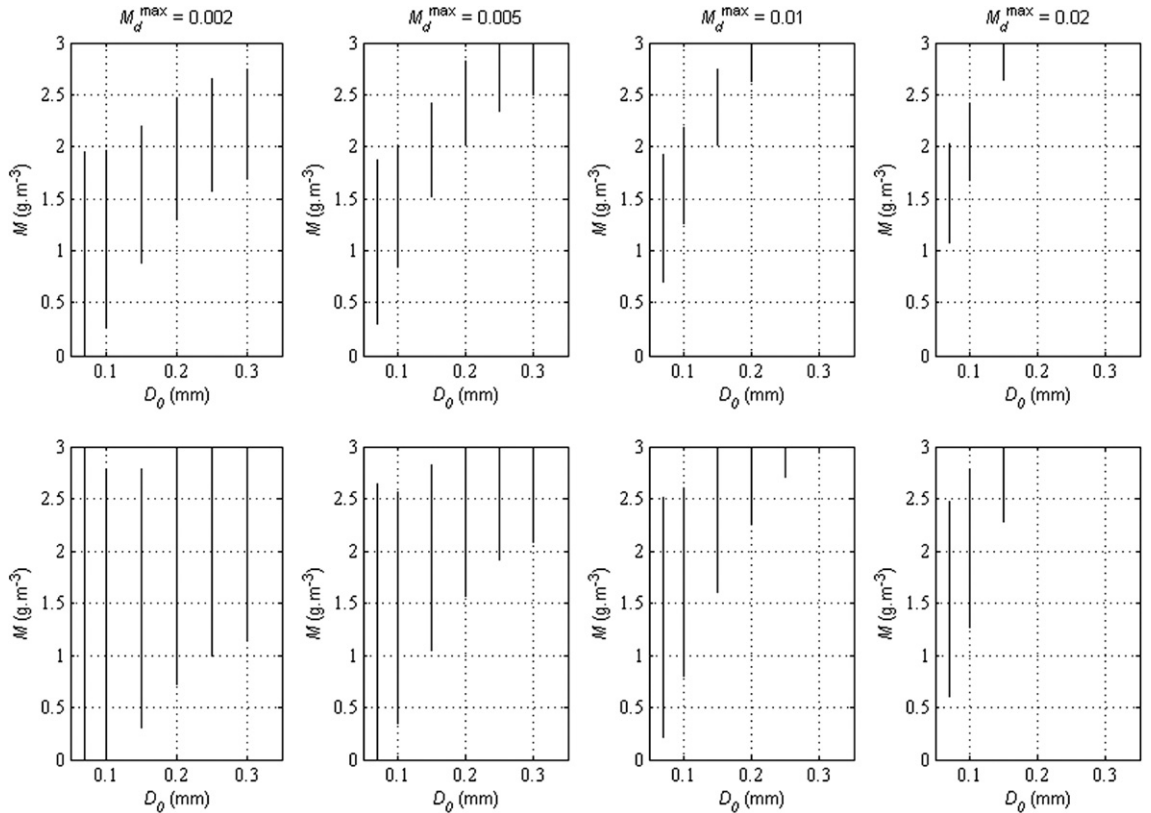


Fig. 10. Intervals of liquid water content  $M$  for which the deviation from the considered reference law  $Z_c=0.026 M_c^{1.61}$  is  $d=5$  and 10% (upper and lower panels respectively) for different drizzle liquid water content  $M_d^{\max}=0.002, 0.005, 0.01,$  and  $0.02$  g m<sup>-3</sup>.

and the lower liquid water content, the more important the scatter of the blue circles and the discrepancy between the two curves. The same remark can be done for a fixed  $D_0$  value as drizzle water content  $M_d^{\max}$  increases. Scattering of blue circles affects the  $r$  correlation coefficient of the associated fitted curve (Fig. 9):  $r$  decreases from about 0.90 to 0.37 as  $D_0$  ( $M_d^{\max}$ ) increases for a given value of  $M_d^{\max}$  ( $D_0$ ); the 95% confidence bound of  $r$  (represented by vertical error bars) follows the same tendency. These effects are the consequence of the main contribution of cloud droplets to  $M$  and the main contribution of drizzle drops to  $Z$ . A high total water content is mainly due to cloud droplets and causes a reflectivity signal high enough to be not distinguishable from that due to drizzle. On the contrary, a low total water content means that cloud droplets are rare. Consequently, their reflectivity is negligible compared to that of drizzle. However, regarding the  $r$  values in association with the number of points considered (about 1200), correlation is quite correct for blue circles so that we can write  $Z=kM^n$ ,  $k$  and  $n$  being calculable coefficient (not shown). The  $Z_c-M_c$  power law can then be used as long as drizzle is characterized by low  $M_d^{\max}$  and  $D_0$ . A possible way to evaluate the validity of the proposed  $Z_c-M_c$  relationship in presence

of drizzle is to calculate, for a given ( $D_0, M_d^{\max}$ ) couple and a given water content  $M$ , the deviation  $d$  (in percentage) between the fitted curve and the reference  $Z_c=0.026 M_c^{1.61}$ , i.e.

