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An LMI formulation for output feedback stabilization in multiple model approach

Mohammed CHADLI, Didier MAQUIN, José RAGOT

Centre de Recherche en Automatique de Nancy, CNRS, UMR 7039
2, Avenue de la forêt de Haye, 54516 Vandœuvre les Nancy – France.
Phone: (33) 3 83 59 57 02, Fax: (33) 3 83 59 56 44

Email: {Mohamed.Chadli, Didier.Maquin, José.Ragot}@ensem.inpl-nancy.fr

Abstract

This paper studies the design of a static output feedback controller for nonlinear systems described by multiple model approach. Motivated by quadratic stabilization result developed for parallel distributed compensation (PDC) controller, an Output PDC (OPDC) controller that corresponds to a nonlinear static output feedback control law is proposed. Both stabilization and pole placement are addressed, firstly by a cone complementarity formulation of the problem and secondly by transformation to linear matrix inequality (LMI) problem. An example is given to illustrate the results.

Keywords: Multiple model approach, nonlinear systems, regulators, Lyapunov method, complementarity problem, LMI.

1 Introduction

There have been several recent studies concerning the stability and the synthesis of controllers for nonlinear systems described by Takagi-Sugeno models [14]. There has been also an increasing interest in the multiple model approach [17,13] which also uses the T-S systems to modeling. Many of these studies use a quadratic Lyapunov function and PDC technique [1] to derive sufficient conditions for the stability and stabilizability [1][5][6][13][16][18]. The stability depends on the existence of a common positive definite matrix guarantying the stability of all local subsystems. The gain of the PDC control which is a nonlinear state feedback controller, can be expressed as the solution of a linear matrix inequality (LMIs) set [18]. Recently a number of control law have been derived from the PDC controller [4][7][11]. For example, a Dynamic PDC (DPDC), which is a dynamic nonlinear control law, is used to stabilize a T-S model [7] while in [4] a Proportional PDC (PPDC) controller which allows to reduce the number of parameters in PDC technique is presented. In [21] a dynamic output feedback controller is proposed for continuous-time T-S systems and in [8] a static output feedback control for switching discrete-time systems is studied. LMIs constraints for pole assignment in LMI regions [12] to achieve desired performances are considered in [2][9] for PDC controller and also in [15] for static output feedback control law.

In this paper, the LMI approach is used to develop a static output feedback controller for nonlinear systems described by T-S models. We propose an Output PDC (OPDC) controller which is useful when only the output of the system is available. Using the quadratic Lyapunov method, sufficient conditions for the global asymptotic stability are derived in LMIs form for OPDC controller.

This paper is organized as follows. Section 2 and 3 recall previous results. Section 4 deals with the OPDC stabilization, firstly by a cone complementarity formulation and secondly by transformation to LMI problem. LMI formulation for pole placement is also considered for this last formulation. Numerical example is given in section 5 to illustrate the result.

Notation: In this paper, we denote the symmetric positive definite matrix X by $X > 0$, the transpose of X by X^T , the conjugate of $z \in \mathbb{C}$ by \bar{z} and the Kronecker product by \otimes .

2 T-S continuous models

A T-S model is based on the interpolation between several LTI local models as follows:

$$\dot{x}(t) = \sum_{i=1}^n \mu_i(z(t))(A_i x(t) + B_i u(t)) \quad (1)$$

where n is the number of sub-models, $x(t) \in \mathbb{R}^p$ is the state vector, $u(t) \in \mathbb{R}^m$ is the input vector, $A_i \in \mathbb{R}^{p \times p}$, $B_i \in \mathbb{R}^{p \times m}$ and $z(t) \in \mathbb{R}^q$ is the decision variable vector.

The choice of the variable $z(t)$ leads to different class of systems. It can depend on the measurable state variables and possibly on the input. In this case, the system (1) describes a nonlinear system. It can also be an unknown constant value, system (1) then represents a linear differential inclusion (LDI)[18]. This variable can also be a function of the measurable outputs of the system, this case will be considered in the section 4.

