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Optimum spin-squeezing in Bose-Einstein condensates with particle losses

Y. Li,^{1,2} Y. Castin,¹ and A. Sinatra¹

¹*Laboratoire Kastler Brossel, ENS, UPMC, 24 rue Lhomond, 75231 Paris Cedex 05, France*

²*Department of Physics, East China Normal University, Shanghai 200062, China*

The problem of spin squeezing with a bimodal condensate in presence of particle losses is solved analytically by the Monte Carlo wavefunction method. We find the largest obtainable spin squeezing as a function of the one-body loss rate, the two-body and three-body rate constants, and the s -wave scattering length.

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Spin squeezed states, first introduced in [1], generalize to spin operators the idea of squeezing developed in quantum optics. In atomic systems effective spins are collective variables that can be defined in terms of two different internal states of the atoms [2] or two orthogonal bosonic modes [3]. States with a large coherence between the two modes, that is with a large mean value of the spin component in the equatorial plane of the Bloch sphere, can still differ by their spin fluctuations. For an uncorrelated ensemble of atoms, the quantum noise is evenly distributed among the spin components orthogonal to the mean spin. However quantum correlations can redistribute this noise and reduce the variance of one spin quadrature with respect to the uncorrelated case, achieving spin squeezing. Besides applications in quantum communication and quantum information [6], these multi-particle entangled states have practical interest in atom interferometry, and high precision spectroscopy [4] where they could be used to beat the standard quantum limit already reached in atomic clocks [5].

Different techniques to create spin squeezed states in atomic systems have been proposed and successfully realized experimentally including transfer of squeezing from light to matter [7] and quantum non demolition measurements of the atomic state [8]. To go further, it was shown that coherent interactions between cold atoms in a bimodal Bose-Einstein condensates [3], can in principle provide a huge amount of entanglement and spin squeezing. It is thus important to determine the ultimate limitations imposed by decoherence to the maximum spin squeezing that can be obtained by this method. Several forms of decoherence may be present in the experiment. In a different context, the effect of a dephasing perturbation to the spin correlations has recently been studied in [9]. In this work we deal with particle losses, an unavoidable source of decoherence in cold atom systems, due in particular to collisions of condensed atoms with the hot background gas, and to three-body collisions followed by a molecule formation.

As shown in [3], bimodal Bose-Einstein condensates realize the one-axis twisting model proposed in [1] to create spin squeezing. This exactly solvable model predicts that a normalized variance of a spin component in the

orthogonal plane as small as $N^{-2/3}$ (N being the number of particles in the system) can be obtained in a time which tends to zero in the $N \rightarrow \infty$ limit. This may give the impression that decoherence will have a negligible effect in the large N limit so that there should be no limit in the amount of squeezing one can reach. We will show that this naive prediction is qualitatively correct for one-body losses (whose rate is independent of N) but it is wrong for two and three-body losses. In particular we find a very simple analytical expression of the maximum squeezing one can get in presence of one, two and three-body losses for a given scattering length once the trap frequency, the atom number, the interaction time and the spin quadrature are optimized.

We consider two spatially separated symmetric condensates a and b prepared in an initial state with N particles and a well defined relative phase [10]

$$|\phi\rangle \equiv \frac{1}{\sqrt{N!}} \left(\frac{e^{i\phi} a^\dagger + e^{-i\phi} b^\dagger}{\sqrt{2}} \right)^N |0\rangle. \quad (1)$$

We assume that $\phi = 0$ initially, correspondingly, the x component of the collective spin $S_x = (a^\dagger b + b^\dagger a)/2$ has a mean value $\langle S_x \rangle = N/2$. Here we assume that no excitation is created during the preparation process and we neglect all the other modes than the condensate modes a and b . When expanded over Fock states $|N_a, N_b\rangle$, the state (1) shows binomial coefficients which, for large N , are peaked around the average number of particles in a and b , $\bar{N}_a = \bar{N}_b = N/2$. We can use this fact to quadratize the Hamiltonian of the system around \bar{N}_a and \bar{N}_b :

$$H_0 = \frac{\hbar\chi}{4} (a^\dagger a - b^\dagger b)^2 \quad (2)$$

where $\chi = (\partial_{N_a} \mu_a)_{\bar{N}_a} / \hbar$ and $\mu_a = \mu_b$ is the chemical potential for each condensate.

