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Charge pumping and noise in a one-dimensional wire with weak electron-electron interactions

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We consider the adiabatic pumping of charge through a mesoscopic one dimensional wire in the presence of electron-electron interactions. A two-delta potential model is used to describe the wire, which allows to obtain exactly the scattering matrix coefficients, which are renormalized by the interactions. Two periodic drives, shifted one from another, are applied at two locations of the wire in order to drive a current through it in the absence of bias. Analytical expressions are obtained for the pumped charge, current noise, and Fano factor in different regimes. This allows to explore pumping for the whole parameter range of pumping strengths. We show that, working close to a resonance is necessary to have a comfortable window of pumping amplitudes where charge quantization is close to the optimum value: a single electron charge is transferred in one cycle. Interactions can improve the situation, the charge Q is closer to one electron charge and noise is reduced, following a $Q(1-Q)$ behavior, reminiscent of the reduction of noise in quantum wires by $T(1-T)$, where T is the energy transmission coefficient. For large pumping amplitudes, this charge vanishes, noise also decreases but slower than the charge.

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I. INTRODUCTION

The suggestion that electrons can be supplied one by one by a mesoscopic circuit has been proposed over two decades ago¹. Instead of applying a constant bias voltage to the system, it is possible to supply a.c. gate voltages which perturb the system periodically. Under certain conditions, the charge transferred from one lead to the other, during one period, can be almost quantized. Adiabatic pumping of electrons could in principle be used in future nanoelectronics schemes based on single electron transfer, and it also has applications to quantum information physics. Over the years, theoretical approaches to this adiabatic pumping based on scattering theory have become available^{2,3,4}. These situations typically describe mesoscopic systems which are large enough, or sufficiently well connected to leads that electronic interactions (charging effects for instance) can be discarded. Scattering theory has been applied⁴ to calculate both the charge and noise in systems in the absence of electron-electron interactions.

On the experimental scene, Coulomb blockade effects have been successfully exploited to achieve pumping with isolated quantum dots⁵. To our knowledge, pumping experiments which are not entirely based on Coulomb blockade, where the shape of the electron wave functions is modified in an adiabatic drive are rather scarce. A recent study⁶ has dealt with the transport through an open quantum dot where such interactions are minimized.

Besides Coulomb blockade physics, the effect of electron-electron interactions in conductors with reduced dimensionality have been discussed by several authors. The case of strong interactions in a one dimensional quantum wire was presented in Ref. 7, using Luttinger liquid theory. Alternatively, Ref. 8 discussed the opposite limit, where the effect of weak interactions can be included in a scattering formulation of pumping using renormalized transmission/reflection amplitudes⁹. However, the results for the pumped charge remain mostly numerical in this work.

The conditions under which pumping amplitude and interactions must be tuned to achieve quantized pumping are not obvious. Many physical parameters enter this problem, such as the amplitude of the pumping potentials, the phase difference between these, the possibility of a constant offset on these potentials, the overall conductance of the unperturbed structure, and to what extent the strength of electron-electron interactions play a role. Analytical results on this issues are highly desirable, as well as information about the noise.

With regard to the experiment of Ref. 6, there is clearly a need for further understanding the role of weak interactions in such mesoscopic systems in the presence of pumping. The purpose of the present work goes in this direction, in the sense that we provide analytical expressions for the pumped charge and the noise for a one dimensional wire in the presence of interactions. This allows us to explore all pumping regimes¹⁰ (weak to strong pumping) and to determine in which manner and to what extent the pumped charge can help to achieve single electron transfer. Besides addressing the question of the ideal conditions for good charge quantization, we shall establish relationships between charge and noise in different regimes. For concreteness, a two delta potential model will be used and interactions will be added on top of it.

II. PUMPED CHARGE AND NOISE

A. Adiabatic pumping in non interacting systems

Here, we recall the formula which was established for the charge transferred during the single period of an adiabatic pumping cycle through a quasi one-dimensional system. The system is in general described by a potential $V(x)$ containing two internal parameters which are modulated periodically. The time dependence is assumed to be sufficiently slow so that, although the scattering matrix depends on time, its variations are minute when an electron is scattered in the mesoscopic wire. The result for the pumped charge is then independent of the frequency ω and, as derived in Ref. 3, is given by

$$Q = \frac{e}{\pi} \int_A \text{Im} \Pi(X_1, X_2) dX_1 dX_2, \quad (1)$$

where the function $\Pi(X_1, X_2)$ is defined as

$$\Pi(X_1, X_2) = \frac{\partial s_{11}^*}{\partial X_1} \frac{\partial s_{11}}{\partial X_2} + \frac{\partial s_{12}^*}{\partial X_1} \frac{\partial s_{12}}{\partial X_2}, \quad (2)$$

s_{1i} ($i = 1, 2$) are the elements of the scattering matrix s associated with $V(x)$, and $X_i = X_i(t)$ are the two external parameters which are adiabatically varying (periodically) with time. \int_A denotes integration within the area encompassed by the contour A (in the X_1 - X_2 plane). The above formula implicitly assumes that thermal effects are not dominant.

Unitarity of the scattering matrix requires that its elements are parametrized in the following way:

$$s(E) = e^{i\phi} \begin{pmatrix} -i\sqrt{R}e^{i\theta} & \sqrt{T} \\ \sqrt{T} & -i\sqrt{R}e^{-i\theta} \end{pmatrix}, \quad (3)$$

where ϕ is the phase accumulated in a transmission event and θ is the phase characterizing the asymmetry between the reflection from the left hand side and from the right hand side of the potential. Conservation of probabilities imposes $R + T = 1$. We assume the quantities \sqrt{R} , \sqrt{T} , θ and ϕ to be functions of the Fermi energy E_F and of the external time-varying parameters $X_i(t)$.

