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Influence of global rotation and Reynolds number on the large-scale features of a turbulent Taylor-Couette flow.

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We study the torque scaling and the angular momentum transport mechanisms in a turbulent flow between two independently rotating cylinders. Depending on the angular velocities ratio, we identify two different states: one with large scale vortical structures in the secondary mean flow, and the other one with no large scale features. At constant shear, varying the angular velocities ratio, we then study the transition between these two states. This transition is smooth and continuous. It resembles a supercritical bifurcation of the secondary mean flow on a turbulent flow.

Introduction. Turbulent flows are present in many applied and fundamental problems, ranging from small scales (engines, mixing) to very large scales (meteorology, convection in the earth mantle, and other astrophysical problems such as momentum transport in accretion disks). One of the several open questions is the emergence of coherent structures in turbulent flows [1]. Another interesting problem concerns the bifurcations that can arise in turbulent regime, for instance the dynamo instability of a magnetic field in a conducting fluid [2], or multistability of the mean flow in von Kármán or Taylor-Couette flows [3, 4].

The Taylor-Couette flow is the flow produced between differentially rotating coaxial cylinders [5]. When only the inner cylinder rotates, the first instability takes the form of toroidal (Taylor) vortices, and with two independent cylinders, there are a host of interesting secondary bifurcations, extensively studied in the last decades [6, 7]. Moreover, it shares strong analogies with Rayleigh-Bénard convection [8, 9] which are useful to explain different torque scalings at high Reynolds numbers [10]. Finally, for some parameters relevant in astrophysical problems the basic flow is linearly stable and can directly transit to turbulence [11].

The turbulent flow structure is not so well known and few measurements are available [12]. In the case of inner cylinder rotating alone, recent direct numerical simulations suggest that vortices-like structures still exist at high Reynolds number ($Re \gtrsim 10^4$) [13, 14], whereas for counterrotating cylinders, the turbulent flow is supposed to be featureless [7]. We address hereafter the question of the transition between a Turbulent Taylor-Vortex Flow and this featureless turbulent flow when varying the global rotation for a constant shear.

Experimental setup. The flow is produced between two coaxial cylinders (see Fig. 1). The inner one is of radius $r_i = 110 \pm 0.05$ mm, and the outer one is of radius $r_o = 120 \pm 0.05$ mm, which gives a gap $d = r_o - r_i = 10$ mm and a gap ratio $\eta = r_i/r_o = 0.917$. Both cylinders can rotate independently. The system is closed at both ends. Top and bottom lids rotate with the outer cylinder. The length of the inner cylinder is $L = 220$ mm, i.e. the axial

aspect ratio is $L/d = 22$. The cylinders are driven by two independent Brushless DC motors (Maxon, 250W), at a rotation rate up to 10Hz. The torque T exerted on the inner cylinder is measured with a rotating torque-meter (HBM T20WN, 2N.m).

For a newtonian fluid of kinematic viscosity ν , with the cylinders rotating at an angular speed of $\omega_{i,o}$, we use the set of parameters defined by Dubrulle *et al.* [15]: a Reynolds number $Re = 2/(1+\eta) |\eta Re_o - Re_i|$ and a “Rotation number” $Ro = (1-\eta) (Re_i + Re_o)/(\eta Re_o - Re_i)$, where $Re_{i,o} = (r_{i,o}\omega_{i,o}d/\nu)$ are the inner and outer Reynolds numbers. With this choice, Re is based on the shear, and can be linked to the Taylor number used by Eckhardt *et al.* [9]: $Ta = \sigma^2 Re^2$, with σ a geometric parameter. At 10Hz in counterrotation, the shear rate is around 1400s^{-1} and $Re \simeq 1.4 \times 10^5$ for pure water. The Rotation number compares the mean rotation to the shear and is the inverse of a Rossby number. The sign of the shear compared to the sign of the angular velocity defines cyclonic (resp. anti-cyclonic) flows $Ro > 0$ (resp. $Ro < 0$). Rotation number is zero in case of perfect counterrotation ($r_i\omega_i = -r_o\omega_o$). Two other relevant values are $Ro_i = \eta - 1 \simeq -0.083$ (resp. $Ro_o = (1-\eta)/\eta \simeq 0.091$) for inner (resp. outer) cylinder rotating alone. Finally, let us recall that the flow is linearly unstable for $-1 < Ro \lesssim Ro_o$ [16].

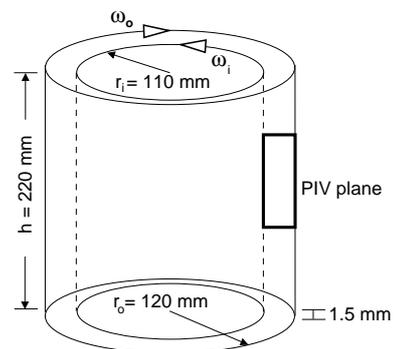


FIG. 1: Sketch of the experimental setup.

We measure the three components of the velocity by stereoscopic PIV in a vertical plane normal to the mean flow (see Fig. 1). The light-sheet thickness is 0.5mm. It is

imaged using two double-frame PCO-cameras, mounted on Scheimpflug adaptors, located at each side of the light sheet. The time between two pulses is $100\mu\text{s}$. The tracer particles are $20\mu\text{m}$ spheres coated with rhodamine. The field of view measures $11\text{mm} \times 25\text{mm}$, corresponding to a resolution of 320×1024 pixels. Special care has been taken concerning the calibration procedure. We use a thin transparent grid, attached to a rotating and translating micro-traverse. The PIV-images are processed with DaVis 7.2 with a final interrogation area of 32×32 pixels (50% overlap), and normalised median filtering as post-processing.