In presence of particle losses, the evolution is ruled by a master equation for the density operator ρ of the system. In the interaction picture with respect to H_0 , with one, two and three-body losses, we have:

$$\frac{d\tilde{\rho}}{dt} = \sum_{m=1}^3 \sum_{\epsilon=a,b} \gamma^{(m)} \left[c_\epsilon^m \tilde{\rho} c_\epsilon^{\dagger m} - \frac{1}{2} \{ c_\epsilon^{\dagger m} c_\epsilon^m, \tilde{\rho} \} \right] \quad (3)$$

where $\tilde{\rho} = e^{iH_0t/\hbar}\rho e^{-iH_0t/\hbar}$, $c_a = e^{iH_0t/\hbar}ae^{-iH_0t/\hbar}$, and similarly for b , $\gamma^{(m)} = K^{(m)} \int d^3r |\phi(r)|^{2m}$, where $K^{(m)}$ is the m -body rate constant and $\phi(r)$ is the condensate wavefunction in one of the two modes. In the Monte Carlo wavefunctions approach [11] we define an effective Hamiltonian H_{eff} and the jump operators $S_\epsilon^{(m)}$

$$H_{\text{eff}} = - \sum_{m=1}^3 \sum_{\epsilon=a,b} \frac{i\hbar}{2} \gamma^{(m)} c_\epsilon^{\dagger m} c_\epsilon^m; \quad (4)$$

$$S_\epsilon^{(m)} = \sqrt{\gamma^{(m)}} c_\epsilon^m. \quad (5)$$

We assume that a small fraction of particles will be lost during the evolution so that we can consider χ and $\gamma^{(m)}$ ($m = 2, 3$) as constant parameters of the model. The state evolution in a single quantum trajectory is a sequence of random quantum jumps at times t_j and non-unitary Hamiltonian evolutions of duration τ_j :

$$|\psi(t)\rangle = e^{-iH_{\text{eff}}(t-t_k)/\hbar} S_{\epsilon_k}^{(m_k)}(t_k) e^{-iH_{\text{eff}}\tau_k/\hbar} S_{\epsilon_{k-1}}^{(m_{k-1})}(t_{k-1}) \dots S_{\epsilon_1}^{(m_1)}(t_1) e^{-iH_{\text{eff}}\tau_1/\hbar} |\psi(0)\rangle. \quad (6)$$

The expectation value of any observable \hat{O} is obtained by averaging over all possible stochastic realizations, that is all kinds, times and number of quantum jumps, each trajectory being weighted by its probability [11]

$$\langle \hat{O} \rangle = \sum_k \int_0^{t_k} dt_1 dt_2 \dots dt_k \sum_{\{\epsilon_j, m_j\}} \langle \psi(t) | \hat{O} | \psi(t) \rangle. \quad (7)$$

We want to calculate spin squeezing. In the considered symmetric case with zero initial relative phase, the mean spin remains aligned to the x axis $\langle S_x \rangle = \langle b^\dagger a \rangle$, and the spin squeezing is quantified by the parameter [3, 4]

$$\xi^2 = \min_\theta \frac{\langle \hat{N} \rangle \Delta S_\theta^2}{\langle S_x \rangle^2}, \quad (8)$$

where $S_\theta = (\cos\theta)S_y + (\sin\theta)S_z$, $S_y = (a^\dagger b - b^\dagger a)/(2i)$, $S_z = (a^\dagger a - b^\dagger b)/2$ and $\hat{N} = a^\dagger a + b^\dagger b$. The non correlated limit yields $\xi^2 = 1$, while $\xi^2 < 1$ is the mark of an entangled state [3, 6]. In all our analytic treatments, it turns out that $\Delta S_z^2 = \langle \hat{N} \rangle / 4$. This allows to express ξ^2 in a simple way:

$$\xi^2 = \frac{\langle a^\dagger a \rangle}{\langle b^\dagger a \rangle^2} \left(\langle a^\dagger a \rangle + A - \sqrt{A^2 + B^2} \right), \quad (9)$$

with

$$A = \frac{1}{2} \text{Re} \left(\langle b^\dagger a^\dagger ab - b^\dagger b^\dagger aa \rangle \right) \quad (10)$$

$$B = 2 \text{Im} \left(\langle b^\dagger b^\dagger ba \rangle \right). \quad (11)$$

