Remote Stabilization via Communication Networks with a Distributed Control Law

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Abstract

In this paper we investigate the problem of remote stabilization via communication networks involving some time-varying delays of known average dynamics. This problem arises when the control law is remotely implemented and leads to the problem of stabilizing an open-loop unstable system with time-varying delay. We use a time-varying horizon predictor to design a stabilizing control law that sets the poles of the closed-loop system. The computation of the horizon of the predictor is investigated and the proposed control law explicitly takes into account an estimation of the average delay dynamics. The resulting closed loop system robustness with respect to some uncertainties on the delay estimation is also considered. Simulation results are finally presented.

Index Terms

Networked control systems, stabilization with time-varying delays, state predictor.

I. Introduction

The networked control systems constitute a new class of control systems including specific problems such as delays, loss of information and data process. The problem studied in this paper concerns the remote stabilization of unstable open-loop systems. The sensor, actuator and system are assumed to be remotely commissioned by a controller that interchanges measurements and control signals through a *lossless* communication network (all lost packets are re-emitted). We

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assume that the communication network has its own dynamics, and that an estimator or a model for the average induced time-delay is available. As an example, the CUMSUM Kalman filter proposed in [1] can be used to estimate the delay from some measurements of the round-trip time or of a single channel delay. Another possibility is to estimate the delay from some established model, such as those proposed in [2], [3], which are derived for local networks where the transfer protocol (TP) is set by the users and where a router (which can possibly inform the emitters of the instantaneous queue length) manages the packets.

Some experimental results [4], [5], [6] on control over networks illustrate the fact that latency and jitter have a crucial effect on the closed-loop performances, while practical solutions can be used to reduce the effect of packet losses to an acceptable level. Our work is then focused on the compensation of the delays induced by the network with a control law that explicitly takes into account an estimation of these delays. We consider also that the remotely controlled system may be unstable, as the teleoperation of open-loop unstable systems with time-varying delays has been scarcely studied yet. Airplane drone and tele-operated vehicles are examples of open-loop unstable and remotely controlled systems.

An interesting survey on time delay systems is proposed in [7], where different control laws are compared. The control approach developed in this paper is based on the design of a state predictor. Compared to other latency compensation methods, such as the one proposed in [8] (based on output feedback and GPS synchronization), the advantage of a control strategy based on the use of a state predictor is to allow for a “pole-placement” on the closed-loop system.

The state predictor is used in [9], [10], [11] to achieve a finite spectrum assignment on systems with delayed output, state or input. The previous works are generalized in [12] with the concept of system reduction (infinite to finite spectrum assignment). The problem of time-varying delays is studied more specifically in [13], which predictor is included in a H^∞ control scheme in [14]. The explicit use of the latency dynamics in the computation of the predictor’s horizon is detailed in [15], [16], where we supposed that a network model was available. These results are first summarized to describe the ideal case and then extended to the case where only an estimation of the network latency is available to set the control law. This is done thanks to an appropriate investigation of the closed-loop system robustness with respect to some latency estimation errors.

This paper is organized as follows. The control problem considered is formulated as the problem of stabilizing a time-delay system with a state predictor which has a time-varying

horizon in section II. The computation of the horizon and the *explicit* use of the average network dynamics is investigated in section III. The robustness of the resulting control setup with respect to some uncertainties on the network model is presented in the section, along with a simulation example.

II. Problem Formulation

Before dealing with a particular transmission protocol dynamics, we aim at exploring how the control design can be elaborated for a system where the transmission delay is considered as an autonomous stable system. More precisely, we consider systems of the form:

$$\dot{x}(t) = Ax(t) + Bu(t - \tau(t)), \quad x(0) = x_0 \tag{1}$$

$$y(t) = Cx(t) \tag{2}$$

$$\dot{z}(t) = f(z(t), u_d(t)), \quad z(0) = z_0 \tag{3}$$

$$\tau(t) = h(z(t), u_d(t)) \tag{4}$$

where $x \in R^n$ is the internal state, $u \in R$ is the control input, $y \in R^m$ is the system output, and A, B, C are matrices of appropriate dimensions. The pairs (A, B) and (A, C) are assumed to be controllable and observable, respectively, but A may be unstable. The signal $u_d(t)$ and the functions $f(\cdot)$ and $h(\cdot)$ are assumed to be some known continuous functions in this nominal case. These hypothesis will be relaxed later on the paper (section IV), where only the estimated dynamics are taken into account. Equation set (3)-(4) describes the internal delay dynamics representing the transmission channel. We assume that all solutions of model (3)-(4), have the following properties for all $t \geq 0$

$$\tau_{max} \geq \tau(t) \geq 0 \tag{5}$$

$$1 - \nu \geq \dot{\tau}(t) \tag{6}$$

where $\tau_{max} \geq 0$ is an upper bound of the time-variation of $\tau(t)$ and $1 > \nu > 0$ is an arbitrarily small constant determined by the delay dynamics. These two conditions on the delay are a direct consequence of the fact that we consider reliable transmission networks. To understand this, first note that the time-delay considered is the latency experienced by the transmitted signal and may be different from the delay measured on the network. From this point of view, $\dot{\tau} = 1$ means that

