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# ON THE MODELING AND ESTIMATION OF NONLINEAR INSTANTANEOUS FREQUENCY AND AMPLITUDE OF NON-STATIONARY SIGNALS

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Abstract: Advanced signal processing methods are helpful in the context of diagnostic and fault detection of induction machines. They provide accurate information about highly nonlinear systems which need complex physical models. We recall and compare some available methods in modeling and/or detection of nonlinear and nonstationary signals in the time and frequency domain. We particularly outline a new method we developed, which is based on a local modeling of the instantaneous amplitude and frequency of the signal. This method reaches significant performance especially when signals are embedded in strong noise and present a high variation of the instantaneous frequency.

Keywords: Non-stationary signal, nonlinear amplitude and frequency modulations, polynomial phase signal, time-frequency distribution, Spectrogram.

## 1. INTRODUCTION

Nowadays, in complex industrial systems, several machines interact between them at dissimilar power levels and classical maintenance methods can be combined with advanced post-processing in order to increase the reliability of the maintenance program and to reduce its cost.

These systems provide different observed signals which can be: nonstationary, with a mixture of different spectral structures, narrow spectral bands and/or wide spectral bands, multi-components, nonlinearly modulated in frequency and/or in amplitude, embedded in different types of noise, white, colored, Gaussian or impulsive noise. Signal processing methods in particular yield an addi-

tional help when applied to these observations.

Section 2 outlines some methods based on spectral and time-frequency analysis.

Section 3 describes a new method already published in (M.Jabloun and N.Martin, July 2007) and based on a local polynomial modeling of both instantaneous amplitude and frequency of nonstationary signals.

These methods have applications in a wide range of fields such as induction machines, mechanics, radar, sonar, seismic and biomedicine.

A comparison with some classical methods is shown in Section 4. Finally Section 5 summarizes our work.

## 2. OVERVIEW OF SIGNAL PROCESSING-BASED METHODS

The modeling and estimation of non-stationary signals from real observations received considerable attention and many methods have already been proposed. We present some of the current methods which can be easily extended and applied in the context of diagnostic and fault detection of induction machines.

In (B.Leprettre and L.Turner, 11-14 June 2007), the authors propose to model the thermal behaviour of an induction motor without considering an exhaustive identification and without a thermal sensor. So, they establish an equivalence between an electrical model, which inputs are motor current and voltage, and a thermal model, which outputs are the stator and rotor temperatures. The parameter estimation of the electrical model is then operating via a harmonic detection in the spectra.

In (F.Perisse and J.P.Lecoite, 13-15 September 2007), the authors present a new monitoring system able to detect slight variations of high frequency resonances in the winding of a working machine fed by an industrial inverter. This system works in association with statistical data on accelerated aging of the magnet wire used to build the machine.

As in (B.Leprettre and L.Turner, 11-14 June 2007), the interest of (M.Chabert and J.Regnier, 11-14 June 2007) is a sensorless estimation. Properties of a given time-frequency transform help the classifying mechanical failures from a monitoring of the stator. In (A.Catherall, 11-14 June 2007), a new time-frequency transform which preserves robustness at low Signal-to-Noise Ratios (SNR) is able to handle signals more complex than the well-known spectrogram. Moreover, this transform can be applied on any signal having a variable instantaneous frequency.

Other general methods based on signal processing, and described in (S.Stankovic, 11-14 June 2007; C.Ioana and A.Quinquis, 11-14 June 2007; F.Castanie, 2006; M.Durnerin, 21 September 1999), are interesting to be used in condition monitoring by making some analogies. A mathematical model classically used in spectral analysis is derived in (S.Stankovic, 11-14 June 2007), and an estimation of the instantaneous frequency of this model allows the estimation of the motion of an object in video sequences. In (C.Ioana and A.Quinquis, 11-14 June 2007), the authors investigate a time-phase representation to be able to estimate a

phase delay. They suggest to filter each time-frequency patterns by a non-unitary time-warping filter. A continuity constraint is then applied on the phase of the signal reconstructed to separate components in the patterns previously extracted. The method can be applied on any modulated signals even if they are multicomponent and embedded in a noisy environment.