Using this parametrization, (see Eq. (2)), we get:

$$\text{Im} \Pi(X_1, X_2) = \frac{1}{2} \left(\frac{\partial T}{\partial X_2} \frac{\partial \theta}{\partial X_1} - \frac{\partial T}{\partial X_1} \frac{\partial \theta}{\partial X_2} \right). \quad (4)$$

B. Inclusion of weak interactions

In the case of weak interactions, the transmission and the reflection amplitudes s_{12} and s_{11} can be calculated in the presence of Coulomb interaction via a renormalization procedure⁹. High energy scales above a given cutoff are eliminated. The high energy cutoff is lowered progressively. The renormalization has to be stopped when the temperature becomes comparable to this cutoff. Finally, if $s_{12}^{(0)}$ and $s_{11}^{(0)}$ denote respectively, the transmission and reflexion coefficient without interactions, s_{12} and s_{11} can be expressed in the form⁹:

$$s_{12} = \frac{s_{12}^{(0)} l^\alpha}{\sqrt{1 + T_0(l^{2\alpha} - 1)}}, \quad (5)$$

$$s_{11} = \frac{s_{11}^{(0)}}{\sqrt{1 + T_0(l^{2\alpha} - 1)}}, \quad (6)$$

where $l = k_B \Theta / D$, where Θ is the temperature, k_B the Boltzmann constant and D the original bandwidth. α is an exponent related to the strength of the Coulomb interaction⁹. Specifically:

$$\alpha = \frac{V_c(2k_F) - V_c(0)}{2\pi v_F}, \quad (7)$$

with $V_c(q)$ the Fourier transform of the Coulomb potential at $q = 2k_F$ and $q = 0$, respectively. $\alpha = 0$ corresponds to the absence of electron-electron interactions. $T_0 = |s_{12}^{(0)}|^2$ represents the conductance of the wire in units of e^2/h . From now on, Q will denote the pumped charge with interactions and Q_0 without interactions. The integrand of Eq. (1) is therefore modified by the presence of the interactions.

C. Two delta potential model and pumped charge

We specialize to the case where the time-dependent perturbations are two δ -like potentials:

$$V_p(x, t) = \frac{\hbar^2}{2m} k_F [X(t) \delta(x + a) + Y(t) \delta(x - a)], \quad (8)$$

with the amplitudes $X(t) = V_i(t)/2k_F$ and $Y(t) = V_f(t)/2k_F$, where $V_i(t)$ and $V_f(t)$ have periodic time evolution with the same period $\tau = 2\pi/\omega$ and k_F is the Fermi wavevector.

The expressions of $s_{12}^{(0)}$ and $s_{11}^{(0)}$ are needed, when the single particle Hamiltonian reads

$$H = -\hbar^2 k^2 / 2m + V_p(x). \quad (9)$$

The elements of the S -matrix in the absence of electron-electron interactions, are given by

$$s_{11}^{(0)} = \frac{[(\bar{Y} - \bar{X}) \sin(2k_F a) - i\{2\bar{X}\bar{Y} \sin(2k_F a) + (\bar{X} + \bar{Y}) \cos(2k_F a)\}]}{D}, \quad (10)$$

$$s_{12}^{(0)} = \frac{1}{D}, \quad (11)$$

with

$$D = \left(1 - 2\bar{X}\bar{Y} \sin^2(2k_F a) + i[\bar{X} + \bar{Y} + \bar{X}\bar{Y} \sin(4k_F a)]\right), \quad (12)$$

and $\bar{X} = X/2$, same for \bar{Y} . $s_{21}^{(0)} = s_{12}^{(0)}$ and $s_{22}^{(0)}$ is obtained by replacing \bar{Y} by \bar{X} in $s_{11}^{(0)}$.

Instead of using the formula giving Q as a double integral³, we prefer to use it as a simple contour integral,^{2,8}

$$Q = \frac{e}{2\pi} \int_0^{2\pi/\omega} \text{Im} \left[\left(\frac{\partial s_{11}}{\partial X} s_{11}^* + \frac{\partial s_{12}}{\partial X} s_{12}^* \right) \frac{dX}{dt} + X \leftrightarrow Y \right] dt. \quad (13)$$

It is possible to simplify this formula. Expressing the matrix elements of $s(E)$ in terms of R , T , θ and ϕ , (Eq. (3)), and switching to variables \bar{X} and \bar{Y} , we get

$$Q_0 = \frac{e}{2\pi} \int_0^{2\pi/\omega} \left[\left(\frac{\partial \phi}{\partial \bar{X}} + R \frac{\partial \theta}{\partial \bar{X}} \right) \frac{d\bar{X}}{dt} + (\bar{X} \leftrightarrow \bar{Y}) \right] dt. \quad (14)$$

Using then the relationship^{11,12} between the Green's function and the s matrix elements, setting $\tilde{s}_{11}^{(0)} = e^{2ik_F a} s_{11}^{(0)}$, $\tilde{s}_{12}^{(0)} = e^{2ik_F a} s_{12}^{(0)}$ and $\tilde{s}_{22}^{(0)} = e^{-2ik_F a} s_{22}^{(0)}$,

$$i \frac{\partial \tilde{s}_{11}^{(0)}}{\partial \bar{X}} = \left(1 + \tilde{s}_{11}^{(0)}\right)^2, \quad (15)$$

$$i \frac{\partial \tilde{s}_{12}^{(0)}}{\partial \bar{X}} = \tilde{s}_{12}^{(0)} \left(1 + \tilde{s}_{11}^{(0)}\right), \quad (16)$$

$$i \frac{\partial \tilde{s}_{11}^{(0)}}{\partial \bar{Y}} = \left(\tilde{s}_{12}^{(0)}\right)^2, \quad (17)$$

$$i \frac{\partial \tilde{s}_{12}^{(0)}}{\partial \bar{Y}} = \tilde{s}_{12}^{(0)} \left(1 + \tilde{s}_{22}^{(0)}\right). \quad (18)$$

