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Depinning transition and scaling of fracture surfaces: the crucial role of disorder

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Can we describe crack propagation as a critical transition ? Such a description would bring the possibility of material-independent predictions for the fracture behavior of heterogeneous materials. Early attempts to model the propagation of a crack front as a depinning line in a random media have apparently failed thus far to reproduce quantitatively the experimental data obtained on fracture surfaces. The ability of designing model heterogeneous materials with a tunable nanostructure lengthscale (phase separated glasses) allows us to observe, that actually, the predictions of depinning models give a correct description of crack propagation: the roughness of fracture surfaces is shown to obey a logarithmic scaling beyond the characteristic size of disorder.

Ever since an early paper by Mandelbrot *et al* [1] about the fractal character of crack surfaces, a growing interest has developed in the understanding of fracture of heterogeneous materials[2, 3, 4]. It was later proposed that not only can crack surfaces be quantitatively described as self-affine fractals but, moreover, that their self-affine character is universal, *i.e* independent of the nature and microscopic details of the material[5, 6]. Such results obviously appealed for a description of crack propagation in terms of critical behaviour. The paradigm of depinning transition of an elastic line through a disordered landscape[7, 8] already used to describe the motion of a wetting contact line or of a ferromagnetic domain wall was applied to brittle fracture[9, 10, 11, 12] but has yet appeared unable to predict the experimental values of the roughness exponents[3, 11, 13, 14, 15]. Here we show that this disagreement between depinning models predictions and measurements on crack surfaces may be only apparent.

Using phase separation in glasses[16], we are able to finely tune the characteristic length of their internal disorder and to design model heterogeneous brittle materials[17]. We show here that fracture surfaces of these glasses obey the logarithmic scaling prediction of depinning models beyond the size of their heterogeneities. This suggests that the disagreement observed thus far may be due to the non-respect in experiments of the main hypothesis of depinning models, namely brittleness and scale separation between heterogeneities and measurement. The present results give a first quantitative validation of the depinning scenario to describe brittle crack propagation through heterogeneous materials.

The fracture of brittle solids is characterized by two rather distinct properties, namely initiation and resistance to crack propagation. The first item is intrinsically dependent on the microstructure and presence of microscopic flaws. The second item is related to the propagation of a large (as compared to the microstructural scale) crack and its interaction with the microstructure. Many studies[1, 3, 5, 6, 13, 18, 19, 20, 21] have dealt with the statistical characterization of both crack front geometry and crack face roughness. It appears that these geometries display a broad spectrum of length scales, revealing moreover an intriguing hidden symmetry called self-affinity, leaving statistically invariant those objects under rescaling with different ratios depending on the direction. More specifically, self-affinity implies that the typical height differences Δh along the surface scales as Δx^ζ when measured over a distance Δx , ζ being a real parameter called the roughness, or Hurst, exponent. It appeared that the roughness of various fractured materials could be described over a wide range of scales with a unique value of the roughness exponent[3, 5, 6] $\zeta \simeq 0.8$.

This apparent universality motivated the development of models based on the depinning of an elastic line through a random landscape[7, 8]. A crack surface can indeed be considered as the trace left by a crack front propagating through a disordered material (see Fig. 1a for a representation of crack front propagation). In such a description, a particular site pinned by a microscopic tough zone may overcome the obstacle with the help of the additional elastic force due to the straining of the structure around the obstacle. This competition between a roughening effect due to the quenched random environment and a smoothing effect due to elastic interactions along the front leads to a rich phenomenology. Below a certain threshold, the front can only propagate over a finite length and then stops; above the threshold, the front can move at a finite, but highly fluctuating, velocity. Moreover, this transition appears to be critical, with a collection of critical exponents (among which the roughness exponent of the elastic front) ruling its behavior close to threshold, with the strong property of universality coming naturally with this observation.

Starting from the description of long range elastic interactions along a rough crack front[22, 23], such models[9, 11] thus intend to capture the geometry and the intermittent dynamics of the crack front as well as characterize the associated depinning transition. The case of interfacial plane cracks was first developed[9] to study the in-plane roughness (*i.e.* in the direction of propagation) of the front in the horizontal plane. The more complex case of a 3D crack front was then discussed[11], allowing the study of the out-of-plane roughness (*i.e.* in the direction normal to the mean plane of propagation) responsible for the fracture surface roughness.