$$d(\%) = 100 \left( \frac{Z - Z_c}{Z_c} \right) \quad (11)$$

where reflectivity are expressed in logarithmic unit (dBZ). In terms of  $M$  ( $\text{g m}^{-3}$ ):

$$d = 100 \left[ \frac{10 \log(kM^n) - 10 \log(0.026M^{1.61})}{10 \log(0.026M^{1.61})} \right] \\ = 100 \frac{\log \left[ \frac{k}{0.026} M^{(n-1.61)} \right]}{10 \log(0.026M^{1.61})} \quad (12)$$

A practical application could be to propose a domain of liquid water content  $M$  for which  $d$  is low enough to consider that  $Z_c-M_c$  relationship is still acceptable. Such information can be deduced from Fig. 10 for an acceptance of discrepancy of  $d=5$  and 10% (up and down figures respectively). Each vertical line in each subplot represents the range values of  $M$  for which the  $Z_c-M_c$  relationship is acceptable for a given  $D_0$  and  $M_d^{\max}$  value. It is to be noted that the ranges of  $M$  is all the less extended

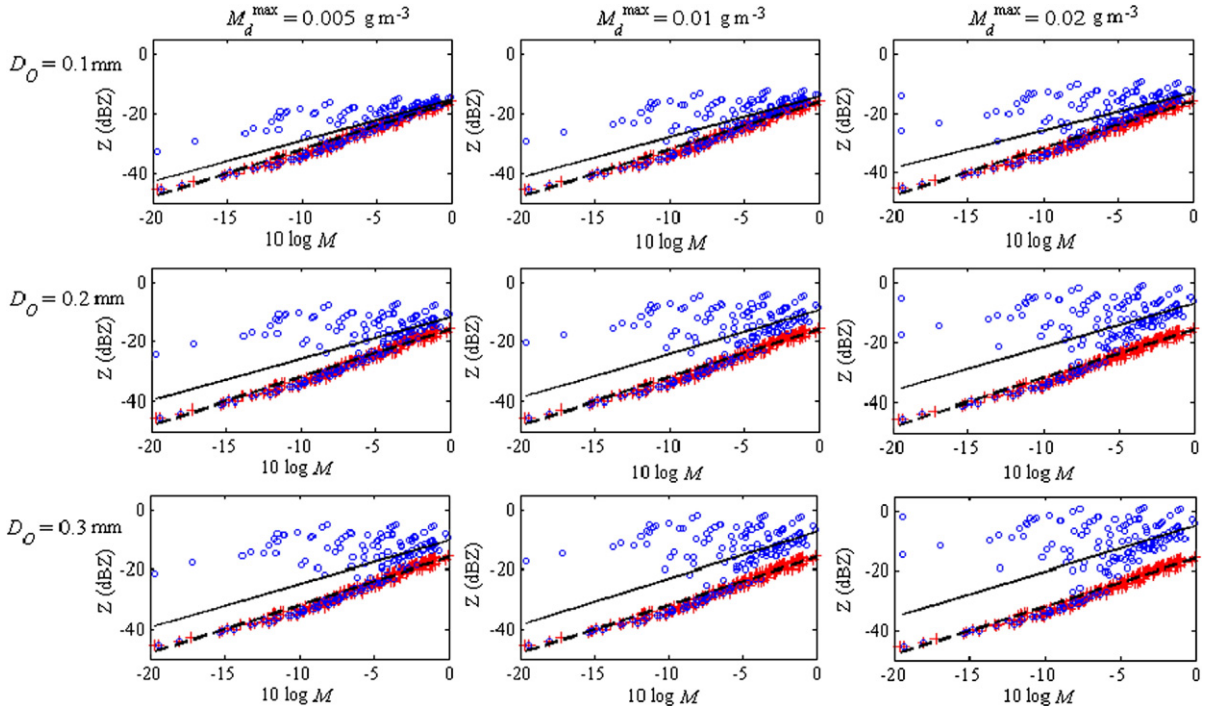


Fig. 11. Same as Fig. 8 but for a congestus cloud.