Then, the following relations are obtained

$$\frac{\partial \phi}{\partial \bar{X}} + R \frac{\partial \theta}{\partial \bar{X}} = |(1 + \tilde{s}_{11}^{(0)})|^2, \quad (19)$$

$$\frac{\partial \phi}{\partial \bar{Y}} + R \frac{\partial \theta}{\partial \bar{Y}} = |\tilde{s}_{12}^{(0)}|^2. \quad (20)$$

This leads to

$$Q_0 = -\frac{e}{2\pi} \int_0^{\frac{2\pi}{\omega}} \left(\left| \frac{\partial \ln s_{11}^{(0)}}{\partial \bar{X}} \right|^2 \frac{d\bar{X}}{dt} + T_0 \frac{d\bar{Y}}{dt} \right) dt. \quad (21)$$

Using the same method for Q , we obtain, in the presence of interactions

$$Q = Q_0 - \frac{e(l^{2\alpha} - 1)}{2\pi} \times \int_0^{\frac{2\pi}{\omega}} \left[\left(\text{Im} \left\{ \frac{\partial \ln s_{11}^{(0)}}{\partial \bar{X}} \right\} - \left| \frac{\partial \ln s_{11}^{(0)}}{\partial \bar{X}} \right|^2 \right) \frac{d\bar{X}}{dt} + \left(\text{Im} \left\{ \frac{\partial \ln s_{11}^{(0)}}{\partial \bar{Y}} \right\} - T_0 \right) \frac{d\bar{Y}}{dt} \right] \frac{T_0}{1 + T_0(l^{2\alpha} - 1)} dt. \quad (22)$$

D. Noise

Ref. 13 studied adiabatic quantum pumping in the context of scattering theory. Their goal was to derive under what conditions pumping could be achieved optimally, in a noiseless manner, with the assumption that the pumping frequency is small compared to the temperature. This enabled the authors to derive expressions not only for the pumped charge per cycle, but also for the pumped noise, the current-current correlation function, averaged over a time which is long compared to the period of the adiabatic drive, at zero frequency. Specifically, the noise is defined from the current-current time correlator:

$$S(t, t') = \frac{1}{2} \langle \delta I(t) \delta I(t') + \delta I(t') \delta I(t) \rangle, \quad (23)$$

with $\delta I = I - \langle I \rangle$. This correlator is then averaged over n periods of the pumping drive with n large, and it is taken at zero frequency by performing an integral over the remaining time argument. Setting $\tau_0 = n 2\pi/\omega$,

$$S(\Omega = 0) = \frac{\omega}{2\pi n} \int_0^{\tau_0} dt \int_0^{\infty} dt' S(t, t'). \quad (24)$$

Ref. 4 extended the results of Ref. 13 to the case where this limiting assumption is relaxed, yielding a complete description of the quantum statistical properties of an adiabatic quantum pump, albeit restricted to small pumping amplitudes. Results made use of the generalized emissivity matrix. These results were generalized to arbitrary pumping amplitudes by Ref. 14. Here, our goal is to address the question whether electron-electron interactions affect the pumping noise and how. We shall be primarily interested in the case where temperature is much lower than the pumping frequency. Note that it should not be too low otherwise, electron-electron interactions can no longer be treated perturbatively⁷. Following formula (14) of Ref. 14 and applying their Eq. (15) without assuming that the pumping amplitudes $X(t)$ are small, it is possible to put the zero frequency noise S for arbitrary pumping amplitudes into the form

$$S(\Omega = 0) = \frac{1}{(2\pi\hbar)^2} \frac{e^2}{\tau_0} \int_0^{\tau_0} dt \int_{-\infty}^{\infty} d(t' - t) \int_{-\infty}^{\infty} f(-\epsilon_1) \int_{-\infty}^{\infty} f(\epsilon_2) \times \text{Tr} \left[s(\epsilon_1, t)^{*T} \sigma_z s(\epsilon_2, t) s(\epsilon_2, t')^* \sigma_z s(\epsilon_1, t') - I \right] e^{i \frac{(t-t')(\epsilon_1 - \epsilon_2)}{\hbar}} d\epsilon_1 d\epsilon_2, \quad (25)$$

where $s(\epsilon_1, t)$ is the 2×2 S -matrix, for an incoming wave at energy $\epsilon_1 + \epsilon_F$ and value of the pumping parameter $X(t)$. τ_0 is a time which is much larger than the period τ . $\epsilon_1 = \hbar^2 k_1^2 / 2m - \epsilon_F$, where k_1 is a wave vector and ϵ_F is the Fermi energy $\epsilon_F = \hbar^2 k_F^2 / 2m$. f is the Fermi-Dirac function. σ_z is the usual Pauli matrix and I the identity two by two matrix. $*T$ is the sign for transposed and conjugate. If s were independent on time, we would have $s^{*T} = s^\dagger$. Here, $s(\epsilon_1, t)$ is taken for fixed t and one takes the transposed and complex conjugate of the matrix, without¹⁴ changing t into $-t$. From now on, we work at zero temperature, so that $f(-\epsilon_1) = 1$ for $\epsilon_1 > 0$ and 0 otherwise. We set $\epsilon'_2 = -\epsilon_2$ so that both ϵ_1 and ϵ'_2 will be positive. M is defined as

$$M(\epsilon_1, \epsilon'_2, t) = s(\epsilon_1, t)^{*T} \sigma_z s(-\epsilon'_2, t). \quad (26)$$