Though qualitatively correct, the results appeared to be quantitatively rather disappointing since these models predict only a logarithmic scaling of the crack surface's out-of-plane roughness [11] (*i.e.* $\zeta^{dep} = 0$ instead of $\zeta \simeq 0.8$). Even in the simpler case of plane crack propagation, the roughness exponent characterising the in-plane roughness of the crack front was found to be significantly lower in models ($\zeta_H^{dep} \simeq 0.4$) [14, 15] than in experiments ($\zeta_H \simeq 0.55$) [13].

The results obtained over the last decade did not really contribute to clarify the debate. Higher order terms were taken into account in the depinning model[24] but apparently did not change the scaling behaviour[25]. The interplay between local damage and crack propagation was proposed but led to controversy at both the theoretical[26, 27, 28, 29] and the experimental[30, 31] levels.

Moreover, in light of the variety of experimental results, the initial claim of universality itself may need to be tempered. It is indeed not clear whether the usually accepted universal roughness exponent $\zeta \simeq 0.8$ [3, 32] is really compatible with measurements on crystalline polymers ($\zeta \simeq 0.84 - 0.89$)[33], sandstone ($\zeta \simeq 0.4 - 0.5$)[18] or recent results obtained on glass ($\zeta \simeq 0.2 - 0.3$)[34]. On top of this when performed on materials with a well defined microstructure such as glass[20], sintered glass beads[21] or some cast aluminium alloys[19], it appears that either the scaling range is very limited or that it does not extend beyond the grain size whereas a truly universal scaling regime should extend beyond the characteristic size of heterogeneities.

We argue here within that the main reason for the apparent discrepancy between experimental results and scaling predictions of depinning models stems from the dubious status in most experiments of a key hypothesis of the models: a clear separation between the scale of disorder and the scale of measurement. To test this argument, we present results of fracture experiments performed on a series of model heterogeneous brittle materials: phase separated glasses[17]. Thermal treatments of various durations allowed the fine tuning of phase domain sizes. Phase separated glasses can thus be considered as model heterogeneous brittle materials since they differ only by one single parameter, the lengthscale of their microstructure, with all other material properties remaining identical.

Phase separated glasses have been prepared from an alkali borosilicate glass of composition (by weight): SiO_2 70%, B_2O_3 25%, Na_2O 2.5% and K_2O 2.5%. The raw materials were mixed together and melted at 1550°C and the melt was then refined over 2 hours in order to obtain a homogeneous glass; after quenching, the glass was annealed for 1h at 630°C in order to relax the internal thermal stresses. At this stage, phase separation had already initiated, attested to by the slightly opalescent character of the glass. The major phase is near-pure silica while the minor phase concentrates other constituents. Heat treatment at 650°C over increasing durations (4h, 16h, 64h) were then applied to obtain phase domains of greater and greater size as shown in Fig. 2 and summarised in Table I. It was thus possible to prepare glasses with controlled phase domain sizes, ranging from around 20 to 100 nm. Further details concerning the preparation of the glass samples as well as a quantitative study of the kinetics of phase separation by atomic force microscopy can be found in Ref. [17].

Stable dynamic fracture experiments on these heterogeneous materials were performed by using a DCDC set-up (Double Cleavage Drilled Compression) (Fig. 1b) and a statistical analysis of the fracture surface roughness carried out from Atomic Force Microscopy (AFM) images (Fig. 1a). From the above mentioned glass, Parallelepipedic shaped samples ($5\text{mm} \times 5\text{mm} \times 25\text{mm}$) with a central hole 1mm in diameter have then been prepared. Stable cracks were then propagated in these by using DCDC mechanical test. Roughness measurements were then immediately performed in areas lying between two crack arrest lines. The surface morphology of all samples has been characterised by AFM height measurement in tapping mode (TM-AFM), using a Nanoscope III A from Digital Instruments with Al coated tip (BudgetSensors - model BS-Tap 300 Al). The Al coating thickness used was 30nm, the resonant frequency 300 kHz and the stiffness constant 40N/m. Images were recorded at a scan frequency between 0.8 and 1Hz for a resolution of 512×512 pixels. For each sample (*i.e.* for each thermal treatment conditions), at least 3 AFM images with 4 different scan areas were performed.

Beyond its imaging ability, AFM has recently been used as a truly quantitative tool to study glass surfaces. It is, for example, possible to obtain a quantitative validation of the description of fused glass surfaces by frozen capillary waves[35] or of the kinetics of phase separation in glasses[17]. In both cases, not only are the expected scaling regimes recovered but the prefactors of the scaling laws can also be extracted and shown to be consistent with the associated physical parameters (diffusivity, interface tension).