Two main problems which are related to the signal nonstationarity definition explain the numerous methods published in the signal processing literature. First is the definition of the concept of instantaneous frequency in the case of nonstationary signals. Second is the reconsideration of the base functions, the exponential functions, which are not adapted to model the evolution in the signal. Moreover, without the stationary hypothesis, the estimation of the energy distribution of nonstationary signals is not so well defined. Actually, a stationary hypothesis makes easier the definition of the correlation function, function of one delay only, and of a power spectral density if the spectrum is continuous. Some developed methods which addressed the analysis of nonstationary signals have assumed a local stationary hypothesis or applied a nonlinear transform in order to make the signal stationary. Others methods have introduced time-varying terms in the exponential functions while others have used more adaptable functions.

In this paper, we just point out some of the methods: spectrogram, Wigner-Ville distribution, the S method and the new method proposed in (M.Jabloun and N.Martin, July 2007).

### 2.1 Spectrogram

The most older time-frequency method, the spectrogram (or sonogram) denoted by  $\mathcal{S}[n, k]$  at time index  $n$  and frequency index  $k$ , of a discrete signal  $s[n]$  of length  $N$  is based on a Short Time Fourier Transform  $\mathcal{STFT}[n, k]$ . The spectrogram assumes a local stationarity over length  $N_w$  and writes

$$\mathcal{S}[n, k] = |\mathcal{STFT}[n, k]|^2, \quad (1)$$

$$\mathcal{STFT}[n, k] = \sum_{m=1}^{N_w} s[m+n] w[m] e^{-2\pi j \frac{m \cdot k}{N_w}}, \quad (2)$$

where  $w[m]$  is a time window of length  $N_w$ . The spectrogram is positive and can be considered as an energy distribution without cross terms. However,  $\mathcal{S}[n, k]$  suffers from a lack of resolution due to the Heisenberg-Gabor inequality.

## 2.2 Wigner-Ville distribution

The Wigner-Ville distribution  $\mathcal{WV}[n, k]$  is a bilinear transform of the signal defined for discrete time  $n$  as:

$$\mathcal{WV}[n, k] = 2 \sum_{m=1}^N s[m+n] s^*[m+n] e^{-4\pi j \frac{m \cdot k}{N}}, \quad (3)$$

where  $*$  denotes the complex conjugate.  $\mathcal{WV}[n, k]$  is the Fourier Transform of the instantaneous correlation  $s[m+n] s^*[m+n]$ . The Wigner-Ville distribution has a perfect localization on linear chirp signal. On the contrary, multicomponent signal generates cross terms. Both time and frequency smoothing were introduced to reduce interferences, what results in a reduction of the resolution (L.Stankovic, Jan 1994).

## 2.3 S method

The S-method introduced in (L.Stankovic, Jan 1994) and used in (S.Stankovic, 11-14 June 2007) was defined by:

$$\mathcal{SM}[n, k] = \sum_{m=1}^{N_w} w[m] \mathcal{STFT}[n, k+m] \mathcal{STFT}^*[n, k-m] \quad (4)$$

where  $\mathcal{STFT}[n, k]$  is defined in (2) and  $w[m]$  is a time window. The S-method is a time-frequency distribution where cross terms are reduced comparing to the Wigner Ville distribution. An appropriate selection of the window  $w[m]$  can make the S-method cross term free.

## 3. LOCAL POLYNOMIAL MODELING OF FREQUENCY AND AMPLITUDE

In (M.Jabloun and N.Martin, July 2007), we propose a new flexible method for modeling and estimating nonstationary signals. We recall the principle of this method.