For zero temperature, S now reads

$$S = \frac{e^2}{(2\pi\hbar)^2} \lim_{\tau_0 \rightarrow \infty} \int_0^{\tau_0} \frac{dt}{\tau_0} \int_{-\infty}^{\infty} dt' \int_0^{\infty} d\epsilon_1 \int_0^{\infty} d\epsilon'_2 \text{Tr} \left\{ M(\epsilon_1, \epsilon'_2, t) M(\epsilon_1, \epsilon'_2, t')^* - I \right\} e^{-i \frac{(t-t')(\epsilon_1 + \epsilon'_2)}{\hbar}}. \quad (27)$$

Now, for large pumping amplitudes, the above formula needs to be rearranged, using the fact that $X(t)$ is a periodic function of period $2\pi/\omega$. Note that the dependence of M on ϵ_1 and ϵ_2 prevents the direct use of fast Fourier transform. Nevertheless, we can use the fact that, for given values of ϵ_1 and ϵ_2 , $M(t)$ and $M(t')$ are periodic functions of t . Switching to Fourier transform

$$\hat{M}_n(\epsilon_1, \epsilon_2) = \frac{\omega}{2\pi} \int_0^{\frac{2\pi}{\omega}} M(\epsilon_1, \epsilon_2, t) e^{-in\omega t} dt, \quad (28)$$

$$M(\epsilon_1, \epsilon_2, t) = \sum_{n=-\infty}^{+\infty} \hat{M}_n(\epsilon_1, \epsilon_2) e^{in\omega t}. \quad (29)$$

Performing the trace, we arrive at

$$S = \frac{e^2}{2\pi\hbar^2} \int_0^\infty d\epsilon_1 \int_0^\infty d\epsilon_2' \sum_{n=-\infty}^{\infty} \left(|\hat{M}_{1,1n}|^2 + 2|\hat{M}_{1,2n}|^2 + |\hat{M}_{2,2n}|^2 - 2\delta_{n,0} \right) \delta\left(\frac{\epsilon_1 + \epsilon_2'}{\hbar} - n\omega\right), \quad (30)$$

where $\delta_{n,0}$ is 1 if $n = 0$ and zero otherwise and $\hat{M}_{i,jn}$ is the (i, j) element of matrix of $\hat{M}_n(\epsilon_1, \epsilon_2')$, where energy dependences have been omitted to ease the notations.

When $\hbar\omega$ is much smaller than ϵ_F , formula (30) can be simplified. In this case, $M(\epsilon_1, \epsilon_2', t)$ will be different from $M(0, 0, t)$ only when ϵ_1 or ϵ_2' are a non negligible fraction of ϵ_F . This occurs because $M(0, 0, t)$ corresponds to matrix M for incident wave and outgoing wave at energy ϵ_F . We denote by ϵ_{1F} typical energies of the order ϵ_F . ϵ_{1F} will correspond to n of the order $(\epsilon_{1F}/\hbar\omega)$, which is very large. The Fourier transform $\hat{M}_n(\epsilon_1, \epsilon_2')$ will decrease exponentially with n for large n . Thus, we can neglect the dependence on ϵ_1 and ϵ_2' and replace them by zero, which amounts to replacing the energies by ϵ_F , except in the argument of the δ function. Under these conditions, we have

$$S \simeq e^2 \frac{\omega}{2\pi} \left[\sum_{n \geq 1} n \left(|\hat{M}_{11n}|^2 + 2|\hat{M}_{12n}|^2 + |\hat{M}_{22n}|^2 \right) \right]. \quad (31)$$

For numerical simulations however, we did not make this simplification and kept the dependence on ϵ_1 and ϵ_2' of Eq. (30).

III. DISCUSSION OF PHYSICAL RESULTS

We now illustrate these formula by computing the charge and noise. The two parameters of the drive X and Y (Eq.(9)) are chosen to vary periodically according to:

$$X = X_0 + \eta \cos(\omega t), \quad (32)$$

$$Y = Y_0 + \eta \cos(\omega t - \phi), \quad (33)$$

where X_0 is a constant offset potential and ϕ a phase difference. Note that X , X_0 and η are all dimensionless, see eq.(8). To ensure maximal pumping, we shall specialize¹⁰ to $\phi = \pi/2$.

A. Zero offset

First, the case without offset $X_0 = Y_0 = 0$ is studied. To look at the influence of interactions, we plot in Fig. 1 the pumped charge in units of e , with interactions and without, versus the amplitude of the drive η , for an interaction parameter $l^{2\alpha} = 0.3$ (moderate electron-electron interactions). There are three regimes: weak pumping, $\eta \ll 1$, intermediate pumping, η of order 1 and large pumping amplitudes, $\eta \gg 1$. The current noise times $2\pi/\omega$, in the limit of small ω , is plotted together on the same figure in units of e^2 . Analytically, for $\eta \ll 1$, Q reads

$$Q = \frac{e}{4} \sin(4k_F a) l^{-2\alpha} \eta^2. \quad (34)$$

As noted in Ref. 8, in the weak pumping regime, charge Q is larger with interactions by a factor $l^{-2\alpha}$, see Fig. 2. Results of Ref. 4 for the noise, valid for weak pumping and no interactions can be adapted in a straightforward fashion to the case with interactions. We find the following formula for the noise for weak pumping.