In the present case of fracture surfaces, it appears that one can identify an apparent scaling regime up to the size of the heterogeneities (Fig. 3a). An indicative power law of exponent $\zeta = 0.8$ is displayed on the graph and reasonably describes the data. This result is consistent with the standard analysis even if the scaling range is limited. More interestingly, beyond the size of heterogeneities, one observes (Fig. 3b) a clear logarithmic scaling as predicted by the depinning model early proposed by Ramanathan *et al* [11, 12].

More specifically Fig. 3 represents the evolution of height differences $\Delta h(\Delta x) = \langle [\Delta h(x + \Delta x) - \Delta h(x)] \rangle_x$ measured on a scale Δx on the crack surface of each of the three types of sample. The results could be collapsed onto a master curve after rescaling by the phase domain size d along the x -axis and by the typical height difference $\Delta h(d)$ along the y -axis. When data are displayed on a log-log scale (Fig. 3a), an apparent power law behaviour is identified,

extending over one decade up to the size of heterogeneity. Such a scaling law, if genuine, could not be explained in the framework of depinning models which predict scaling regimes above the typical scale of disorder. However if data are represented on a simple linear-log scale, we obtain excellent agreement between our results and the logarithmic scaling predicted by depinning models above the characteristic scale of the heterogeneities. Depinning models thus seem to give an excellent account of the scaling of the roughness of fracture surfaces of brittle heterogeneous materials.

The common experimental observation of a wide scaling regime on fracture surfaces with roughness exponents differing from the depinning predictions may therefore result from the non-respect of the hypothesis of this simple model: scale separation between disorder and experiments or perfect brittleness. This may be the case for ductile materials or very heterogeneous materials presenting heterogeneities over a wide range of scales. In particular the “classical” scaling regime characterized by a roughness exponent $\zeta \simeq 0.8$ may correspond to the existence of additional microscopic mechanisms (*e.g.* damage or other non-linear behaviour) over a more limited range of scales. The present results also suggest that experiments displaying a scaling regime only up to the characteristic size of disorder may be revisited to test whether they exhibit the logarithmic scaling of roughness expected beyond that characteristic size.

This first quantitative validation of the depinning scenario to describe brittle crack propagation through heterogeneous materials thus provides a framework to extend the use of critical transitions concepts to fracture, for instance in order to better predict fluctuations and homogeneous behaviour from nano- to macro-scale.

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| Heat treatment duration | 4h | 16h | 64h |
|-------------------------|-------------------|-------------------|-------------------|
| Phase domain size | 28.2 ± 2.5 nm | 56.3 ± 9.3 nm | 92.2 ± 9.4 nm |

TABLE I: Size d of phase domains (estimated by the correlation length at half height) *vs* duration of thermal treatment at 650°C. See Ref. [17] for quantitative details about the kinetics of phase separation on these glasses.