The normalized activation function $\mu_i(z(t))$ in relation with the i^{th} sub-model is such that:

$$\begin{cases} \sum_{i=1}^n \mu_i(z(t)) = 1 \\ \mu_i(z(t)) \geq 0 \quad \forall i \in \{1, \dots, n\} \end{cases} \quad (2)$$

The global output of T-S model is interpolated as follows:

$$y(t) = \sum_{i=1}^n \mu_i(z(t)) C_i x(t) \quad (3)$$

where $y(t) \in \mathbb{R}^l$ is the output vector and $C_i \in \mathbb{R}^{l \times p}$. More detail about this type of representation can be found in [1][14]. It should be point out that at a specific time, only a number s of local models are activated, depending on the structure of the activation functions $\mu_i(\cdot)$.

3 Previous results

The PDC controller [1][6], which is nonlinear in general, is described by:

$$u(t) = \sum_{i=1}^n \mu_i(z(t)) K_i x(t) \quad (4)$$

Substituting (4) in (1), we obtain the closed-loop continuous T-S model:

$$\dot{x}(t) = \sum_{i=1}^n \sum_{j=1}^n \mu_i(z(t)) \mu_j(z(t)) \tilde{A}_{ij} x(t) \quad (5)$$

where

$$\tilde{A}_{ij} = A_i + B_i K_j \quad (6)$$

For PDC controller design, it is supposed that the system (1) is locally controllable, i.e. the pairs $(A_i, B_i), \forall i \in \{1, \dots, n\}$ are controllable.

The stability conditions for system (5) are formulated by theorem 1 and for less of conservatism by theorem 2. In order to simplify the notation of the forthcoming equations, lets us denote:

$$L(\tilde{A}_{ij}, P) = \left(\frac{\tilde{A}_{ij} + \tilde{A}_{ji}}{2} \right)^T P + P \left(\frac{\tilde{A}_{ij} + \tilde{A}_{ji}}{2} \right) \quad (7)$$

Theorem 1. The closed-loop continuous T-S model described by (5) is globally asymptotically stable if there exist a symmetric matrix $P > 0$ such that

$$L(\tilde{A}_{ii}, P) < 0 \quad \forall i \in \{1, \dots, n\} \quad (8a)$$

$$L(\tilde{A}_{ij}, P) \leq 0 \quad \forall i < j \in \{1, \dots, n\} \quad (8b)$$

and $\mu_i(z(t)) \mu_j(z(t)) \neq 0$.

Proof: see [1] ■

For less of conservatism, the following constraints take into account the number of local models s simultaneously activated [1]:

$$L(\tilde{A}_{ii}, P) + (s-1)R < 0 \quad \forall i \in \{1, \dots, n\} \quad (9a)$$

$$L(\tilde{A}_{ij}, P) - R \leq 0 \quad \forall i < j \in \{1, \dots, n\} \quad (9b)$$

Where R is a symmetric semi-definite positive matrix. In the PDC technique, which is a state feedback law, the conditions (8) and (9) are easy to convert into an LMI problem [1].

4 Static output stabilization using OPDC

In the sequel, we assume that $z(t)$ is a function of the measurable outputs of the system, $z(t) = h(y(t))$ and the following assumption holds:

Assumption 1 : The matrix $C_i = C, \forall i \in \{1, \dots, n\}$ is full row rank

The OPDC is a nonlinear static output feedback which shares the same activation functions as the T-S model (1):

$$u(t) = \sum_{i=1}^n \mu_i(z(t)) F_i y(t) \quad (10)$$

where $F_i \in \mathbb{R}^{m \times l}$ is the local output feedback controller to determine.