$$S = e^2 l^{-2\alpha} \eta^2 \frac{\omega}{2\pi}. \quad (35)$$

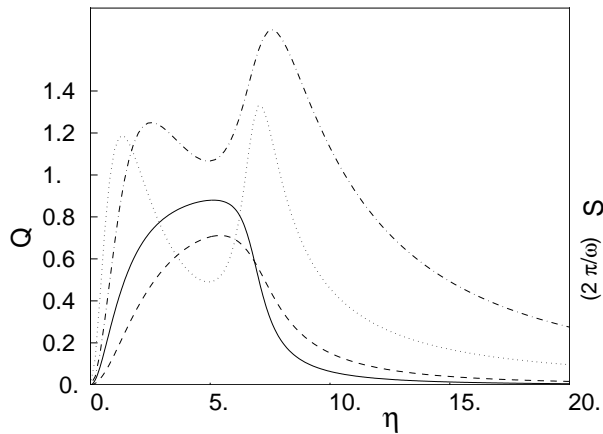


FIG. 1: Q , charge with interactions, (solid line), Q_0 , charge without interactions, (dashed line), both in units of e . S , noise with interactions, (dotted line) and S_0 without, (dashed dotted line), multiplied by $2\pi/\omega$, in units of e^2 , vs. η . Essential parameters are $X_0 = 0$, no offset, $l^{2\alpha} = 0.3$, $k_F a = 0.5$ and $\hbar\omega = 10^{-2}\epsilon_F$.

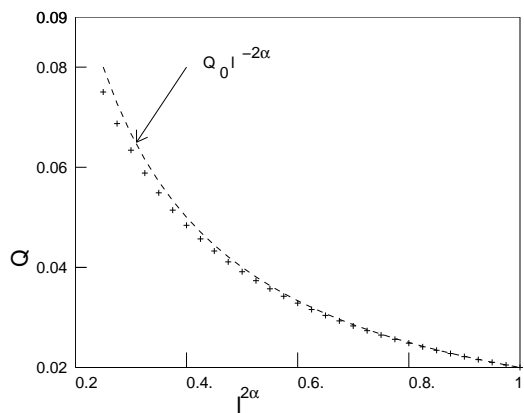


FIG. 2: Q , charge with interactions, (pluses), and $Q_0 l^{-2\alpha}$, (upper dashed line), both in units of e vs. $l^{2\alpha}$, for $\eta = 0.3$, $X_0 = 0$, $k_F a = 0.5$ and $\hbar\omega = 10^{-2}\epsilon_F$.

The noise is thus increased by the same factor as the current. The Fano factor, defined as the ratio $S/e\langle I \rangle$, is $4/\sin(4k_F a)$ and remains independent of the interactions, as long as we remain in the weak pumping regime. This corresponds to the very left part of Fig. 1, for η smaller than 0.25, typically.

At intermediate pumping amplitudes, Q reaches a maximum value Q_{max} which is again larger than its non-interacting analog Q_{0max} . This maximum is of the order of the single electron charge, but less than it. Meanwhile, the noise decreases. This is a reminder of the reduction of the noise by a factor $T(1-T)$, where T is the energy transmission coefficient for electrons in quantum wires. This explains why the noise exhibits a first maximum around η close to 1, since Q gets closer to one electron charge, noise will decrease. Then, for moderate amplitudes, η around 6, charge decreases and passes through the value $0.5e$, this corresponds then to the second maximum of the noise.

For large, but not very large pumping amplitudes, typically $\eta = 10$, Q remains smaller than Q_0 but behaves in the same way, namely as η^{-3} , as noted in Ref. 10. As a function of the interaction parameter, Q behaves as $Q_0 l^{2\alpha}$. See Fig. 3. For very large pumping amplitudes, (typically order 100 or more), Q becomes practically equal to Q_0 , $Q - Q_0$ behaves as η^{-4} , see appendix for details.

For the noise, we found numerically that S and its analog without interactions, S_0 , both decrease as η^{-2} , much slower than the charge. See below for analytical derivations. As concerns now the interaction dependence of the noise, S is always smaller than S_0 , but for very large η , S tends towards S_0 . More precisely, for large but still reasonable η , of the order 10 typically, S is almost equal to $S_0 l^{2\alpha}$, whereas for very large η , of the order 100 or more, S and S_0 are practically the same. This is not surprising since Q and Q_0 are then also practically equal in the end.

This dependence on the interaction parameter is shown in fig. 4. We have to plot $(2\pi/\omega) S \eta^2$, vs. $l^{2\alpha}$, but the overall factor $2\pi/\omega$ is unimportant; the product $S \eta^2$ can be compared to both $l^{2\alpha} S_0 \eta^2$ and to $S_0 \eta^2$ for $\eta = 15$ and $\eta = 200$. For $\eta = 15$, we see that $\eta^2 S$ is fairly well approximated by $\eta^2 S_0 l^{2\alpha}$. On the contrary, for $\eta = 200$, such a fit

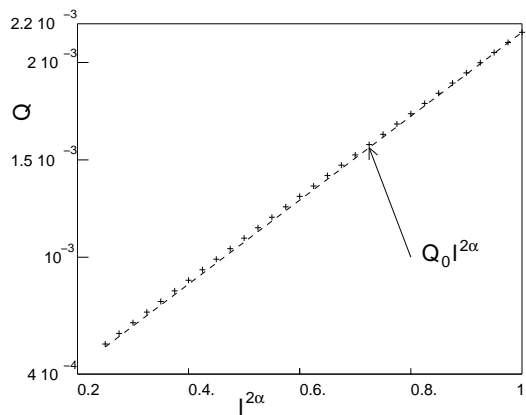


FIG. 3: Q , charge with interactions, (pluses), and $Q_0 l^{2\alpha}$, (lower dashed line), both in units of e , vs. $l^{2\alpha}$, for $\eta = 15$, $X_0 = 0$, $k_F a = 0.5$ and $\hbar\omega = 10^{-2}\epsilon_F$. In this regime, Q is approximately larger than Q_0 by a factor $l^{-2\alpha}$.