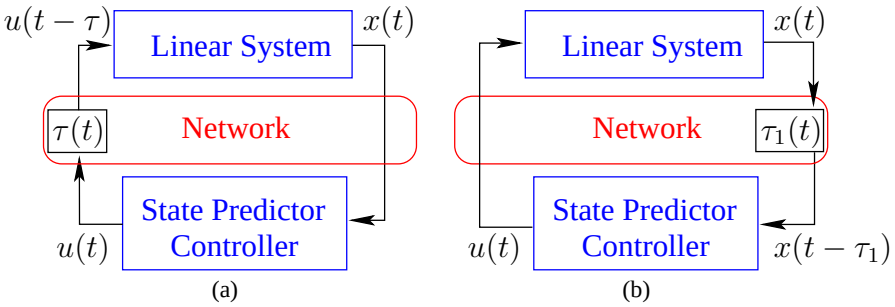

Fig. 1. Time-delay on the actuator (a) and measurement (b) signals.

the signal considered is blocked in the communication link indefinitely since the latency grows as fast as the current time t, which contradicts the lossless data property.

The control setup is presented on Figure 1(a). This specific location of the delay, between the control setup and the system, is motivated by the fact that most of the destabilizing effect and technical difficulties to solve the problem come from this delay location. Indeed, if we consider an induced delay $\tau_1(t)$ located between the system and the control setup, as in Figure 1(b), then we can set the control law

$$\begin{aligned} u(t) &= -K\left[e^{A\tau_1}x(t-\tau_1(t)) + e^{At}\int_{t-\tau_1}^{t} e^{-A\theta}Bu(\theta)d\theta\right] \\ &= -Kx(t) \end{aligned}$$

where $\tau_1(t)$ is estimated or directly measured. Keeping track of the control input during the time $[t-\tau_1,\, t]$, the resulting closed-loop system has the dynamics

$$\dot{x}(t) = (A - BK)x(t)$$

and the remote stabilisation problem reduces to a traditional pole placement problem. An error in the predictor computation only introduces a consideration on the robustness with respect to some disturbances on the input signal. A setup with two delays is studied in an observer-based control scheme in [15] but will not be presented here.

A. Control design

Due to the inherent time-variation of the delay considered here, it is not possible to design a controller that imposes an invariant closed-loop spectrum. Instead, under certain weak conditions, we are able to set the eigenvalues of a *time-varying shifted* system, or equivalently we transform

the time-invariant delayed unstable open-loop system, into a stable time-varying linear system. The control design proposed here is similar to the one used in [13] in an adaptive control context. The system transformation is done by replacing the current time t by the shifted time coordinate $t + \delta(t)$ in (1), which results in

$$x'(t + \delta(t)) = Ax(t + \delta(t)) + Bu(t + \delta(t) - \tau(t + \delta(t))), \tag{7}$$

where $x'(\cdot)$ is the derivative of $x(\cdot)$ with respect to its argument and $\delta(t)$ is a bounded and positive time-depending function. Defining $\delta(t)$ as

$$\delta(t) = \tau(t + \delta(t)) \tag{8}$$

and considering first the problem of state feedback stabilization, the eigenvalues of the time-varying shifted system (7) are set with the control input

$$\begin{aligned} x(t+\delta) &= e^{A\delta}\left[\, x(t) + e^{At}\int_t^{t+\delta} e^{-A\theta}Bu(\theta - \tau(\theta))d\theta \,\right] \\ u(t) &= -Kx(t + \delta(t)). \end{aligned} \tag{9}$$

The resulting closed-loop equation is then

$$x'(t + \delta(t)) = (A - BK)x(t + \delta(t)) = A_{cl}x(t + \delta(t)) \tag{10}$$

where A_{cl} is the closed loop state matrix, that can be made Hurwitz by the controllability hypothesis on the (A, B) pair.

B. Stability analysis

The stability analysis of the time-varying system (10) and the resulting constraints on the dynamics of $\delta(t)$ is detailed in the following Lemma, which proof is given in [16].