Taking into account the expression (10), the closed loop model (1) becomes:

$$\dot{x}(t) = \sum_{i=1}^n \sum_{j=1}^n \mu_i(z(t)) \mu_j(z(t)) \bar{A}_{ij} x(t) \quad (11)$$

where

$$\bar{A}_{ij} = A_i + B_i F_j C, \quad \forall i, j \in \{1, \dots, n\} \quad (12)$$

The synthesis of OPDC controller for T-S model can be done using the results (8) and (9) by simply replacing \tilde{A}_{ij} by \bar{A}_{ij} . We obtain respectively:

$$L(\bar{A}_{ii}, P) < 0, \quad \forall i \in \{1, \dots, n\} \quad (13a)$$

$$L(\bar{A}_{ij}, P) \leq 0 \quad \forall i < j \in \{1, \dots, n\} \quad (13b)$$

and

$$L(\bar{A}_{ii}, P) + (s-1)R < 0, \quad \forall i \in \{1, \dots, n\} \quad (14a)$$

$$L(\bar{A}_{ij}, P) - R \leq 0 \quad \forall i < j \in \{1, \dots, n\} \quad (14b)$$

However the obtained equations are no longer linear with regard to the unknown matrices P and $F_i, \forall i \in \{1, \dots, n\}$. So, the solution is not guaranteed to belong to a convex domain and the classical tools for solving sets of matrix inequalities cannot be used. It constitutes the major difficulty of output feedback design.

In the following, sufficient conditions are given to ensure asymptotic stability of (11), firstly by transformation to cone complementarity problem in section 4.1 and secondly by variables changes in section 4.2.

4.1 Cone complementarity formulation

Constraints (13) give sufficient conditions for global asymptotic stability of (11). But they are BMIs in P_i and $F_i, \forall i \in \{1, \dots, n\}$ and hard to linearize. However, it is possible to solve the problem locally by using iterative methods. For example the path-following method to locally solve a BMI is proposed in [10] and used in [13]. For the global resolution of BMIs one will consult the reference [19].

Another way to deal with the static output feedback control for T-S models is to transform the synthesis of the output feedback into a cone complementarity problem. Consider the following LMI constraints of the form

$$F(V, W, Z) \geq 0, \quad \begin{pmatrix} V & I \\ I & W \end{pmatrix} \geq 0 \quad (15)$$

where V, W are symmetric matrix variables of same size, Z is matrix variables and $F(\cdot)$ is a symmetric matrix-valued, affine function. The corresponding cone complementarity problem is

$$\text{Minimize } \text{Tr}(VW) \text{ subject to (15)}$$

This section proposes an algorithm based on a cone complementarity problem [3]. The method uses the following lemma.

Lemma 1 (Elimination lemma, [18])

Let $G \in \mathbb{R}^{p,p}, U \in \mathbb{R}^{p,m}$ and $V \in \mathbb{R}^{p,l}$. Let \tilde{U} and \tilde{V} be respectively the orthogonal complements of U and V i.e. for U , $U^T \tilde{U} = 0$ and the matrix $(U \tilde{U})$ is of maximum rank. Then the following three propositions are equivalent:

- i) $G + UXV^T + VX^T U^T < 0$, holds for some $X \in \mathbb{R}^{m,l}$
- ii) $\tilde{U}^T G \tilde{U} < 0$ et $\tilde{V}^T G \tilde{V} < 0$
- iii) $G - \sigma VV^T < 0$ et $G - \sigma U U^T < 0$, holds for some $\sigma \in \mathbb{R}$. ■

A- Controller elimination:

In the following, the procedure elimination uses only the equivalence between the 1st and the 3rd proposition. The constraints (13a) can be rewritten as follow

$$G(P) + U(P)F_i V^T + V F_i^T U(P)^T < 0 \quad (16)$$

with

$$G(P) = A_i^T P + P A_i, \quad U(P) = P B_i, \quad V = C^T$$

Applying the elimination lemma to (16), we obtain after elimination of the variable F_i

$$A_i^T P + P A_i - \sigma_i P B_i B_i^T P < 0 \quad (17a)$$

$$A_i^T P + P A_i - \sigma_i C^T C < 0 \quad (17b)$$

To guarantee equivalence between the 3 propositions of lemma 1, we must respect the strict form of those inequalities. Indeed, by applying the same procedure of elimination as before to the strict form of (13b), we obtain after elimination of the variable F_j

$$(A_i + A_j + B_j F_j C)^T P + P(A_i + A_j + B_j F_j C) - \sigma_{ij} P B_j B_j^T P < 0 \quad (18a)$$

$$(A_i + A_j + B_j F_j C)^T P + P(A_i + A_j + B_j F_j C) - \sigma_{ij} C^T C < 0 \quad (18b)$$