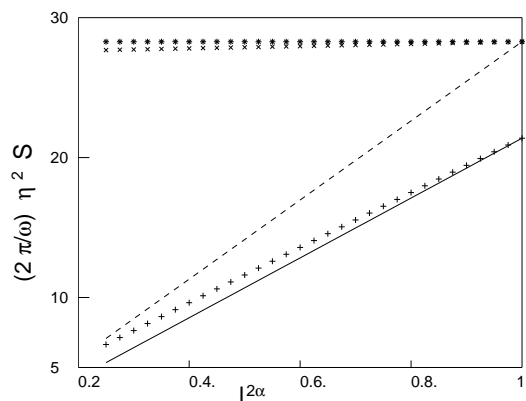


FIG. 4: $(2\pi/\omega)\eta^2 S$, noise with interactions scaled by η^2 and by the period, (+) and $(2\pi/\omega)\eta^2 S_0 l^{2\alpha}$, lower solid line, for moderate $\eta = 15$. Also illustrated by the top two curves, is the very large pumping amplitude regime, $(2\pi/\omega)\eta^2 S$ (x) and $(2\pi/\omega)\eta^2 S_0$ (*), for very large η ; $\eta = 100$. The top two curves are close to each other. An attempt to fit $\eta^2 S$ by $l^{2\alpha}\eta^2 S_0$ (dashed line) for $\eta = 100$, clearly fails for this regime of pumping amplitudes. $k_F a = 0.5$ and $\hbar\omega = 10^{-2}\epsilon_F$ for all cases.

fails and instead, $\eta^2 S$ is almost equal to $\eta^2 S_0$.

The results at large η can be derived from analytical formula for the charge and noise. An expansion for large η is performed, X and Y behave as η , except at particular points where X or Y are zero. See Appendix A for details.

B. Non-zero offset

We now turn to the case where X_0 is non-zero, which enables to have regions where Q is almost quantized. There are basically three cases, according to the value of $k_F a$.

The first case corresponds to $k_F a = n\pi/2$ (rigorously), where n is an integer. In this case, it is impossible to pump anything. The reason is given below. The second case corresponds to the case where $k_F a$ is small but non-zero, 0.1 typically. We first describe the behavior, then give numerical illustrations and last provide analytical justifications. In this case, the charge is almost zero up to $\eta = X_0$. It rises quickly around $\eta = \sqrt{2}X_0$ and reaches a value close to quantized e for a wide range of values of η . This is the quantized region of η . The width of this region can be shown to scale approximately as $(k_F a)^{-1}$. After the end of this region, Q and Q_0 first decrease abruptly and for even larger values of η , decrease slower, as η^{-3} . The noise in the quantized region and around it seems to be well approximated by $Q(1-Q)$, reminiscent of the noise for fermions in narrow quantum wires. However, this does not last when η becomes noticeably out of the region of almost quantized charge, since Q and Q_0 behave as η^{-3} , whereas S and S_0 decay only as η^{-2} .

Fig. 5 shows noise and charge with and without interactions, versus η for $k_F a = 0.1$. Fig. 6 shows a comparison

between S and a least square fit of the form $CQ(1-Q)$, in the quantized charge regime, where C is the only adjustable parameter. See captions for details. We now turn to analytical justifications of the previous assertions.

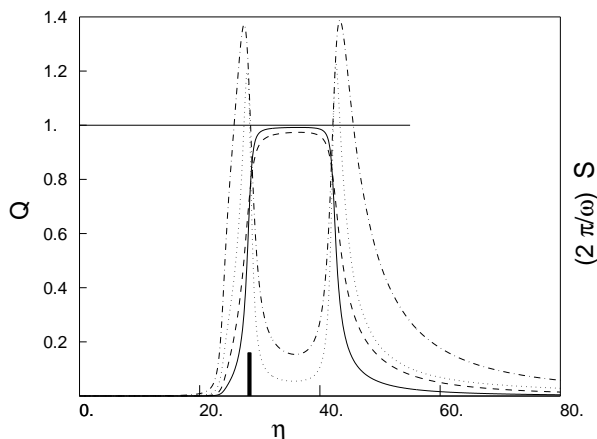


FIG. 5: Q , (solid line), curve with a plateau for η between 28 and 43, (the quantized region), Q_0 , (dashed line). The two curves with two spikes located at η around 28 and at η around 43 represent $(2\pi/\omega)S$, (dotted line) and $(2\pi/\omega)S_0$, (dashed dotted line), vs. η . Essential parameters are $X_0 = 20$, $l^{2\alpha} = 0.3$, $k_F a = 0.1$ and $\hbar\omega = 10^{-2}\epsilon_F$. Electron charge e is set to 1. The solid horizontal line of ordinate 1 and the thick vertical line at $\eta = X_0\sqrt{2} \simeq 28.28$ are guides to the eye.