Lemma 2.1: Assume that $\exists\delta(t)$ satisfying (8), such that the control law (9) applied to system (7) leads to the closed-loop form (10). Then if the following conditions hold:

$i)$ All the real parts of the eigenvalues of A_{cl} are in the open left hand side of the complex plane,

$ii)$ $\infty > \delta_M \geq \delta(t) \geq 0$,

$iii)$ $\infty > \rho > \dot{\delta}(t) > -1$ with ρ an arbitrarily large positive constant.

then, $\lim_{t\to\infty} ||x(t + \delta(t))|| = 0 \quad \forall\ t + \delta(t) \geq \delta_0$ with $\delta_0 = \delta(0)$ and for all bounded values of $x(\delta_0)$. Furthermore, the state $x(t + \delta(t))$ is exponentially stable.

The stability result of the pervious lemma is applied to the system considered thanks to the following proposition.

Note that the hypotheses (ii) and (iii) of the previous Lemma are always satisfied for the delay models defined by (3)-(4) and satisfying the conditions (5)-(6). Indeed, hypothesis (ii) is clearly satisfied from the definition of $\delta(t)$ and (5) while (iii) is obtained from (6). More precisely, taking the time-derivative of (8) and from the fact that $\dot{\tau}(t) \neq 1 \, \forall t$, we can write

$$\dot{\delta}(t) = \frac{d\tau(\zeta)/d\zeta}{1 - d\tau(\zeta)/d\zeta}$$

Hypothesis (iii) is then satisfied if

$$-1 < \frac{d\tau(\zeta)/d\zeta}{1 - d\tau(\zeta)/d\zeta} < \rho.$$

The left part of this inequality clearly always holds since

$$d\tau(\zeta)/d\zeta - 1 < d\tau(\zeta)/d\zeta \Leftrightarrow -1 < \frac{d\tau(\zeta)/d\zeta}{1 - d\tau(\zeta)/d\zeta}$$

and the right part is also satisfied since (6) implies

$$\frac{1}{\nu} \geq \frac{1}{1 - d\tau(\zeta)/d\zeta} \quad and \quad \frac{d\tau(\zeta)/d\zeta}{1 - d\tau(\zeta)/d\zeta} < \frac{1-\nu}{\nu}$$

Choosing $\rho = \dfrac{1-\nu}{\nu}$ finally ensures that ρ is finite, from the properties of ν.

We can then conclude on the stability of the closed loop system with the following corollary, which summarizes the previous discussion.

Corollary 2.1: The control law (9) applied to the system (1)-(4), where the delay satisfies (5)-(6), has a bounded solution and the system trajectories exponentially decrease to zero.

III. Computation of $\delta(t)$ and use of the time-delay model

The computation of the control law implies to continuously solve (8) for $\delta(t)$ and to keep a history of the past control inputs during a time-interval $[t-\tau(t), t]$. The existence of a solution to this equation implies that $\tau(\cdot)$ satisfies (5)-(6). It is solved analytically (for specific delay models) or numerically (time consuming) in [16]. A more convenient and efficient way to compute $\delta(t)$ is to use directly the delay dynamics. This is achieved by first defining the function

$$s(t) = \hat{\delta}(t) - \tau(t + \hat{\delta}(t)) \tag{11}$$

where $\hat{\delta}(t)$ is the computed estimate of $\delta(t)$. The idea is to find a variation law for $\hat{\delta}$ such that the manifold $s(t) = 0$ is rendered attractive and invariant, consequently ensuring that $\hat{\delta}$ converges asymptotically to δ. In order to prevent for the numerical instabilities, the dynamics of $s(t)$ is defined as

$$\dot{s}(t) + \lambda s(t) = 0 \tag{12}$$

where λ is a positive constant. Taking the derivative of (11) with respect to time and substituting $\dot{s}$ in (12), we obtain

$$\dot{\hat{\delta}} - \tau'(\hat{\zeta})(1 + \dot{\hat{\delta}}) + \lambda(\hat{\delta} - \tau(\hat{\zeta})) = 0 \tag{13}$$

where $\hat{\zeta}(t) = t + \hat{\delta}(t)$ and $\tau'(\cdot)$ is the derivative of $\tau(\cdot)$ with respect to its argument. From the previous equation, (12) is satisfied if $\tau'(\cdot) \neq 1$ and the variation law $\dot{\hat{\delta}}(t)$ is set with

$$\dot{\hat{\delta}}(t) = -\frac{\lambda\hat{\delta}}{1 - \tau'(\hat{\zeta})} + \frac{\tau'(\hat{\zeta}) + \lambda\tau(\hat{\zeta})}{1 - \tau'(\hat{\zeta})} \tag{14}$$

This *explicit* expression for the dynamics of $\hat{\delta}(t)$ then ensures that the approximate $\hat{\delta}(t)$ converges to the desired value $\delta(t)$, and that the function $s(t)$ exponentially converges to zero. The convergence speed can be set arbitrarily fast by choosing λ sufficiently small, and we directly use the delay dynamics ($\tau(\hat{\zeta})$ and $\tau'(\hat{\zeta})$ are given by (3)-(4)). To illustrate the computation of $\dot{\hat{\delta}}$, consider the case where $\tau(t) = z(t)$: (14) is then set using $\tau(\hat{\zeta}) = z(\hat{\zeta})$ and $\tau'(\hat{\zeta}) = z'(\hat{\zeta}) = f(z(\hat{\zeta}), u_d(\hat{\zeta}))$.