Therefore, let us consider the observed noisy signal  $y[n]$  defined as follows

$$y[n] = s[n] + e[n], \quad \text{for } 0 \leq n \leq N-1, \quad (5)$$

$$s[n] = A[n] e^{j\Phi[n]}, \quad (6)$$

where  $s[n]$  represents the noise-free signal and  $e[n]$  a white complex Gaussian noise with zero mean and unknown variance  $\sigma^2$ .  $N$  is the total sample number and  $j$  is the complex number verifying  $j^2 = -1$ .  $A[n]$  and  $\Phi[n]$  are the Instantaneous Amplitude (IA) and phase respectively. The Instantaneous Frequency (IF) is defined by a numerical derivation of  $\Phi[n]$

$$F[n] = \frac{1}{2\pi} (\Phi[n+1] - \Phi[n]). \quad (7)$$

IA and IF are both time-varying functions and IF verifies  $0 < F[n] < \frac{F_s}{2}$  in relation to Shannon's theorem,  $F_s$  being the sampling frequency. To remove the ambiguity in the definition of the amplitude and phase of the signal model (6), we assume  $\Phi[n]$  is non-discontinuous and  $A[n]$  is real and positive (L.Cohen and D.Vakman, 1999). The modeling and estimation of both the instantaneous amplitude  $A[n]$  and phase  $\Phi[n]$  are the objective of the method.

Unlike (S.Peleg and B.Friedlander, 1995) which considered phase polynomial models requiring a large number of parameters to approximate the entire phase, we use a local approach. We assume that the signal non-stationarity could be piecewise modeled by low-order polynomials on short-time windows called segments. We apply a strategy for extracting short-time segments from a long-time signal. This extraction is not ordered through time, the length the position and the number of segments are estimated using any non-negative Time-Frequency Distribution (Spectrogram for example). To reduce the estimation error, the first segment corresponds to the highest-energy part of the signal. Then a segmentation process was followed in positive or negative time progression in order to estimate the next high-energy part of the signal. Readers can refer to (M.Jabloun and N.Martin, July 2007) for more details.

In the strategy proposed, we start by finding short-time segments whose lengths are suitable for approximating locally both the IA and IF by low-order polynomial models. The lengths vary from one segment to another and segments can overlap. On each segment, the following polynomial models are assigned to the local time variation of the IA and IF:

$$\begin{aligned} A[n] &= \sum_{m=0}^p a_m g_m[n], \\ F[n] &= \sum_{m=0}^q f_m g_m[n], \end{aligned} \quad (8)$$

for  $n$  belongs to the Segment considered.  $p$  and  $q$  are the polynomial approximation orders of local  $A[n]$  and  $F[n]$  respectively and  $g_m[n]$  is a discrete polynomial of order  $m$ . The real coefficients of the decomposition of local  $A[n]$  and  $F[n]$  on the discrete polynomial base  $(g_m[k])_{m=0, \dots, \max(p, q)}$  are  $a_m$  and  $f_m$  respectively. The local phase related to the segment considered is calculated by numerical integration up to  $2\pi$  of the local frequency and this local phase is referenced to the segment center in order to reduce the estimation error (D.Rife and R.Boorstyn, 1974). Therefore, on each segment, we have to estimate for each local model a vector

of  $p + q + 3$  parameters

$$\boldsymbol{\theta} = [a_0, a_1, \dots, a_p, \varphi_0, f_0, f_1, \dots, f_q]^T, \quad (9)$$

where  $\varphi_0$  denotes the initial phase referenced to the segment center and  $T$  is the transpose. To reduce the number of parameters to be estimated for each segment the polynomial approximation orders  $p$  and  $q$  are limited. The segment length is selected in (M.Jabloun and N.Martin, July 2007) such as  $p$  and  $q$  satisfying

$$0 \leq \min(p, q) \leq \max(p, q) \leq 3. \quad (10)$$

In (M.Jabloun and N.Martin, July 2007), we calculated an orthonormal discrete polynomial base  $g_m[n]$  by applying the Gram-Schmidt procedure. The comparison with other polynomial bases shows that the orthonormality property enhances the estimation efficiency by significantly reducing the parameter coupling.