With one-body losses only, the problem is exactly solvable. Following a similar procedure as in [12], we get

$$\xi^2(t) = \frac{1 + \frac{1}{4}(N-1)e^{-\gamma t} [\tilde{A} - \sqrt{A^2 + B^2}]}{\left[\frac{\gamma^2 + \chi[\gamma \sin(\chi t) + \chi \cos(\chi t)]e^{-\gamma t}}{\gamma^2 + \chi^2} \right]^{2N-2}} \quad (12)$$

with $\gamma \equiv \gamma^{(1)}$ and

$$\tilde{A} = 1 - \left[\frac{\gamma^2 + 2\chi[\gamma \sin(2\chi t) + 2\chi \cos(2\chi t)]e^{-\gamma t}}{\gamma^2 + 4\chi^2} \right]^{N-2}$$

$$\tilde{B} = 4 \sin \chi t \left[\frac{\gamma^2 + \chi[\gamma \sin(\chi t) + \chi \cos(\chi t)]e^{-\gamma t}}{\gamma^2 + \chi^2} \right]^{N-2}.$$

The key points are that (i) H_{eff} is proportional to \hat{N} so it does not affect the state, and (ii) a phase state $|\phi\rangle$ is changed into a phase state with one particle less after a quantum jump, $c_{a,b}|\phi\rangle \propto |\phi \mp \chi t/2\rangle$ where t is the time of the jump, the relative phase between the two modes simply picking up a random shift $\mp \chi t/2$ which reduces the squeezing.

When two and three-body losses are taken into account, an analytical result can still be obtained by using a constant loss rate approximation [12]

$$H_{\text{eff}} \simeq - \sum_{m=1}^3 \sum_{\epsilon=a,b} \frac{i\hbar}{2} \gamma^{(m)} \bar{N}_\epsilon^m \equiv -\frac{i\hbar}{2} \lambda. \quad (13)$$

We verified by simulation (see Fig.1) that this is valid for the regime we consider, where a small fraction of particles is lost at the time at which the best squeezing is achieved. In this approximation, the mean number of particles at time t is

$$\langle \hat{N} \rangle = N \left[1 - \sum_m \Gamma^{(m)} t \right]; \quad \Gamma^{(m)} \equiv (N/2)^{m-1} m \gamma^{(m)} \quad (14)$$

where $\Gamma^{(m)} t$ is the fraction of lost particles due to m -body losses. Spin squeezing is calculated from (9) with

$$\langle b^\dagger a \rangle = \frac{e^{-\lambda t}}{2} \cos^{N-1}(\chi t) \tilde{N} F_1 \quad (15)$$

$$A = \frac{e^{-\lambda t}}{8} \tilde{N} (\tilde{N} - 1) [F_0 - F_2 \cos^{N-2}(2\chi t)] \quad (16)$$

$$B = \frac{e^{-\lambda t}}{2} \cos^{N-2}(\chi t) \sin(\chi t) \tilde{N} (\tilde{N} - 1) F_1 \quad (17)$$

where the operator $\tilde{N} = (N - \partial_\alpha)$ acts on the functions

$$F_\beta(\alpha) = \exp \left[\sum_{m=1}^3 2\gamma^{(m)} t e^{\alpha m} \frac{\sin(m\beta\chi t)}{m\beta\chi t \cos^m(\beta\chi t)} \right], \quad (18)$$

and all expressions should be evaluated in $\alpha = \ln \bar{N}_a$.

We want to find simple results for the best squeezing and the best squeezing time in the large N limit. In the absence of losses [1] the best squeezing and the best squeezing time in units of $1/\chi$ scale as $N^{-2/3}$. We then

set $N = \varepsilon^{-3}$ and rescale the time as $\chi t = \tau \varepsilon^2$. We expand the results (12) and (15-17) for $\varepsilon \ll 1$ keeping $\Gamma^{(m)}/\chi$ constant, and we obtain in both cases

$$\xi^2(t) \simeq \frac{1}{N^2(\chi t)^2} + \frac{1}{6}N^2(\chi t)^4 + \frac{1}{3}\Gamma_{\text{sq}}t, \quad (19)$$

with

$$\Gamma_{\text{sq}} = \sum_m \Gamma_{\text{sq}}^{(m)} \quad \text{and} \quad \Gamma_{\text{sq}}^{(m)} = m\Gamma^{(m)}. \quad (20)$$