The influence of the dynamics of $s(t)$ introduced in (11) on the closed-loop system is studied with the following lemma, which is a synthesis of the results presented in [17].

Lemma 3.1: Consider the closed-loop system described by

$$x'(t + \delta) = A_{cl}x(t + \delta) + BK[x(t + \delta) - x(t + \hat{\delta})], \quad x(0) = x_0 \tag{15}$$

with $\hat{\delta}$ obtained from (14). If

- $\tau(t)$ satisfies the properties (5)-(6),
- A_{cl} is a Hurwitz matrix,
- $0 < \lambda < \dfrac{1 - \nu}{2|\hat{\delta}(0) - \tau(\hat{\delta}(0))|}$,

then the trajectories of $x(t + \delta)$ are asymptotically stable.

Proof: (Outline) The previous lemma is established from the fact that the stability of the transformed system

$$\Sigma_t : x'(\zeta) = (A-BK)x(\zeta) + BKA\int_{-\epsilon_\delta}^{0} x(\zeta+\theta)d\theta - (BK)^2\int_{-2\epsilon_\delta}^{-\epsilon_\delta} x(\zeta+\theta)d\theta$$

where $\epsilon_\delta(t) = \delta(t) - \hat{\delta}(t)$ and $\zeta(t) = t + \delta(t)$, implies the stability of (15). This transformed system is obtained using the Leibniz-Newton formula

$$x'(\zeta) = (A-BK)x(\zeta) + BK\int_{-\epsilon_\delta}^{0} x'(\zeta+\theta)d\theta$$

The behaviour of Σ_t is then investigated thanks to the Lyapunov-Krasovskii functional [18]

$$\begin{aligned} V(x(\zeta)) &= x(\zeta)^T P x(\zeta) \\ &\quad + \frac{1}{1-\bar{\epsilon}_\delta}\int_{-\epsilon_\delta}^{0}\left[\int_{\zeta+\theta}^{\zeta} x(\mu)^T S x(\mu) d\mu\right] d\theta \\ &\quad + \frac{\alpha}{1-2\bar{\epsilon}_\delta}\int_{-2\epsilon_\delta}^{-\epsilon_\delta}\left[\int_{\zeta+\theta}^{\zeta} x(\mu)^T S x(\mu) d\mu\right] d\theta \end{aligned}$$

with P, S some positive definite matrices, $\bar{\epsilon}_\delta \doteq \sup_t \dot{\epsilon}_\delta(t)$ and $0 < \alpha < \dfrac{1-2\bar{\epsilon}_\delta}{\bar{\epsilon}_\delta}$. Taking the time-derivative of this functional along the system trajectories of (14)-(15) ensures, if the hypotheses of the lemma are satisfied, the stability of the system considered. ■

The previous lemma is now applied to the proposed control scheme in the following theorem.

Theorem 3.1: Consider the system (1) with (A, B) a controllable pair. Assume that the delay dynamics (3)-(4) is such that (5)-(6) hold for all t, then the feedback control law (9) based on the estimated predictor's horizon $\hat{\delta}(t)$ which dynamics are described by (14) with

$$\begin{aligned} \tau'(\hat{\zeta}) &= \frac{dh}{d\hat{\zeta}}(z(\hat{\zeta}), u_d(\hat{\zeta})) \\ \frac{dz}{d\hat{\zeta}}(\hat{\zeta}) &= f(z(\hat{\zeta}), u_d(\hat{\zeta})), \quad z(0) = z_0 \end{aligned}$$

and λ satisfying the conditions stated in lemma 3.1, ensures that the trajectories of $x(t)$ decrease asymptotically to zero.

Proof: First note that the time-shifted system

$$x'(t+\delta) = Ax(t+\delta) + Bu(t)$$

with $u(t) = -Kx(t+\hat{\delta})$ writes as (15) by adding and subtracting $BKx(t+\delta)$ to the previous dynamic equation. Thanks to Lemma 3.1 and conditions (5)-(6), the proposed control law then

allows for a pole placement on the time-shifted system described by the state $x(t+\delta)$ and A_{cl} in (10) is made Hurwitz with a proper choice of K. Therefore, the time-shifted state converges asymptotically with the proposed control law. Finally, the stability of $x(t)$ is deduced from the fact that the system (1)-(2) is linear and its states cannot diverge in finite time. ■

IV. Robustness Analysis

The aim of this section is to study the robustness of the system (1)-(4) stabilized by the state feedback (9) with respect to some delay uncertainties. These uncertainties are due to the difference that may exist between the delay model (3)-(4) and the true delay induced by the communication channel.