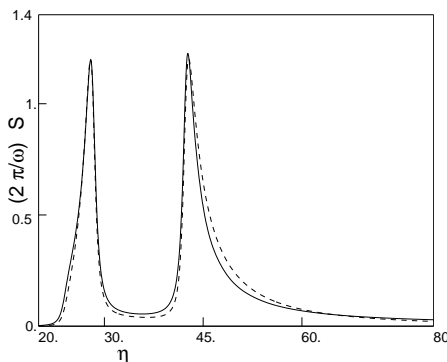


FIG. 6: S (solid line), in units of e^2 , and best fit of the form $CQ(1-Q)$ (dashed line) in the interval $20 \leq \eta \leq 45$, for the same situation as Fig. 5. The fit no longer works in the large pumping regime, for η between 45 and 80 here.

We now explain why pumping is impossible for $k_F a = n\pi/2$. Since $\sin(2k_F a) = 0$, the scattering matrix now depends only on a single parameter, the combination $(\overline{X} + \overline{Y})$, see eqs. (10-12), so we denote by $s_{ij}^{(0) \prime}$ its derivative with respect to $\overline{X} + \overline{Y}$. Thus,

$$Q_0 = \frac{e}{\pi} \int_0^{2\pi} \text{Im} \left[\left(\sum_{j=1}^2 s_{1j}^{(0) \prime}(\overline{X} + \overline{Y}) s_{1j}^{(0) *}(\overline{X} + \overline{Y}) \right) \right] \frac{d}{dt}(\overline{X} + \overline{Y}) dt = 0, \quad (36)$$

because the bracket is just $(1/2) d/d(\overline{X} + \overline{Y}) (|s_{11}^{(0)}|^2 + |s_{12}^{(0)}|^2)$. Then, we look at the case when $k_F a$ is close to $n\pi/2$, but different from it. Clearly, when $\sin(2k_F a)$ is small, for $\eta|\sin(k_F a)| < 1$, we will be back to the former case, so Q_0 can start to level noticeably from 0 only for η values larger than a critical value η_{c1} , which is proportional to $1/|\sin(2k_F a)|$, independently of X_0 . There is at least another scale, namely X_0 . For X_0 large, (typically larger than 10) and η smaller than X_0 , the pumping contour is a circle which does not enclose the origin and both transmission $s_{12}^{(0)}$ and derivatives of the reflexion coefficients, $\partial s_{11}^{(0)}/\partial X$ are small. Q_0 will remain very small. Thus, to have a significant Q_0 , one needs $\eta > \max(X_0, 1/|\sin(2k_F a)|)$, where $\max(x, y)$ is the larger of x and y . For even larger η , when terms like $\sin(2k_F a)\overline{X}\overline{Y}$ dominate over terms linear in \overline{X} or \overline{Y} , i.e. $(|\sin(2k_F a)|\eta) \gg 1$, it is possible to

expand in η^{-1} and we are back to the large pumping regime where Q decays as η^{-3} . So, for $|\sin(2k_F a)|$ smaller than X_0 , there will be a region between $\max(X_0, C_1/\sin(2k_F a))$ and $C_2/\sin(2k_F a)$, where C_1 and C_2 are constants, where Q_0 is appreciable. These very qualitative arguments however do not prove that the charge is almost quantized in this interval, whose width is of the order $|\sin(2k_F a)|^{-1}$.

The third case corresponds to $k_F a$ not close to $n\pi/2$. In this case, for large X_0 , Q is almost zero except in the vicinity of a value η_c , which is very near $\sqrt{2}X_0$; numerically it seems that η_c is always a little less than this value. The maximum pumped charge is of order e but no longer close to one electron charge. Noise has a double peak structure around η_c . A rough qualitative picture of this can be seen in Eq. (1) because, as soon as the integration contour does not get close to the point $X(t) = Y(t) = 0$, for any t , the integrand in Eq. (13) is very small. An example is shown in Fig 7.

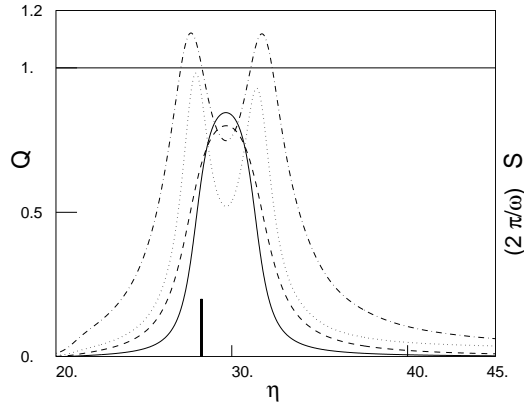


FIG. 7: Same as Fig. 5, except that $k_F a = 0.4$. The solid horizontal line of ordinate 1 and the thick vertical line at abscissa $X_0\sqrt{2}$ are just guides to the eye.

IV. CONCLUSION

We have studied the influence of weak electron-electron interactions on pumped charge and noise in the adiabatic regime, within the two delta potential barrier model. Analytical results were obtained for the charge and noise. Results were analyzed numerically for local pumping fields with a harmonic dependence. Without any voltage offset, at weak pumping amplitudes, interactions tend to enhance the pumped charge, as $l^{-2\alpha}$, where l is the interaction parameter. For fairly large pumping amplitudes, it is exactly the reverse, Q and Q_0 both decrease as η^{-3} , but Q remains smaller than Q_0 by a factor $l^{2\alpha}$. At very large pumping amplitudes, Q and Q_0 are practically the same. As to the pumping noise, at weak amplitudes, it increases with interactions, but in the same way as the charge, so that, the Fano factor remains constant, independent of the interactions. For moderate pumping amplitudes, noise has a double peak structure around the maximum of pumped charge. For large amplitudes, the noise decreases slower than the charge, as η^{-2} , and for very large amplitudes, noise with and without interactions become approximately the same.