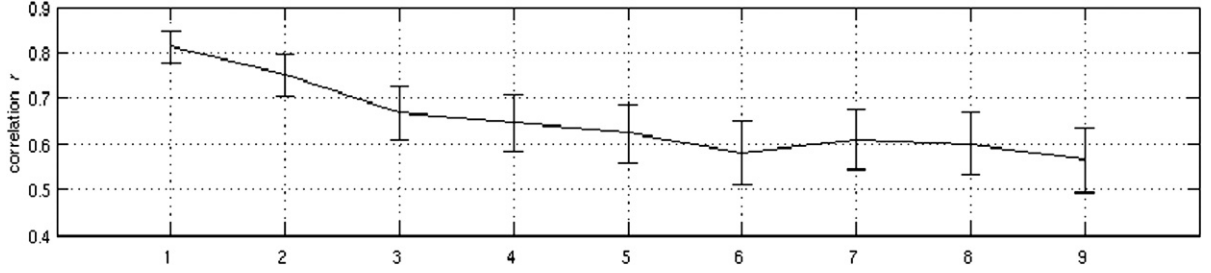


Fig. 12. Same as Fig. 9 but for a congestus cloud.

as  $D_0$  and  $M_d^{\max}$  is high. For example, for  $D_0=0.2$  mm, as  $d=5\%$ ,  $M$  ( $\text{g m}^{-3}$ ) is included in the intervals [1.3, 2.5], [2.0, 2.8] and [2.6, 3] for  $M_d^{\max}=0.002$ , 0.005 and  $0.01$   $\text{g m}^{-3}$ , respectively. The absence of line at a given  $D_0$  and/or  $M_d^{\max}$  values means that the  $Z_c-M_c$  relationship is never verified whatever the water content. For instance, for  $M_d^{\max}=0.02$   $\text{g m}^{-3}$  and  $D_0$  higher than  $0.15$  mm, there is no domain of  $M$  for which  $d<5\%$ . Thus, considering  $D_0=0.2$  mm, for an accepted deviation  $d=5\%$ , the  $Z_c-M_c$  relationship can be applied only if

$M_d^{\max}<0.01$   $\text{g m}^{-3}$  under particular conditions on  $M$  indicated by the black vertical lines. Note that for  $d=10\%$ , conditions on  $M$  are less restrictive since  $M$  intervals are obviously larger.

A similar analysis has been done for a cumulus congestus for which  $M \leq M_d^{\max}=1$   $\text{g m}^{-3}$  and  $N_r=400-600$   $\text{cm}^{-3}$ . Figs. 11, 12 and 13 are respectively the analog of the above Figs. 8, 9 and 10. The first one shows quite similar scatter plots than for the above case: reflectivity values ranges from  $-45$  to  $0$  dBZ and lots of