A. Problem formulation

In order to study the robustness of the control setup with respect to delay uncertainties, we investigate their influence on the dynamics of the closed-loop system. The dynamics of the estimated delay $\hat{\tau}$ is obtained from

$$\dot{\hat{z}}(t) = f_e(\hat{z}(t), u_{de}(t)), \quad \hat{z}(0) = \hat{z}_0 \tag{16}$$

$$\hat{\tau}(t) = h_e(\hat{z}(t), u_{de}(t)) \tag{17}$$

where $f_e(\cdot)$ and $h_e(\cdot)$ are some continuous functions, $\hat{z}$ is the internal state of the model and u_{de} is an exogenous input to this model, possibly including some network measurements. An example of such dynamics is provided by the Kalman filter updates in [1], determined by the combination of Kalman filtering and CUMSUM change-detection that sets the delay estimation strategy. Another possibility is to use some network models, such as the one provided in [2], which relate the dynamics of the emitters window size and of the routers queue length to the network protocol (the TCP case is investigated in the referred work).

The estimated delay satisfies the conditions $\hat{\tau}_{max} \geq \hat{\tau}(t) \geq 0$ and $\sup \dot{\hat{\tau}}(t) = \hat{\nu} < 1$. Considering that such a model exists and is compared to the actual network induced delay with the error parameters ϵ_M and $\bar{\epsilon}_M$ defined as $\{\epsilon_M, \bar{\epsilon}_M\} \doteq \sup_t\{\epsilon(t), \dot{\epsilon}(t)\}$, where $\epsilon(t) \doteq \tau(t) - \hat{\tau}(t)$, the aim of this section is to determine if, for a chosen feedback gain K, the closed-loop system remains stable when $\{\epsilon_M, \bar{\epsilon}_M\} \neq \{0, 0\}$. The predicted state feedback is computed from the

delay model and the resulting closed-loop system writes as

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t-\tau(t)) \\ u(t) &= -Ke^{A\hat{\delta}(t)}\Big[x(t) \\ &\quad + e^{At}\int_t^{t+\hat{\delta}(t)} e^{-A\theta}Bu(\theta-\hat{\tau}(\theta))d\theta\Big] \end{aligned}$$

where $\hat{\delta}(t) = \hat{\tau}(t+\hat{\delta}(t))$ is the prediction horizon computed from (16)-(17) ($\hat{\tau} \neq \tau \Rightarrow \hat{\delta} \neq \delta$).

The controller output $u(t)$ can be expressed, equivalently, as

$$u(t) = -Kx(t+\hat{\delta}(t)) + \Delta_u(t) \tag{18}$$

where

$$\begin{aligned} \Delta_u(t) \doteq -Ke^{A(t+\hat{\delta}(t))}\int_t^{t+\hat{\delta}(t)} & e^{-A\theta}B[u(\theta-\hat{\tau}(\theta)) \\ & -u(\theta-\tau(\theta))]d\theta \end{aligned}$$

The resulting closed-loop system is then defined by the functional differential equation

$$x'(t+\delta(t)) = Ax(t+\delta(t)) - BKx(t+\hat{\delta}(t)) + B\Delta_u(t) \tag{19}$$

While a direct Lyapunov-Krasovskii analysis (similar to the one used in the previous section) of this problem is very conservative [19], some more interesting results can be obtained by neglecting the effect of Δ_u in the previous dynamics.

Indeed, (18) can be expressed equivalently as $u(t) - \Delta_u(t) = -Kx(t+\hat{\delta}(t))$, which can be considered as a functional equation with $x(\cdot)$ as an input. If the delay is small, $|\Delta_u(t)|/|u(t)|$ is small, and the dynamics of the functional equation is stable and fast converging. The effect of Δ_u can then be easily ignored. The same conclusion holds when the estimation error is small, since

- Δ_u is proportional to the difference $u(\theta-\hat{\tau}(\theta)) - u(\theta-\tau(\theta))$ and is bounded since there is no singularity in the system and the integration is carried on a finite-time horizon,
- $|u(t)|$ is proportional to $|x(t+\hat{\delta}(t))|$,

which implies that $|\Delta_u|$ is proportional to the distance $|x(\theta) - x(\theta-\tau(\theta)+\hat{\delta}(\theta-\tau(\theta)))|$, where $\theta \in [t,\, t+\hat{\delta}(t)]$. If we suppose that this distance is sufficiently small to ensure the robustness of

the origin stability, or if the delay is sufficiently small, then the stability of (19) can be deduced from the stability of

$$x'(t+\delta(t)) = Ax(t+\delta(t)) - BKx(t+\hat{\delta}(t)) \tag{20}$$

Note that this is a qualitative result based on the vanishing perturbation theory [20]. From a physical point of view, it is equivalent to consider that the main disturbing effect of the delay estimation error acts on the fundamental dynamics of (19).