In the same way we obtain after elimination of the variable F_j of (18):

$$\begin{cases} (A_i + A_j)^T P + P(A_i + A_j) - \sigma_{ij} P B_j B_j^T P - \tilde{\sigma}_{ij} P B_j B_j^T P < 0 \\ (A_i + A_j)^T P + P(A_i + A_j) - \sigma_{ij} P B_j B_j^T P - \tilde{\sigma}_{ij} C^T C < 0 \\ (A_i + A_j)^T P + P(A_i + A_j) - \sigma_{ij} C^T C - \bar{\sigma}_{ij} P B_j B_j^T P < 0 \\ (A_i + A_j)^T P + P(A_i + A_j) - \sigma_{ij} C^T C - \bar{\sigma}_{ij} C^T C < 0 \end{cases} \quad (19)$$

Applying the Schur complement [18] to (19) and multiplying (17a) in the left and the right by $Q = P^{-1}$, we obtain from (19) and (17) the following system which is equivalent to the strictly form of (13):

$$\begin{cases} Q A_i^T + A_i Q - \sigma_i B_i B_i^T < 0 \\ A_i^T P + P A_i - \sigma_i C^T C < 0 \\ \begin{pmatrix} Q(A_i + A_j)^T + (A_i + A_j)Q - \sigma_{ij} B_j B_j^T & B_j \\ B_j^T & \tilde{v}_{ij} I \end{pmatrix} < 0 \\ \begin{pmatrix} Q(A_i + A_j)^T + (A_i + A_j)Q - \sigma_{ij} B_j B_j^T & Q C^T \\ C Q & \tilde{v}_{ij} I \end{pmatrix} < 0 \\ \begin{pmatrix} (A_i + A_j)^T P + P(A_i + A_j) - \sigma_{ij} C^T C & P B_j \\ B_j^T P & \bar{v}_{ij} I \end{pmatrix} < 0 \\ \begin{pmatrix} (A_i + A_j)^T P + P(A_i + A_j) - \sigma_{ij} C^T C & C^T \\ C & \bar{v}_{ij} I \end{pmatrix} < 0 \end{cases} \quad (20a)$$

$$\forall i < j \in \{1, \dots, n\} \text{ and } \mu_i(z(t)) \mu_j(z(t)) \neq 0$$

$$PQ = I \quad (20b)$$

Where $I \in \mathbb{R}^{p,p}$ is the identity matrix. It should be noted that conditions (20a) are LMIs in $P \in \mathbb{R}^{p,p}, Q \in \mathbb{R}^{p,p}$ and scalars $\sigma_i, \sigma_{ij}, \tilde{v}_{ij} = (\tilde{\sigma}_{ij})^{-1}, \bar{v}_{ij} = (\bar{\sigma}_{ij})^{-1}$ but the equation (20b) is non convex. This equality can be enforced by imposing $\text{Tr}(PQ) = p$ with the additional constraint

$$\begin{bmatrix} P & I \\ I & Q \end{bmatrix} \geq 0 \quad (21)$$

To solve such a problem a convergent algorithm is proposed in [3]. This heuristic is based on a linear approximation of $\text{Tr}(PQ)$ by $\text{Tr}(P_0 Q + Q_0 P)$ where P_0 and Q_0 is a particular solution of the LMI constraints (20a)-(21) (see Annex). We then obtain the following cone complementarity problem:

$$\text{Minimize } \text{Tr}(P_0Q + Q_0P) \text{ subject to (20a)-(21)} \quad (22)$$

B- Controller reconstruction:

Assuming that P and Q satisfying (22) are founded, we can now compute an appropriate controller by solving the LMI problem (13) in $F_i, \forall i \in \{1, \dots, n\}$. Which allows us to propose the following result.