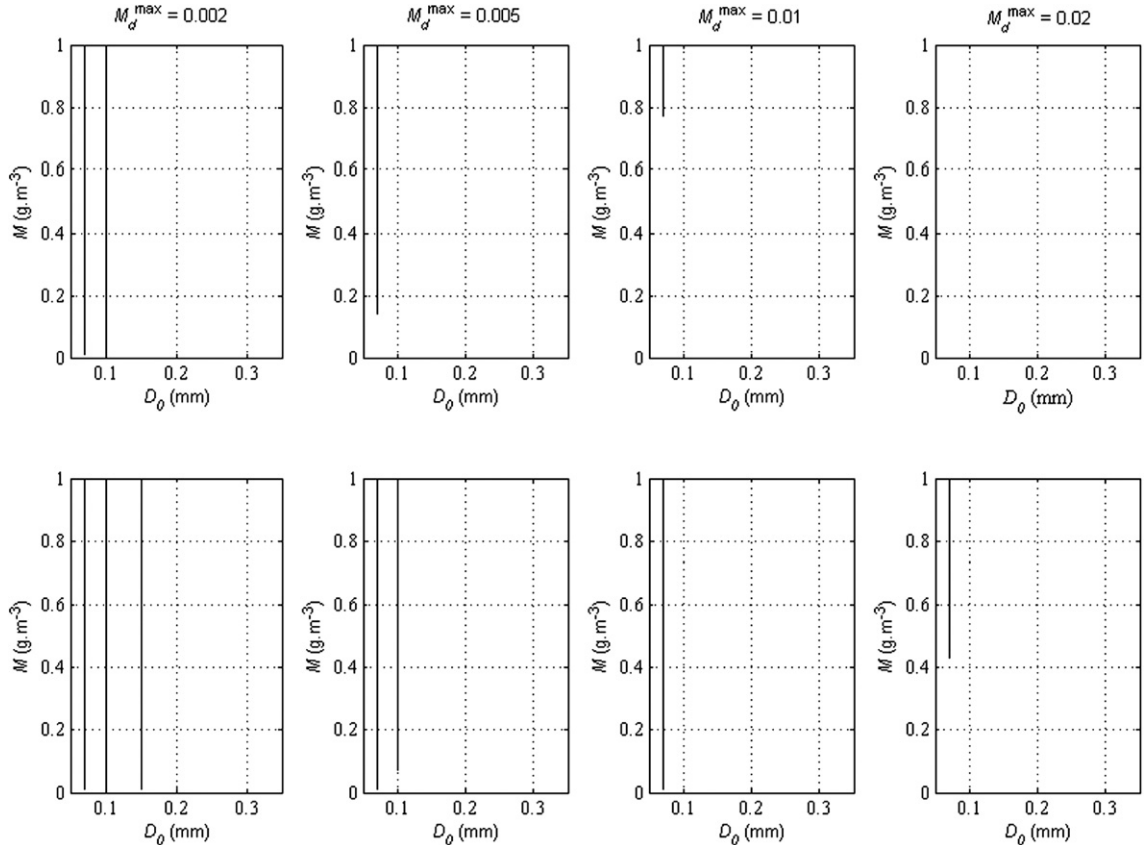


Fig. 13. Same as Fig. 10 but for a congestus cloud.

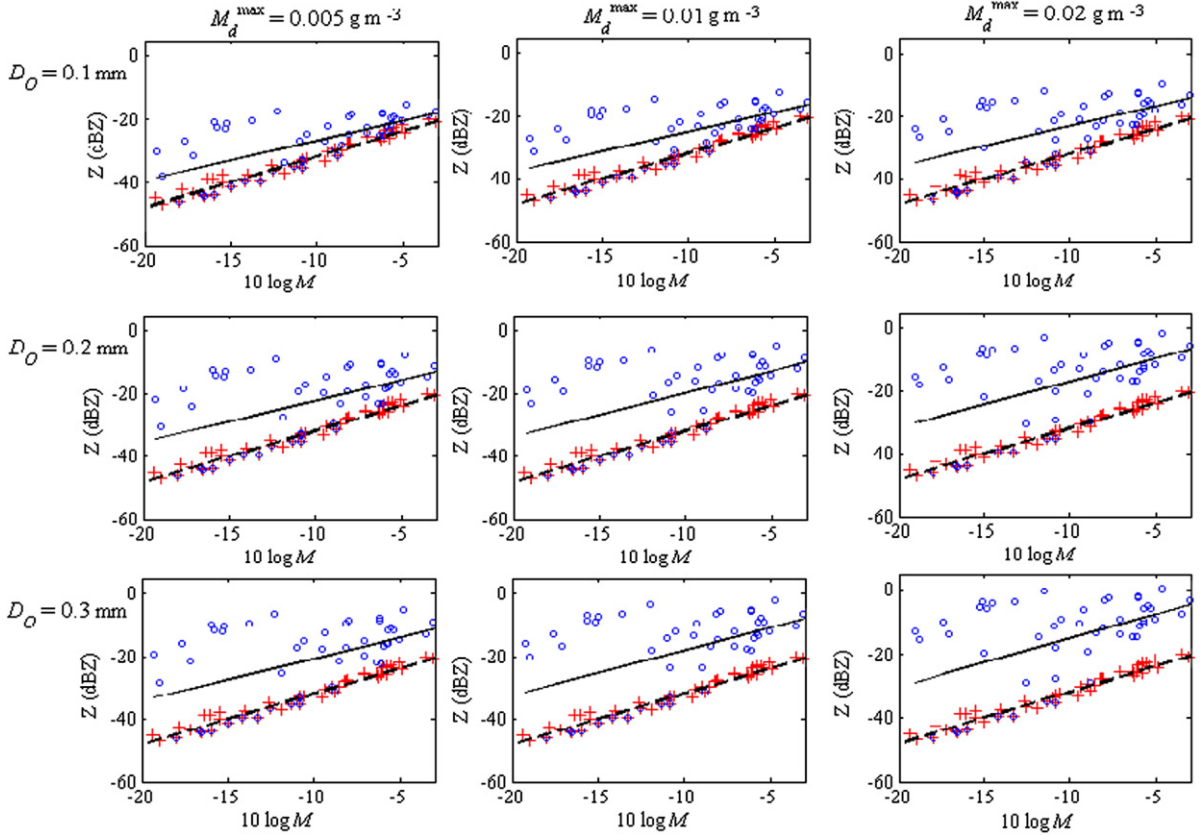


Fig. 14. Same as Fig. 8 but for small cumulus clouds (humilis and mediocris).