The local model parameters (8) are estimated using a maximum-likelihood procedure known to possess optimality property. This is equivalent to minimization of the Least-Square (LS) function for Gaussian noises and thus results in the following multivariate nonlinear equation

$$\hat{\boldsymbol{\theta}} = \arg \min_{\boldsymbol{\theta} \in \mathbb{R}^{p+q+3}} \ell_{\text{LS}}(\boldsymbol{\theta}), \quad (11)$$

$$\text{with } \ell_{\text{LS}}(\boldsymbol{\theta}) = \sum_{n \in \text{Segment}} |y[n] - s[n]|^2. \quad (12)$$

Direct minimization of (11) is extremely difficult due to the high non-linearity of the cost function (12) and the parameter number, a stochastic optimization technique based on the simulated annealing method is used. Simulating annealing has an analogy with thermodynamics where metal cools and anneals. In the same way, after an initialization of the parameter vector to estimate, an iterative loop controlled by a scalar referred to as a temperature generates a new candidate of the vector (9) that minimizes the cost function, namely the least square function (12). A statistical significance test relied on the assumption that the error is normally distributed stops the algorithm.

Finally the estimated segments are then merged in order to reconstruct the whole amplitude, frequency, phase and signal.

The method proposed is of course more complex and needs more computing time than the classical methods presented in section 2. Nevertheless, in the case of more complex signals with highly nonlinear amplitude and frequency, the results obtained defend this complexity (see results in

Section 4). The method has been extended to multicomponent signals (M.Jabloun *et al.*, 2005).

#### 4. SIMULATION RESULTS AND COMPARISON

We consider an academic noise-free signal  $s[n]$  with a time-varying frequency modulation and sinusoidal amplitude defined by:

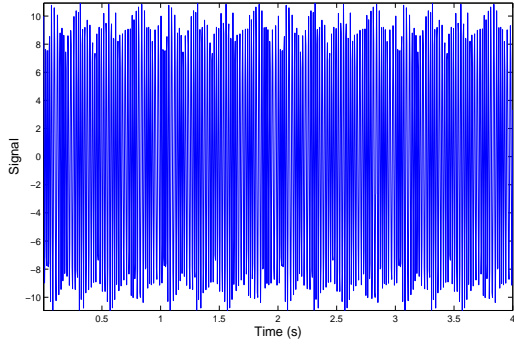
$$A[n] = 10 + 1 \sin\left(\frac{8\pi n}{F_s}\right); \quad (13)$$

$$F[n] = 50 + 5 \sin\left(\frac{16\pi n}{F_s}\right) + 10 \sin\left(\frac{14\pi n}{F_s}\right); \quad (14)$$

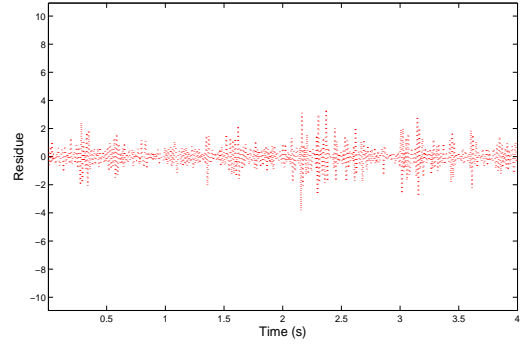
$F_s$  is the frequency sampling and is equal 256 Hz. The sample number is 1024.  $s[n]$  is embedded in noise of 20 dB. The variation of the instantaneous frequency  $F[n]$  are such that the analysis by the methods mentioned in Section 2 is not acceptable except for the S-method. From, the spectrogram we can thought that two modulated component are present in the signal which is not the case. The S-method give the behaviour of the frequency modulation but the estimation errors are important. Figure 1 shows the noise-free signal superposed to estimation error of  $s[n]$  obtained using the local approach proposed in (M.Jabloun and N.Martin, July 2007). Figures 2(a), (b), (c) and (d) display the results given by the spectrogram, the Wigner-Ville distribution, the S-method and the local approach proposed in (M.Jabloun and N.Martin, July 2007) respectively.