Theorem 2. Suppose that there exist symmetric positive definite matrices $P \in \mathbb{R}^{p \times p}$ and $Q \in \mathbb{R}^{p \times p}$ such that the objective (22) is achieved such that $\text{Tr}(P_0Q + Q_0P) = 2p$. Then there exist a nonlinear output feedback $u(t) = \sum_{i=1}^n \mu_i(z(t)) F_i y(t)$ that stabilises globally asymptotically the T-S model (11) where F_i is the solution of LMIs (13). ■

The numerical example given in section 5 illustrates the method. It is significant to note that the transformation of the constraints (14) into a cone complementarity problem is not direct and requires additional nonconvex constraints due to the presence of the matrix R .

4.2 LMI formulation for synthesis

In the following we present another method to transform the BMIs conditions (13) and (14) in P and $F_i, \forall i \in \{1, \dots, n\}$ into LMIs conditions which are used to design directly a stabilizing OPDC.

Theorem 3. Suppose that there exist matrices N_i, M and Q such that

$$Q > 0$$

$$QA_i^T + A_iQ + C^T N_i^T B_i^T + B_i N_i C < 0 \quad (23a)$$

$$Q(A_i + A_j)^T + (A_i + A_j)Q + C^T (N_j^T B_i^T + N_i^T B_j^T) + (B_i N_j + B_j N_i)C \leq 0 \quad (23b)$$

with

$$CQ = MC \quad (23c)$$

$\forall i < j \in \{1, \dots, n\}$ and $\mu_i(z(t))\mu_j(z(t)) \neq 0$. Then there exist a nonlinear output feedback $u(t) = \sum_{i=1}^n \mu_i(z(t)) F_i y(t)$ that stabilises globally asymptotically the T-S model (11) with

$$F_i = N_i C C^T (C Q C^T)^{-1} \quad \forall i \in \{1, \dots, n\} \quad (24)$$

Proof: The inequality (13a) is equivalent to

$$Q(A_i + B_i F_i C)^T + (A_i + B_i F_i C)Q < 0 \quad (25)$$

where $P^{-1} = Q > 0$. With the changes of variables

$$CQ = MC \text{ and } F_i M = N_i \quad (26)$$

The inequality (17) becomes

$$QA_i^T + A_iQ + (B_i N_i C)^T + B_i N_i C < 0 \quad (27)$$

The LMIs (23b) are obtained from (13b) using the same changes of variables (26). Since the matrix C is assumed full row rank, we deduce from (26) that there exist a non-singular matrix $M = C Q C^T (C C^T)^{-1}$ and then $F_i = N_i M^{-1}$. ■

For less of conservatism the following result tack into account the number of local model simultaneously activated (s).

Theorem 4. Suppose that there exist matrices N_i, M, S and Q such that

$$Q > 0, S \geq 0$$

$$QA_i^T + A_iQ + C^T N_i^T B_i^T + B_i N_i C + (s-1)S < 0 \quad (28a)$$

$$Q(A_i + A_j)^T + (A_i + A_j)Q + C^T (N_j^T B_i^T + N_i^T B_j^T) + (B_i N_j + B_j N_i)C - 2S \leq 0 \quad (28b)$$

with

$$CQ = MC \quad (28c)$$

$\forall i < j \in \{1, \dots, n\}$ and $\mu_i(z(t))\mu_j(z(t)) \neq 0$. Then there exist a nonlinear output feedback $u(t) = \sum_{i=1}^n \mu_i(z(t)) F_i y(t)$ that stabilises globally asymptotically the T-S model (11) where F_i is defined in (24).

Proof: It can be easily established using the same step as theorem 3.