For equal loss rates $\Gamma^{(m)}$, the larger m , the more the squeezing is affected. Introducing the squeezing $\xi_0^2(t)$ in the no-loss case, the above result can be written as

$$\xi^2(t) = \xi_0^2(t) \left[1 + \frac{1}{3} \frac{\Gamma_{\text{sq}}t}{\xi_0^2(t)} \right]. \quad (21)$$

This shows that (i) the fact that only a small fraction of atoms is lost at the best squeezing time does not imply that the correction on the squeezing due to losses is small; (ii) the more squeezed the state is, the more sensitive to the losses. This was expected as a larger amount of correlation is then lost in each loss event. In presence of losses, the best squeezing time and the corresponding squeezing are

$$t_{\text{best}} = \left[\frac{f(C)}{2} \right]^{1/3} \frac{N^{-2/3}}{\chi}, \quad (22)$$

$$\xi^2(t_{\text{best}}) = \left[\frac{1}{f(C)^{2/3}} + \frac{f(C)^{4/3}}{24} + \frac{Cf(C)^{1/3}}{3} \right] \left(\frac{2}{N} \right)^{2/3} \quad (23)$$

$$f(C) = \sqrt{C^2 + 12} - C; \quad C = \frac{\Gamma_{\text{sq}}}{2\chi}. \quad (24)$$

In order to find optimal conditions to produce spin squeezing in presence of losses and set the ultimate limits of this technique, from now on, we assume that the number of particles is large enough for the condensates to be in the Thomas-Fermi regime so that

$$\mu = \frac{1}{2}\hbar\bar{\omega} \left(\frac{15Na}{2a_0} \right)^{2/5}, \quad (25)$$

where $a_0 = \sqrt{\hbar/M\bar{\omega}}$ is the harmonic oscillator length, M is the mass of a particle and $\bar{\omega}$ is the geometric mean of the trap frequencies,

$$\chi = \frac{2^{3/5}3^{2/5}}{5^{3/5}} \left(\frac{\hbar}{M} \right)^{-1/5} a^{2/5}\bar{\omega}^{6/5}N^{-3/5} \quad (26)$$

$$\Gamma^{(1)} = K^{(1)} \quad (27)$$

$$\Gamma^{(2)} = \frac{15^{2/5}}{2^{2/5}7\pi} \left(\frac{\hbar}{M} \right)^{-6/5} a^{-3/5}\bar{\omega}^{6/5}N^{2/5}K^{(2)} \quad (28)$$

$$\Gamma^{(3)} = \frac{15^{4/5}}{2^{19/5}7\pi^2} \left(\frac{\hbar}{M} \right)^{-12/5} a^{-6/5}\bar{\omega}^{12/5}N^{4/5}K^{(3)}. \quad (29)$$

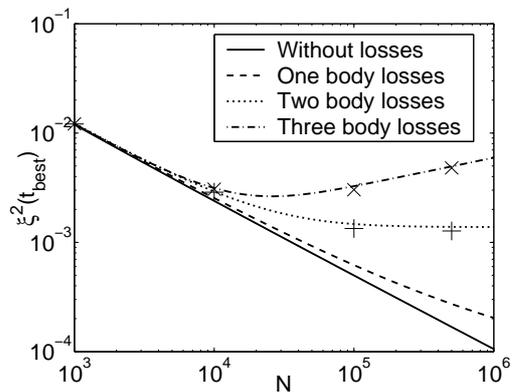


FIG. 1: Spin squeezing obtained by a minimization of ξ^2 over time, as a function of the initial number of particles, without loss of particles (solid line), with one-body losses (dashed line), with two-body losses (dotted line), with three-body losses (dash-dotted line) respectively. Parameters: $a = 5.32\text{nm}$, $\bar{\omega} = 2\pi \times 200\text{Hz}$, $K^{(1)} = 0.1\text{s}^{-1}$, $K^{(2)} = 10^{-21}\text{m}^3/\text{s}$ [13], $K^{(3)} = 6 \times 10^{-42}\text{m}^6/\text{s}$ [14]. The points with symbols (crosses for three-body and plus for two-body losses) are results of a full numerical simulation with 400 Monte Carlo realizations.