B. Proposed solution

We consider here the *small gain* approach for time-delay systems proposed in [21], applied to the stability analysis of

$$\begin{aligned} x'(\zeta) &= Ax(\zeta) - BKx(\zeta - \epsilon_\delta(t)) \\ &= (A-BK)x(\zeta) + BK\left[x(\zeta) - x(\zeta-\epsilon_\delta(t))\right] \end{aligned} \tag{21}$$

where $\epsilon_\delta(t) \doteq \delta(t) - \hat{\delta}(t)$ and $\zeta = t + \delta(t)$. The previous equation is first written as a function of the average, constant value of the error ϵ_a (i.e. $\epsilon_a = [\max \epsilon(t) - \min \epsilon(t)]/2$) thanks to the relationship

$$x(\zeta - \epsilon_\delta(t)) = x(\zeta - \epsilon_a) - \int_{\zeta-\epsilon_\delta(t)}^{\zeta-\epsilon_a} x'(\theta)d\theta$$

Note that the average and maximum values of $\epsilon_\delta(t)$ are the same as those of $\epsilon(t)$. The dynamics of the resulting system is then

$$\begin{aligned} x'(\zeta) &= Ax(\zeta) - BKx(\zeta-\epsilon_a) \\ &\quad + BK\int_{\zeta-\epsilon_\delta(t)}^{\zeta-\epsilon_a} \left[Ax(\theta) - BKx(\theta-\epsilon_\delta(\theta))\right] d\theta \end{aligned}$$

with $x(\theta) = \phi(\theta)$, $\theta \in [t_0 - \epsilon_M,\, t_0]$, $(t_0,\, \phi) \in \mathbb{R}^+ \times \mathcal{C}^\nu_{n,\,\epsilon_M}$. The integral term in the previous equality is considered as an uncertainty and the closed-loop system writes as

$$y_{sg} = G(u_{sg}), \quad u_{sg} = \Delta(y_{sg})$$

where $y_{sg} = [y_1\, y_2]^T$, $u_{sg} = [u_1\, u_2]^T$, and G and Δ are defined as

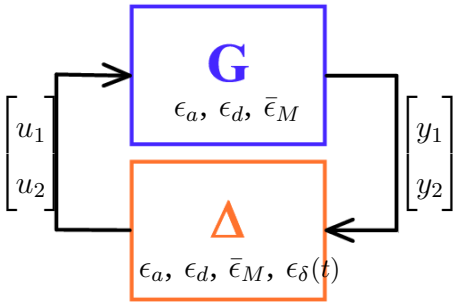

Fig. 2. Small gain formulation.

$$G \ : \ \begin{cases} x'(\zeta) &= Ax(\zeta) - BKx(\zeta - \epsilon_a) + \epsilon_d BKu_2(\zeta) \\ y_1(t) &= \dfrac{1}{\sqrt{1-\bar{\epsilon}_M}} x(t) \\ y_2(t) &= Ax(t) - BKu_1(t) \end{cases} \tag{22}$$

$$\Delta \ : \ \begin{cases} u_1(t) &= \Delta_1 y_1(t) = \sqrt{1-\bar{\epsilon}_M}\, y_1(t - \epsilon(t)) \\ u_2(t) &= \Delta_2 y_2(t) = \dfrac{1}{\epsilon_d} \displaystyle\int_{t-\epsilon_\delta(t)}^{t-\epsilon_a} y_2(\theta) d\theta \end{cases} \tag{23}$$

where $\epsilon_d \doteq max\{\epsilon_M - \epsilon_a; \epsilon_a - \epsilon_m\}$ and $\epsilon_m \doteq \inf_t(\epsilon(t))$. The interconnection between G and Δ is presented in figure IV-B. Note that this specific formulation aims at separating the expressions with constant (in G) and time dependent (in Δ) values of $\epsilon_\delta(t)$. The stability of the interconnected system is obtained by showing that the gain of both subsystems G and Δ are less then one. The main advantage of this formulation is that the stability of the closed loop system is inferred from the stability of G, which is a system with a *constant* time-delay. More precisely, we first consider the following result [21]

$$\gamma_0(\Delta_{kX_k}) \leq 1, \text{ for all non-singular matrices } X_k \in \mathbb{R}^{n\times n},\ k = 1, 2,$$

where $\gamma_0(\cdot)$, the gain of the system considered, and Δ_{kX_k} are defined respectively as

$$\gamma_0(H) = \inf\{\gamma |\ ||Hf||_2 \leq \gamma ||f||_2, \text{ and}$$
$$\Delta_{kX_k} f = X_k \Delta_k (X_k^{-1} f), \text{ for all } f \in L_{2+}\}.$$

L_{2+} denotes the set of functions $f : \overline{\mathbb{R}}_+ \to \mathbb{R}^n$, $\overline{\mathbb{R}}$ being the closed set of square integrable reals, i.e., $\int_0^\infty ||f(t)||^2$ is well defined and finite. We can then conclude on the stability of (21) by applying the following proposition.