As emphasised in Ref. 8, interactions tend to make resonances sharper, which is conducive to obtaining an almost quantized pumped charge. However, it is not sufficient to enclose a resonance, it is also necessary that the pumping contour does not go too far from the resonance. Otherwise, the noise is appreciable and the signal Q can even be very small.

In the case of constant offset X_0 , the behavior depends if we are close to a resonance, $k_F a = n\pi/2$ in this model. Close to a resonance, there is a region of almost quantized pumped charge where the noise seems to follow a $Q(1-Q)$ behavior, reminiscent of the noise reduction in quantum wires for good transmission by a $T(1-T)$ factor, where T is the modulus of the energy transmission coefficient. Interactions do help in having a charge closer to e and to reduce the noise. However, it does not change the range of pumping amplitudes, where quantized charge is observed, i.e. the width of the quantized region practically does not depend on the interactions. Qualitative arguments seem to indicate that this width scales as $\sin(2k_F a)^{-1}$ close to $k_F a = n\pi/2$. Further from the resonance, the maximum charge which can be pumped becomes of order, but less than e . Moreover, the region of quantized charge shrinks to a narrow window of pumping amplitudes around a value close to $X_0\sqrt{2}$.

In summary, our study of noise shows that interactions tend to increase the quality of pumping. However, two conditions need to be met; first, to operate at certain wavevectors and second to have a pumping contour which

encircles the resonant point, passing not very far from it. Otherwise, only noise is produced and the quantization of the charge is not achieved. These restrictions were not pointed out in previous works.c

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APPENDIX A

In this appendix, we consider the limit of large pumping. In order to explain why, for offset $X_0 = 0$, Q_0 behaves like η^{-3} at large η , we use Eq. (21). Let us look first at the terms involving $d\bar{X}/dt$.

\bar{X} , \bar{Y} , as well as their time derivatives will be of order η , except at isolated particular points. The term $|\partial \ln s_{11}^{(0)}/\partial \bar{X}|^2$ can be expanded in powers of $1/\bar{X}$ and $1/\bar{Y}$:

$$\left| \frac{\partial \ln s_{11}^{(0)}}{\partial \bar{X}} \right|^2 = \frac{1}{\bar{X}^2} - \frac{\cotan(2k_F a)}{\bar{X}^3} + \frac{3 \cotan^2(2k_F a) - 1}{4\bar{X}^4} - \frac{1 + \cotan^2(2k_F a)}{4\bar{X}^2 \bar{Y}^2} + \frac{\cotan^2(2k_F a) - 1}{2\bar{X}^3 \bar{Y}} + o(\eta^{-5}). \quad (A1)$$

All terms multiplied by $d\bar{X}/dt$ and then, integrated over one period give zero. Note that a term like $1/(\bar{X}^4 \bar{Y})$, which is in η^{-5} , would not give 0. Thus, for large η , the term proportionnal to $d\bar{X}/dt$ in Q , behaves at least as η^{-3} .

For the term involving $d\bar{Y}/dt$, the situation is simpler. $d\bar{Y}/dt$ goes as η , but since D goes as η^2 , T_0 goes as η^{-4} , thus this term is at least in η^{-3} .

Now the remainder of contributions to $Q - Q_0$ behave at least like η^{-4} for large η ; it can be seen from (22). For large η , since T_0 goes as η^{-4} , the quantity $T_0/(1 + T_0(A^{2\alpha} - 1))$ is practically equivalent to $T_0 \sim \eta^{-4}$. $Im\{\partial \ln s_{11}^{(0)}/\partial \bar{X}\}$ goes as η^{-1} for large η . The same holds for $Im\{\partial \ln s_{11}^{(0)}/\partial \bar{Y}\}$. In the integral in the r.h.s. of (22), $Im\{\partial \ln s_{11}^{(0)}/\partial \bar{X}\}$ is larger, by η , than $|\partial \ln s_{11}^{(0)}/\partial \bar{X}|^2$. Then, $Im\{\partial \ln s_{11}^{(0)}/\partial \bar{X}\}$ is of order η^{-1} whereas T_0 is of order η^{-4} . Finally, the integrand is at least of order $Im\{\partial \ln s_{11}^{(0)}/\partial \bar{X}\}(d\bar{X}/dt)T_0$, or $Im\{\partial \ln s_{11}^{(0)}/\partial \bar{Y}\}(d\bar{Y}/dt)T_0$, which are both at least of order $\eta^{-1} \times \eta \times \eta^{-4} \sim \eta^{-4}$.

We now evaluate the behavior of the noise for large η . When $\omega \ll \epsilon_F$, only the low order Fourier components of $\eta(t)$ are important. ϵ_1 and ϵ'_2 will be much smaller than ϵ_F . At $\epsilon_1 = \epsilon'_2 = 0$, $s_{11}^{(0)} \simeq -e^{-2ik_F a}(1 + i/\bar{X} + o(\eta^{-2}))$ and $s_{22}^{(0)}$ is the same but \bar{X} is replaced by \bar{Y} . $s_{12}^{(0)} = -(1/2\bar{X}\bar{Y})(1 + i \cotan(2k_F a)) + o(\eta^{-3})$. As a result, in Eq. (30), $s(\epsilon_1, t)^* \sigma_z s(\epsilon_2, t) = \sigma_z + o(\eta^{-2})$, the same holds for $s(\epsilon_2, t')^* \sigma_z s(\epsilon_1, t')$, so that the trace is $o(\eta^{-2})$, which yields S of order η^{-2} , at least.

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