blue circles are concentrated on the upper right part of each scatter plot. However, dispersion is greater because of the lower number of blue circles (about 500) due to a smaller size of the modeled cumulus congestus. As for Fig. 8, the fitted  $(Z, M)$  data can be represented by a linear function which is all the closer to the reference  $Z_c-M_c$  relationship as  $D_0$  is smaller and  $M_d^{\max}$  is lower. As  $D_0$  and/or  $M_d^{\max}$  increase, the gap between the fitted line and the reference one uniformly increases whatever the water content  $M$  whereas for the above considered extreme congestus the curves tend to converge for high  $M$ . This difference can be explained by the lower cloud liquid water content of the congestus, so that drizzle has a more important influence. Correlation coefficient  $r$  (Fig. 12) ranges from about 0.85 to 0.55 and follows the same tendency as for the extreme congestus one. But, confidence bounds at 95% are greater. Range of  $M$  values for which the  $Z_c-M_c$  relationship can be accepted with a deviation  $d$  of 5 and 10% (Fig. 13, up and down respectively) is now considerably reduced: for  $d=5\%$  (10%) only 4 (7) intervals of  $M$  can be identified whereas for the extreme congestus case, we had respectively

19 and 20 intervals. For example, for  $M_d^{\max}=0.002, 0.005,$  and  $0.01 \text{ g m}^{-3}$ , the  $Z_c-M_c$  relationship can not be applied whatever the  $M$  value if  $D_0$  exceeds 0.1, 0.07, and 0.07 mm respectively for  $d=5\%$  and 0.15, 0.1, and 0.07 mm respectively for  $d=10\%$ . Note that for  $M_d^{\max}=0.02 \text{ g m}^{-3}$  the  $Z_c-M_c$  relationship can never be applied unless for  $d=10\%$  if  $D_0$  is lower than 0.07 mm.

For the mediocris and humilis cumuli ( $M < 0.5 \text{ g m}^{-3}$ ,  $N_t < 400 \text{ cm}^{-3}$ ), drizzle has obviously a greater influence than previously since the values of  $M$  involved are not yet negligible with respect to  $M_c$ . The number of blue circles in Fig. 14 is low (about 80) due to the smallest dimension of this kind of cloud. Correlation coefficient on these circles is on the whole greater than 0.4 but the confidence bounds at 95% are very large (not shown). In this context, the  $Z_c-M_c$  relationship can practically never be applied even if we accept a deviation  $d=10\%$  (not shown). Table 3 summarizes the condition for the application of the  $Z_c-M_c$  relationship. Such information can be practically useful if radar observations are combined with measurements by a vertically pointed radiometer. The former measures the reflectivity  $Z$



Table 3

Domain of liquid water content value  $M$  for which, for a given  $D_0$  and  $M_d^{\max}$ , radar reflectivity  $Z$  differs from the cloud reference value  $Z_c = 0.026 M_c^{1.61}$  from  $d=5$  and 10%