Proposition 4.1: [21] The input-output stability of system (21) is ensured if the scaled small gain problem

$$\gamma_0(G_X) < 1 \text{ for } X = diag(X_1\ X_2),$$
$$X_1, X_2 \in \mathbb{R}^{n\times n} \text{ non singular,}$$

has a solution, where G is described by (22).

Consequently, we have to find the sufficient conditions that the estimation error has to fulfil in order to ensure that the gain of G is bounded by one. This is done with the following proposition

Proposition 4.2: Consider the system G described by

$$\begin{aligned} \dot{x}(t) &= A_0x(t) + A_1x(t-r) + Eu(t) \\ y(t) &= G_0x(t) + G_1x(t-r) + Du(t), \end{aligned}$$

a given set of non-singular matrices $\mathcal{X}$, and $\mathcal{Z} \doteq \{X^TX | X \in \mathcal{X}\}$. There exists a $X \in \mathcal{X}$ such that $\gamma_0(G_X) < 1$ if there exists a $Z \in \mathcal{Z}$ and real matrices $P = P^T$, Q_p, S_p, $R_{pq} = R_{qp}^T$, $p = 0, 1, \ldots, N$, $q = 0, 1, \ldots, N$ such that the following LMIs are satisfied:

$$\begin{pmatrix} \tilde{\Delta} & -\tilde{D}^s & -\tilde{D}^a \\ -\tilde{D}^{sT} & R_d + S_d & 0 \\ -\tilde{D}^{aT} & 0 & 3S_d \end{pmatrix} > 0, \quad \begin{pmatrix} P & \tilde{Q} \\ \tilde{Q}^T & \tilde{R} + \tilde{S} \end{pmatrix} > 0$$

where

$$\begin{aligned} \tilde{Q} &\doteq (Q_0\ \ Q_1\ \ \ldots\ \ Q_N), \quad \tilde{S} \doteq \frac{1}{h}\, diag(S_0\ \ S_1\ \ \ldots\ \ S_N) \\ \tilde{R} &\doteq \begin{pmatrix} R_{00} & R_{01} & \ldots \\ R_{01}^T & \ddots & \\ \vdots & & R_{NN} \end{pmatrix}, \quad h = \frac{r}{N} \\ \tilde{\Delta} &\doteq \begin{pmatrix} \Delta_{00} & Q_N - PA_1 - G_0^TZG_1 & -PE - G_0^TZD \\ (*) & S_N - G_1^TZG_1 & -G_1^TZD \\ (*) & (*) & Z - D^TZD \end{pmatrix} \\ \Delta_{00} &\doteq -PA_0 - A_0^TP - Q_0 - Q_0^T - S_0 - G_0^TZG_0 \\ \tilde{D}^s &\doteq \begin{pmatrix} D_1^s & \ldots & D_N^s \end{pmatrix} \end{aligned}$$

$$
\begin{aligned}
D_p^s &\doteq \begin{pmatrix} \frac{h}{2}A_0^T(Q_{p-1}+Q_p)+\frac{h}{2}(R_{0,p-1}+R_{0,p}) \\ -(Q_{p-1}-Q_p) \\ \frac{h}{2}A_1^T(Q_{p-1}+Q_p)-\frac{h}{2}(R_{N,p-1}+R_{N,p}) \\ \frac{h}{2}E^T(Q_{p-1}+Q_p) \end{pmatrix} \\
\tilde{D}^a &\doteq \begin{pmatrix} D_1^a & \dots & D_N^a \end{pmatrix} \\
D_p^a &\doteq \frac{h}{2}\begin{pmatrix} -A_0^T(Q_{p-1}-Q_p)-(R_{0,p-1}-R_{0,p}) \\ -A_1^T(Q_{p-1}-Q_p)+(R_{N,p-1}-R_{N,p}) \\ E^T(Q_p-Q_{p-1}) \end{pmatrix} \\
R_d &\doteq \begin{pmatrix} R_{d11} & R_{d12} & \dots \\ R_{d12}^T & \ddots & \\ \vdots & & R_{dNN} \end{pmatrix} \\
R_{dpq} &\doteq h(R_{p-1,q-1}-R_{pq}) \\
S_d &\doteq diag\,(S_{d1} \;\; S_{d2} \;\; \dots \;\; S_{dN}), \quad S_{dp} \doteq S_{p-1}-S_p
\end{aligned}
$$