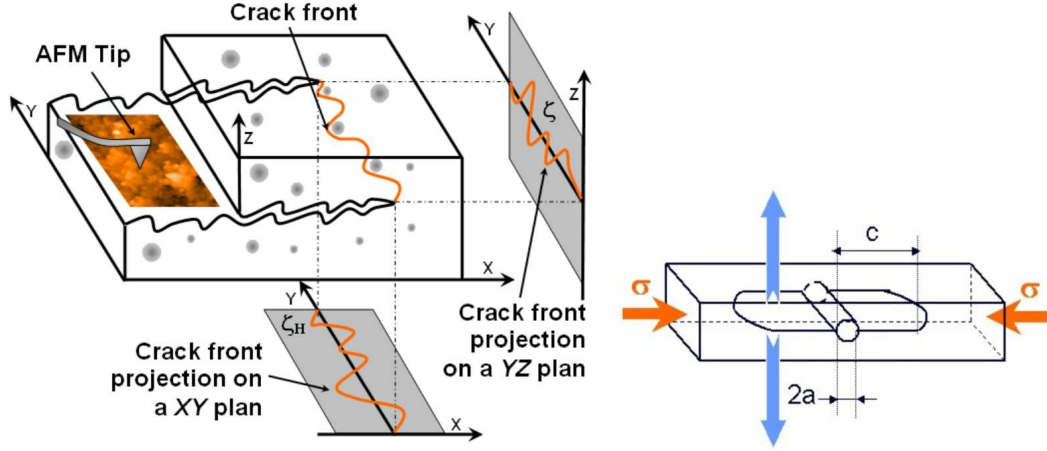


FIG. 1: **Schematic view of the experimental set-up.** **a**, sketch of a propagating crack front pinned by heterogeneities (here figured as grey dots). The front develops a roughness both in the out-of-plane direction, visible on the $y-z$ plane projection and in the direction of propagation, visible on the $x-y$ plane projection. When scaling invariant, this roughness is characterized by an exponent ζ in the out-of-plane direction and an exponent ζ_H in the direction of propagation. AFM measurements are performed on the crack surface. **b**, geometry of the DCDC mechanical test. Two longitudinal cracks propagate from the cylindrical hole

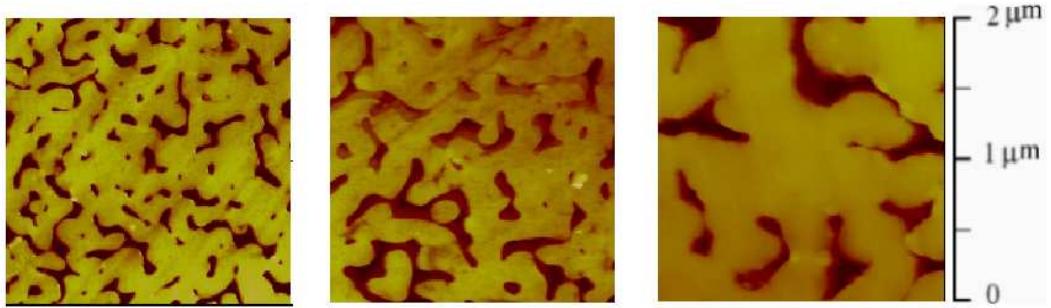


FIG. 2: **AFM images obtained after acid etching on glass samples annealed at 650°C over 4h, 16h and 64h respectively.** Under annealing, alkali borosilicate separate into a near pure silica phase and another phase concentrating all other components. The latter phase can be eliminated by acid etching, the height contrast obtained on AFM images thus reveals the phase contrast (see also Ref. [17]).

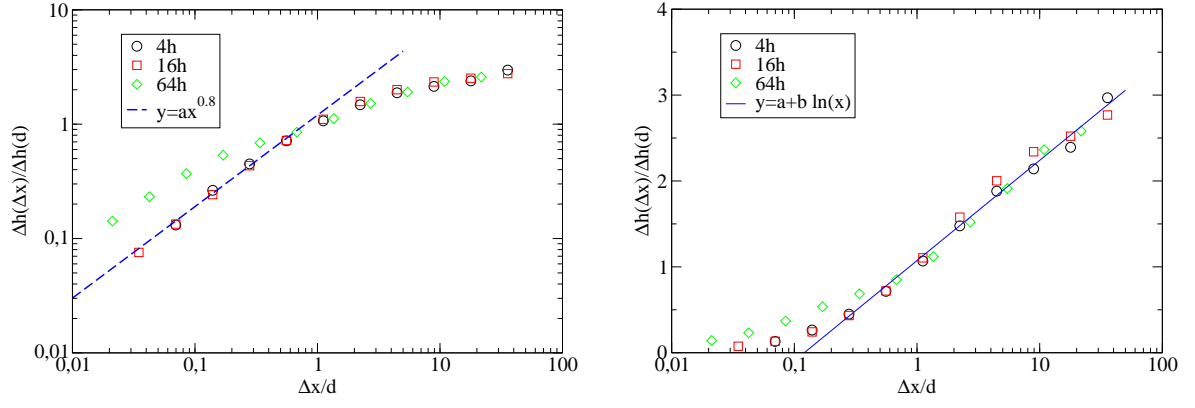


FIG. 3: **Roughness of fracture surfaces after rescaling by the phase domain size d along the x -axis and the typical roughness $\Delta h(d)$ at size d along the y -axis.** **a**, in log-log scale an apparent self-affine regime with exponent $\zeta = 0.8$ may be identified up to the the size of the phase domains. An indicative power law of exponent 0.8 is displayed. **b**, in linear-log scale, a simple logarithmic regime appears to describe the surface roughness as predicted by depinning models. The straight line corresponds to a logarithmic fit performed on the 3 sets of data for lengths larger than the domain size.