$M_d^{\max}$ ( $\text{g m}^{-3}$ )	$D$	0.002		0.005		0.01		0.02	
		5%	10%	5%	10%	5%	10%	5%	10%
		<hr/>							
$D_0$ (mm)									
$M_c^{\max} = 3 \text{ g m}^{-3}$ extreme congestus	0.07	<2	All	0.3–1.9	<2.6	0.7–1.9	0.3–2.5	1.1–2	0.6–2.5
	0.1	<2	<2.8	0.9–2	0.3–2.6	1.3–2.2	0.8–2.6	1.6–2.4	1.3–2.7
	0.15	0.9–2.2	0.3–2.8	1.5–2.4	1–2.8	2–2.7	>1.6	>2.6	>2.3
	0.2	1.4–2.5	>0.7	2–2.8	>1.6	>2.6	>2.3	No	No
	0.25	1.6–2.6	>1.1	>2.4	>1.9	No	>2.7	No	No
	0.3	1.7–2.7	>1.1	>2.5	>2.1	No	No	No	No
$D_0$ (mm)									
$M_c^{\max} = 1 \text{ g m}^{-3}$ congestus	0.07	All	All	>0.15	All	>0.75	All	No	>0.4
	0.1	All	All	No	>0.1	No	No	No	No
	0.15	No	All	No	No	No	No	No	No
	0.2	No	No	No	No	No	No	No	No
	0.25	No	No	No	No	No	No	No	No
	0.3	No	No	No	No	No	No	No	No
$D_0$ (mm)									
$M_c^{\max} < 0.5 \text{ g m}^{-3}$ mediocris–humilis	0.07	No	All	No	>0.1	No	No	No	No
	0.1	No	All	No	No	No	No	No	No
	0.15	No	0.3–2.8	No	No	No	No	No	No
	0.2	No	>0.7	No	No	No	No	No	No
	0.25	No	>1	No	No	No	No	No	No
	0.3	No	>1.1	No	No	No	No	No	No

The interval depends on the kind of cumulus cloud considered and identified by  $M_c^{\max}$ , “All” means that all  $M$  values between 0 and  $M_c^{\max}$  satisfy the  $d$  value chosen whereas “No” means that no value of  $M$  can be regarded.

which can be directly related to  $D_0$  and the latter provides the total liquid water content  $M$ . A measure of the couple  $(Z, M)$  can thus be compared with the information on Figs. 10 or 13 so that the validity and applicability of the relationship (7) can be determined.

## 5. Conclusion

In this paper, the influence of microphysical drizzle parameters on the relationship between cloud reflectivity  $Z_c$  and cloud liquid water content  $M_c$  is studied through simulations of radar observations at 3 GHz of different cloud types. These last ones consist of cumulus clouds, characterized by their maximum liquid water content  $M_d^{\max}$  and their total droplet concentration  $N_r$ . Superimposed is a drizzle field characterized by a maximum drizzle water content  $M_d^{\max}$  and a drizzle drop median volume diameter  $D_0$ . Four kinds of cumulus clouds (humilis, mediocris, congestus, extreme congestus) and various superimposed drizzle fields are constructed. Although simple, the present warm cloud

modeling is quite reasonable and realistic. Further, it is original and interesting to use simulations to answer questions about what would see a radar in terms of liquid hydrometeor reflectivity.

In order to make the paper consistent, the relationship

$$Z_c = 0.026 M_c^{1.61}$$

is first proposed. This one is shown to match well previous empirical relationships relative to cumulus and stratocumulus clouds. In addition, it extends the domain of validity of the  $Z_c$ – $M_c$  relationships towards high water content (till  $3 \text{ g m}^{-3}$ ).

This relation is used as a reference for looking at the influence of drizzle sized drops on the correlation between radar reflectivity and liquid water content. As already shown by previous empirical studies, drizzle drops disable such relationships since they are characterized by relatively high diameters and low concentrations with respect to cloud droplets so that the signal of reflectivity is carried by drizzle drops whereas liquid

water content is supported by cloud droplets. Several thresholds in terms of radar observables (reflectivity, Doppler velocity) have been proposed to discriminate drizzle-free clouds from drizzle clouds. This work gives some indications on the microphysical conditions under which the correlation between reflectivity and liquid water content is broken down. Precisely, considering a cumulus cloud characterized by a maximum liquid water content  $M_c^{\max}$  and a total droplet concentration  $N_t$ , we look at the values of the  $(D_0, M_d^{\max})$  couples which permits to use the  $Z_c-M_c$  relationship with a deviation lower or equal than 5 and 10%. It is found that the larger  $D_0$ , and/or  $M_d^{\max}$ , the more restrictive the application of the  $Z_c-M_c$  relationship. For small cumuli as humilis and mediocris, such a relationship can never be applied under the deviation considered whatever the  $M$  values considered, whereas for cumulus congestus or extreme congestus, some useful values of  $M$  are provided. These microphysical information are complementary of those previously proposed regarding reflectivity or Doppler velocity. Further, they are practically useful if a radar and a radiometer are associated since the couple  $(Z, M)$  can be measured.

In this study, none experimental data are used since not available. The authors are conscious that data would be useful and fruitful for studying sensitivity of the  $Z_c-M_c$  relationship. This work should be then considered as exploratory and suggestive of further investigations by means of in-situ measurements or numerical simulations.

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