Remarks:

1. Since C is assumed full row rank, to respect constraint (28c) it suffices to impose particular structure to matrix Q dependent on matrix C .
2. In case where $F_i = F, \forall i \in \{1, \dots, n\}$ the synthesis of linear static output feedback $u(t) = Fy(t)$ can be reduced to find matrices N and Q such that $\forall i \in \{1, \dots, n\}$:

$$\begin{pmatrix} QA_i^T + A_iQ + C^T N^T B_i^T + B_i N C & 0 \\ 0 & -Q \end{pmatrix} < 0 \quad (29)$$

$$\text{with } CQ = MC \quad (30)$$

3. Another control law derived from OPDC controller which is useful when the input matrices are linearly independent, i.e. $B_i = \alpha_i B, \alpha_i > 0, i \in \{1, \dots, n\}$, is considered in [15]. The modified OPDC control law has the form
- $$u(t) = \sum_{i=1}^n \mu_i(z(t)) \alpha_i F_i y(t) \left(\sum_{i=1}^n \mu_i(z(t)) \alpha_i \right)^{-1}$$
- and leads to less conservative stability conditions than the above results (see [15]).

4.3 LMI formulation for pole-placement

In order to achieve some desired transient performance, a pole placement should be considered. For many problems, exact pole assignment may not be necessary, it suffices to locate the pole of the closed loop system in a sub-region of the complex left half plane [2][12][20]. This section discusses a pole assignment in LMI regions.

Definition 1 [12]. A subset D of the complex plane is called an LMI region if there exist a symmetric matrix $\alpha = (\alpha_{ij}) \in \mathbb{R}^{p \times p}$ and a matrix $\beta = (\beta_{ij}) \in \mathbb{R}^{p \times p}$ such that $D = \{z \in \mathbb{C}: f_D(z) < 0\}$

where $f_D(z) = (\alpha_{ij} + \beta_{ij}z + \beta_{ji}\bar{z}), \forall i, j \in \{1, \dots, p\}$.

Theorem 5 [12]. A matrix A is D -stable if and only if there exists a symmetric positive definite matrix X such that

$$M_D(A, X) < 0$$

where $M_D(A, X) = \alpha \otimes X + \beta \otimes (AX) + \beta^T \otimes (AX)^T$ ■

For example, a disk region D_d centered at $(-q, 0)$ with radius $r > 0$ can be obtained by taking the matrices α and β as follows:

$$\alpha = \begin{pmatrix} -r & q \\ q & -r \end{pmatrix} \text{ and } \beta = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

which give the expression of this sub-region:

$$f_{D_d}(z) = \begin{pmatrix} -r & z+q \\ \bar{z}+q & -r \end{pmatrix} \quad (31)$$

As it is shown in figure 1, this region which include conic region, allows to fix a lower bound on both the exponential decay rate: $-q+r$ and the damping ratio: $\xi_{\min} = \sqrt{1-r^2}/q^2$ ($r < q$) of the closed-loop response

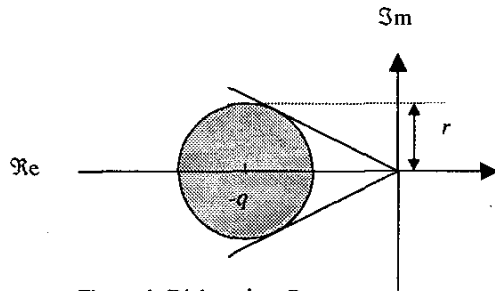


Figure 1. Disk region D_d

Since the prescribed LMI region (31) will be added as supplementary constraint to these in (23) and (28) it should be noted that it only suffices to locate the poles of the dominant term in the prescribed LMI regions, i.e. the case of $i = j$. It follows that the system (11) is D_d -stable if there exists a matrix $Q > 0$ such that

$$\begin{pmatrix} -rQ & qQ + \bar{A}_{ii}Q \\ qQ + QA_{ii}^T & -rQ \end{pmatrix} < 0 \quad (32)$$

With the same changes of variables (26), the equation (32) leads to the following LMI formulation:

$$\begin{pmatrix} -rQ & qQ + A_iQ + B_iN_iC \\ qQ + QA_i^T + (B_iN_iC)^T & -rQ \end{pmatrix} < 0 \quad (33)$$