We first analyze the dependence of squeezing on the initial number of particles, separating for clarity one, two and three-body losses. Fig.1 shows the best squeezing $\xi^2(t_{\text{best}})$ as a function of N when only one kind of losses is present. Parameters correspond to ^{87}Rb atoms in the $|F = 1, m_F = -1\rangle$ state. The curve without losses is also shown for comparison. According to Fig.1, one-body losses do not change qualitatively the picture without losses and we have $\xi^2(t_{\text{best}}) \propto N^{-4/15}$ for $N \rightarrow \infty$. In the same limit, with two-body losses, $\xi^2(t_{\text{best}})$ is independent of N . With three-body losses, $\xi^2(t_{\text{best}}) \propto N^{4/15}$ for $N \rightarrow \infty$, implying that there is a finite optimum number of particles for squeezing.

We now turn to a full optimization of squeezing over $\bar{\omega}$ and N in the simultaneous presence of one, two and three-body losses. To this end, we note that the square brackets in Eq.(23) is an increasing function of C , we can then optimize $\xi^2(t_{\text{best}})$ by minimizing C with respect to $\bar{\omega}$. Under the conditions $K^{(1)} \neq 0$ and $K^{(3)} \neq 0$, the minimum of C , C_{min} , is obtained for $\Gamma_{\text{sq}}^{(3)} = \Gamma_{\text{sq}}^{(1)}$ yielding

$$\bar{\omega}^{\text{opt}} = \frac{2^{19/12}7^{5/12}\pi^{5/6}}{3^{5/12}15^{1/3}} \frac{\hbar}{M} \frac{a^{1/2}}{N^{1/3}} \left(\frac{K^{(1)}}{K^{(3)}} \right)^{5/12}. \quad (30)$$

It turns out that C_{min} is proportional to N and $\xi^2(t_{\text{best}}, \bar{\omega}^{\text{opt}})$ is a decreasing function of N . The lower bound for ξ^2 , reached for $N \rightarrow \infty$ is then

$$\inf_{t, \bar{\omega}, N} \xi^2 = \left(\frac{5\sqrt{3}M}{28\pi\hbar a} \right)^{2/3} \left[\sqrt{\frac{21}{2}(K^{(1)}K^{(3)})} + 2K^{(2)} \right]^{2/3}. \quad (31)$$

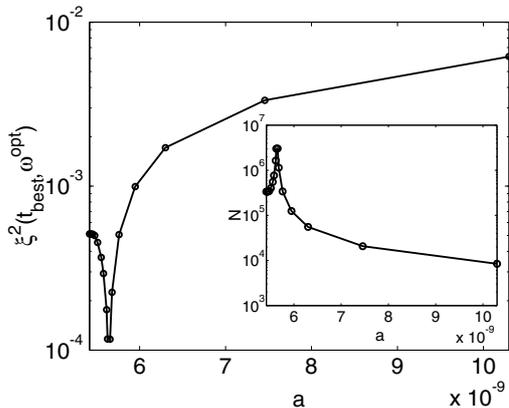


FIG. 2: Spin squeezing $\xi^2(t_{\text{best}})$ optimized with respect to $\bar{\omega}$ as a function of the scattering length a , when the magnetic field is varied on the left side of the $B_0 = 1007.4\text{G}$ Feshbach resonance of ^{87}Rb . The inset shows the number of particles for each point, calculated for $\eta = 10\%$. We took $a(B) = a_{\text{bg}}[1 - \Delta B/(B - B_0)]$ with $a_{\text{bg}} = 5.32\text{nm}$, $\Delta B = 0.21\text{G}$. The three-body rate constant $K^{(3)}(B)$ is taken from [15].