This result is applied to the system considered by using $h = \frac{\epsilon_a}{N}$ and

$$
\begin{cases}
A_0 = A, \quad A_1 = -BK, \quad E = [0_{n\times n} \;\; \epsilon_d BK] \\
G_0 = \begin{bmatrix} \frac{1}{\sqrt{1-\bar{\epsilon}_M}}\, I_n \\ A \end{bmatrix}, \quad G_1 = 0_{2n\times n} \\
D = \begin{bmatrix} 0_{n\times n} & 0_{n\times n} \\ -BK & 0_{n\times n} \end{bmatrix}
\end{cases} \tag{24}
$$

from which we can compute the maximum average estimation error on the delay, for a given maximum variation of this error.

Example 4.1: Consider the "T" shape inverted pendulum described by the dynamics

$$
\dot{x} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -18.78 & 0 & 14.82 & 0 \\ 0 & 0 & 0 & 1 \\ 56.92 & 0 & -15.18 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 7.52 \\ 0 \\ -8.82 \end{bmatrix} u(t-\tau)
$$

The controller gain K is chosen such that the poles of the matrix A_{cl} are $[-8+0.5i; -8-0.5i; -16; -32]$. The results obtained in proposition 4.2 are applied with the relationships (24)

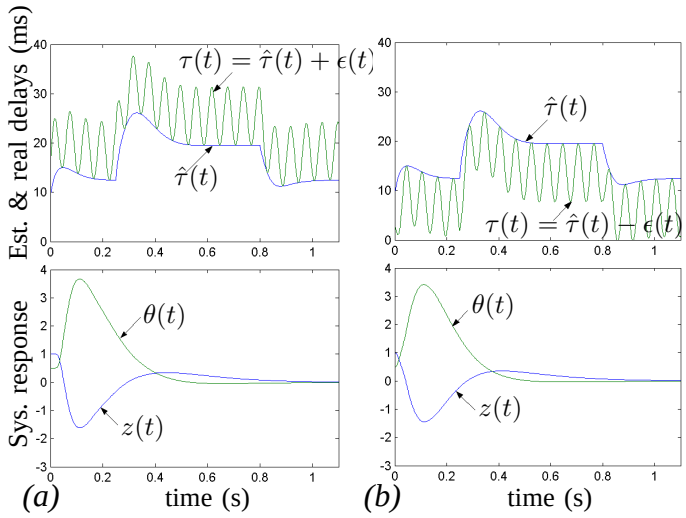

Fig. 3. Influence of the delay estimation error.

and $\epsilon_d = 2\epsilon_a$. The estimated delay average $\hat{\tau}(t)$ is based on the model proposed in [2] and depicted in figure 3(a).

This example aims at illustrating the fact that the closed-loop system remains stable if the error fits within the bounds estimated in this section. We suppose that the error and estimated delay maximum variations are the same: $\bar{\epsilon}_M = \hat{\nu} = 0.6167$, which gives $\epsilon_a = 5.9ms$. The error trial function is $\epsilon(t) = \epsilon_a + \epsilon_a \sin\left(\frac{\bar{\epsilon}_M}{\epsilon_a} t\right)$ and we study the system response when the actual delay is $\tau(t) = \hat{\tau}(t) + \epsilon(t)$ or $\tau(t) = \hat{\tau}(t) - \epsilon(t)$. The system response to a non zero initial condition is presented in figure 3(a) in the first case and in figure 3(b) in the second case. The oscillating delay $\tau(t)$ applies to the data travelling from the control setup to the system, while the estimation $\hat{\tau}(t)$ is used to compute the predictor horizon. The time evolutions of the pendulum angle $\theta(t)$ and position $z(t)$ illustrate the sensitivity of the system to the estimation error.

This simulation result illustrates the capability of the proposed control law to stabilize the system considered when the error satisfies the conditions established in this section. Note that the closed-loop system fails to stabilize if ϵ_a is increased by $2\,ms$.

V. Conclusions

In this paper we have investigated the problem of remote stabilization via communication networks, which is formulated as the problem of stabilizing an open-loop unstable system with a time-varying delay with known dynamics. The proposed controller results in an exponentially

converging closed-loop system, under weak assumptions. The controller is based on a $\delta(t)$-step ahead predictor, where $\delta(t)$ is the solution of the implicit equation $\delta - \tau(t+\delta) = 0$, which is shown to be solved if the time delay is bounded. A dynamic solution of this equation is detailed, allowing for the explicit use of the average network dynamics in the control law. The robustness of the control law with respect to time-delay uncertainties is also studied and a LMI formulation allows to compute the maximum admissible bounds on the delay estimation error. We have presented a simulation showing the capability of this controller to robustly stabilize a system when the average delay is estimated and the actual delay satisfies some computed error bounds.

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