5 Numerical example

Consider the T-S model (1) where $s = n = 2$

$$A_1 = \begin{bmatrix} 2 & -10 \\ 1 & 0 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad C = [1 \quad 0] \quad (34a)$$

$$A_2 = \begin{bmatrix} 1 & -10 \\ 1 & 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 10 \\ 0 \end{bmatrix}, \quad C = [1 \quad 0] \quad (34b)$$

$$\mu_1(y(t)) = \frac{1 - \tanh(y(t))}{2}, \quad \mu_2(y(t)) = \frac{1 + \tanh(y(t))}{2} \quad (34c)$$

- The resolution of the cone complementarity problem (22) with the convergent algorithm given in annex allows to compute :

$$P = \begin{pmatrix} 0.1361 & 0.0352 \\ 0.0352 & 1.5596 \end{pmatrix}, \quad Q = \begin{pmatrix} 7.3896 & -0.1666 \\ -0.1666 & 0.6450 \end{pmatrix} \quad (35)$$

which verify the constraint (20b) i.e. $PQ = I$.

With P defined in (35), the synthesis of the static output controller is obtained from the feasible LMIs (13) in $F_i, \forall i \in \{1, \dots, n\}$:

$$F_1 = -7.1128, \quad F_2 = -2.2239 \quad (36)$$

Example of simulation of the closed loop T-S model (34) is presented in figure 2 with the stabilizing OPDC control law $u(t) = (\mu_1(y(t))F_1 + \mu_2(y(t))F_2)y(t)$ and F_1 and F_2 are defined in (36).

- To compute the stabilizing OPDC controller by using the results of section 4.2, the matrix Q must be diagonal accordingly of the remark 1 and the structure of C given in (34). This constraint is shown to be conservative for certain examples.

6 Conclusion

This paper presents static output feedback controller for nonlinear system described by T-S models. We have shown that the OPDC controller can be designed by two methods. The first one applies a cone complementarity formulation while the second uses a direct convex formulation of the initial BMIs problem. Also pole placement in LMI form is considered for the second method. It should be noted that the cone complementarity formulation is less conservative than the second formulation. The cause of the conservatism of the second method is due to the assumption 1 (C is full row rank). This assumption requires a particular structure for the matrix Q and consequently to the Lyapunov function $V(x(t)) = x(t)^T Q^{-1} x(t)$.

Annex : Linearization algorithm [3]

1. Find a feasible point P_0 and Q_0 that satisfy the LMI constraints (20a)-(21). If there are one, exit. Otherwise set $k = 0$.
2. Set $V_k = P_k$, $W_k = Q_k$ and find P_{k+1}, Q_{k+1} that solve the LMI problem:

$$\text{Minimize } \text{Tr}(V_k Q + W_k P) \text{ subject to (20a) and (21).}$$
3. If the objective has reached a stationary point, stop. Otherwise, set $k = k + 1$ and go to step 2.

This heuristic is guaranteed to converge to a stationary point. For more detail about this algorithm see [3].

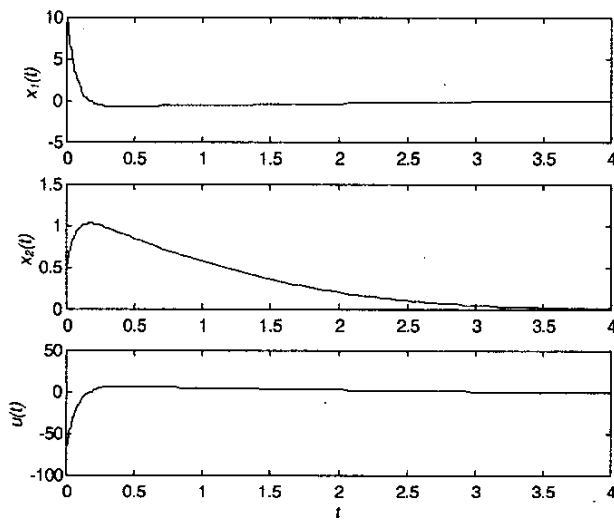


Figure 2. Closed loop model (34) with $x(0) = (10, 0.5)$

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