#### 5. CONCLUSION

We give some insights in time-frequency analysis. Particularly, modeling of highly nonstationary signals having both non-linear amplitude and non-linear frequency modulations is considered and illustrated on academic signals.

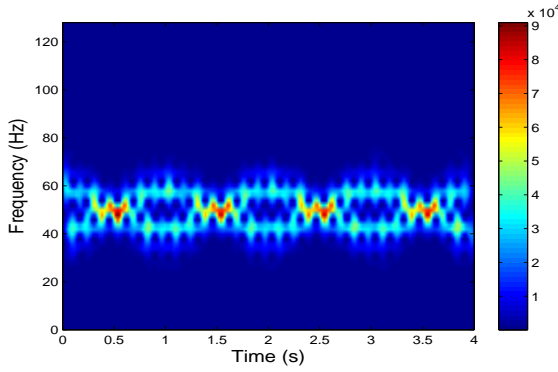


(a)

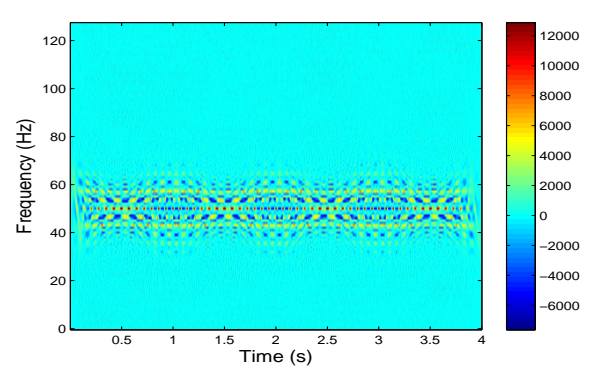


(b)

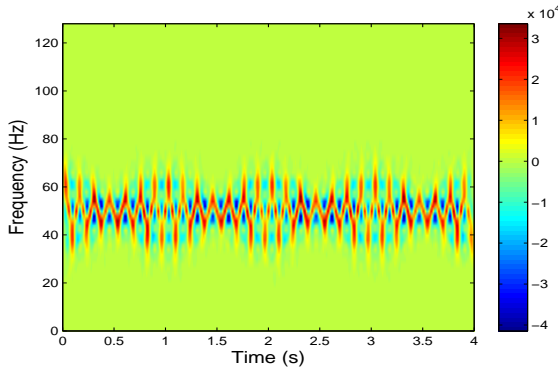
Fig. 1. (a) An academic signal (—) processed by the proposed approach based on a piecewise modeling of the signal nonstationarity by low-order polynomials on short-time segments. (b) The residue (:) obtained on the estimation of  $s[n]$ .



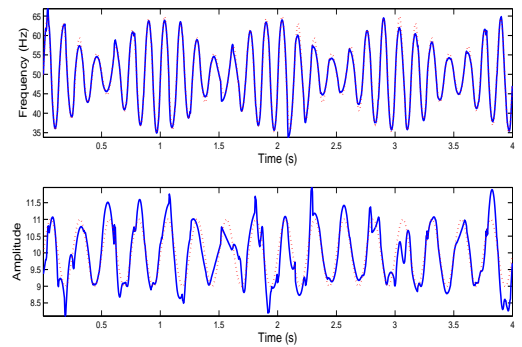
(a) Spectrogram of the noisy signal calculated with a Hamming window with  $N_w = 128$  and a STFT of 128 points.



(b) Wigner-Ville distribution of the noisy signal.



(c) S-method applied to the noisy signal.



(d) Local polynomial modeling applied to the noisy signal: (·) original curves of the IA and IF versus (—) estimated curves.

Fig. 2. An academic signal processed by some time-frequency analysis methods: the spectrogram, Wigner-Ville distribution and S-method compared to the approach based on a piecewise modeling of the signal nonstationarity using low-order polynomials on short-time segments.

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