In practice, one can choose $N = N_\eta$ in order to have $\xi^2 = (1 + \eta) \inf \xi^2$ (e.g. $\eta = 10\%$), and then calculate the corresponding optimized frequency $\bar{\omega}^{\text{opt}}$ with (30). For a suitable choice of the internal state, in an optical trap, the two-body losses can be neglected $K^{(2)} = 0$. One can get in this case very simple formulas for the optimized parameters and squeezing. Consider $\eta = 10\%$, then

$$N_\eta \simeq \frac{10.296}{[K^{(1)}K^{(3)}]^{1/2}} \frac{\hbar a}{M}, \quad (32)$$

$$t_{\text{best}} \simeq 0.40 \left[\frac{M}{\hbar K^{(1)}} \right]^{2/3} \left[\frac{K^{(3)}}{a^2} \right]^{1/3}, \quad (33)$$

$$\xi^2 \simeq 0.514 \left[\frac{MK^{(1)}}{\hbar} \right]^{1/3} \left[\frac{MK^{(3)}}{\hbar a^2} \right]^{1/3}. \quad (34)$$

We now ask whether, in presence of one and three-body losses only, we can use a Feshbach resonance to change the scattering length and improve the squeezing, taking into account that $K^{(3)}$ is also affected. In Fig.2 we plot the squeezing parameter vs the scattering length a . Predicted values of $K^{(3)}$, as a function of a , are taken from [15] for ^{87}Rb in the state $|F = 1, m_F = 1\rangle$ and $K^{(1)} = 0.01\text{s}^{-1}$. We calculate $\bar{\omega}^{\text{opt}}$ and the number of particles needed for $\eta = 10\%$ for each point in the curve. The dip giving large squeezing corresponds to a strong decrease in $K^{(3)}$ around 1003.5G ($K^{(3)} \simeq 2 \times 10^{-44}\text{m}^6/\text{s}$). Close to the Feshbach resonance the squeezing gets worse as $K^{(3)}$ increases (even if in the figure we do not enter the regime $K^{(3)} \sim \hbar a^4/m$).

Finally we consider the problem of the survival time of a spin squeezed state in presence of one-body losses. We imagine that the system evolves within two periods: in the first period, which lasts a time T_1 , the system is squeezed in presence of interactions ($\chi \neq 0$), one and

three-body losses; and in the second period, which lasts a time T_2 , the interaction is stopped ($\chi = 0$), for example by opening the trap, and the system evolves under the influence of one-body losses. As T_2 can be arbitrarily long, we use the exact solution for the second period, while for the first period T_1 , on the order of t_{best} , we use a constant loss rate approximation (13). We obtain

$$\begin{aligned} \xi^2(T_2) &= \frac{1}{4} \frac{\langle \hat{N}(T_1) \rangle^2}{\langle S_x(T_1) \rangle^2} - \left[\frac{1}{4} \frac{\langle \hat{N}(T_1) \rangle^2}{\langle S_x(T_1) \rangle^2} - \xi^2(T_1) \right] e^{-\gamma^{(1)} T_2} \\ &\simeq 1 - [1 - \xi^2(T_1)] e^{-\gamma^{(1)} T_2}. \end{aligned} \quad (35)$$

In the second line of Eq.(35) we used the fact that for $T_1 \simeq t_{\text{best}}$, $\langle S_x \rangle \simeq \langle \hat{N}(T_1) \rangle / 2$. This result shows that the spin squeezing can be kept some time after the interactions have been stopped. To give an example, for ^{87}Rb parameters with bare scattering length $a = 5.32\text{nm}$, $K^{(1)} = 0.01\text{s}^{-1}$, $K^{(2)} = 0$, $K^{(3)} = 6 \times 10^{-42}\text{m}^6/\text{s}$, in optimized conditions (32)-(34) $N = 1.6 \times 10^5$ and $\bar{\omega}^{\text{opt}} = 2\pi \times 15.24\text{Hz}$, $\xi^2 = 8.2 \times 10^{-4}$ is reached at $T_1 = t_{\text{best}} = 6.5 \times 10^{-2}\text{s}$, and a large amount of squeezing $\xi^2 \simeq 0.01$ is still available after a time of $T_1 + T_2 = 1\text{s}$.

In conclusion, within the scheme proposed by [3] we have found analytically the maximum spin squeezing obtainable in a bimodal condensate in presence of losses, see Eq.(31). This bound can be approached with realistic atom numbers in rather weak traps. LKB is a unit of ENS and UPMC associated to CNRS. We acknowledge discussions with the atom chip team of Jakob Reichel. Y. Li acknowledges support from the ENS/ECNU exchange program.

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