Scaling of the torque. We present in Fig. 2 the dimensionless torque G/Re^2 where $G = T/(\rho L\nu^2)$ as a function of Re for three particular Rotation numbers, corresponding to inner cylinder rotating alone, exact counterrotation and outer cylinder rotating alone. At low Re , the three curves collapse on a Re^{-1} curve. This characterizes the laminar regime where the torque is proportional to the shear on which the Reynolds number is based.

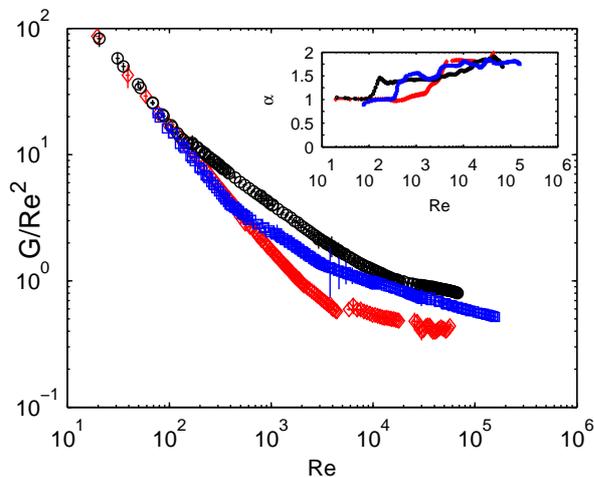


FIG. 2: Dimensionless torque G/Re^2 vs. Re for $Ro_i = \eta - 1$ (black \circ), $Ro_c = 0$ (blue \square) and $Ro_o = (1 - \eta)/\eta$ (red \diamond). Relative error on Re : $\pm 5\%$, absolute error on torque: $\pm 0.02\text{N.m}$. Inset: local exponent α such that $G \propto Re^\alpha$, computed as $d\log(G)/d\log(Re)$, for $Ro_i = \eta - 1$ (black), $Ro_c = 0$ (blue) and $Ro_o = (1 - \eta)/\eta$ (red).

For $Ro_i = \eta - 1$, one can then notice a transition to a different regime at $Re_{ci} \simeq 140$. This corresponds to the first instability, and is very close to the theoretical threshold of $Re = 150$ [16]. Laminar Taylor vortices grow, and the dimensionless torque is then supposed to scale as $Re^{-1/2}$ ($\alpha = 3/2$), which is the case here (see Inset in Fig. 2). For exact counterrotation ($Ro_c = 0$), the first instability threshold is $Re_{cc} \simeq 400$. It is a little bit lower than the theoretical prediction $Re = 515$ [16]; which is probably due to our finite aspect-ratio. The Taylor-Couette flow with outer cylinder rotating alone ($Ro_o = (1 - \eta)/\eta$) is linearly stable whatever Re . The experimental flow is still laminar at $Re \simeq 4000$.

Increasing further the Reynolds number, the local exponent increases (see Inset in Fig. 2). For $Ro_i = \eta - 1$, it varies between $\alpha \simeq 1.5$ at $Re \simeq 200$ and $\alpha \simeq 1.8$ at $Re \simeq 10^5$. The order of magnitude of these values agree with the results of Lathrop *et al* [10], though a direct comparison is difficult, owing to the different gap ratio of the experiments. The bigger the gap ratio is, the closer to 2 the local exponent is supposed to be. Dubrulle and Hersant [8] attribute the increase of α to logarithmic corrections, whereas Eckhardt *et al* [9] attribute the increase of α to a balance between a boundary-layer/hairpin contribution in $Re^{3/2}$ and a bulk contribution in Re^2 . The case of perfect counterrotation shows a plateau at $\alpha \simeq 1.5$ and a sharp increase of the local exponent to $\alpha \simeq 1.75$ at $Re_{tc} \simeq 3200$, possibly tracing back a secondary transition. The local exponent then seems to increase gradually to the asymptotically expected $\alpha = 2$ value. Finally, for $Ro_o = (1 - \eta)/\eta$, the flow directly transit from a laminar state to a turbulent state with a local exponent around $\alpha = 1.77$ at $4000 \lesssim Re_{to} \simeq 5000$. Please note that the dimensional values of the torque when only the outer cylinder rotates are very small and difficult to measure accurately.

Owing to these different first thresholds and to the fact that the local exponents in “turbulent regimes” ($Re \geq 10^4$) are then equal within ± 0.1 , the torque with inner cylinder rotating alone is greater than the torque in counterrotation, this latter being greater than the torque for outer cylinder alone. We now address the question of the transition between these different states at a constant Reynolds number.

Mean flow bifurcation. The presence of vortices-like structures at high Reynolds number ($Re \gtrsim 10^4$) in turbulent Taylor-Couette flow with inner cylinder rotating alone is confirmed in our experiment through stereoscopic PIV measurements [17]. The time-averaged flow show strong secondary mean flow in the shape of vortices. The time-averaged angular velocity profile is almost flat and the angular momentum transport is then mainly ensured by these coherent structures: their contribution corresponds to 75% of the total transport, the remaining 25% being done by correlated fluctuations. On the contrary, in the case of perfect counterrotation, there is no mean secondary flow and the angular momentum is completely transported by highly fluctuating structures more similar to plumes. In that case, the correlation coefficient between radial and azimuthal fluctuations is 0.4 [17].

To characterize the transition between these two regimes, we first look at global torque measurements. We plot in Fig. 3 the dimensionless torque normalised by the torque for inner cylinder rotating alone as a function of the Rotation number, for a constant Reynolds number in the turbulent regimes ($Re = 1.4 \times 10^4$). We first recover the hierarchy of torques: torque in counterrotation ($Ro_c = 0$) is approximately 80% of $G(Ro_i = \eta - 1)$ and the torque with outer cylinder rotating alone

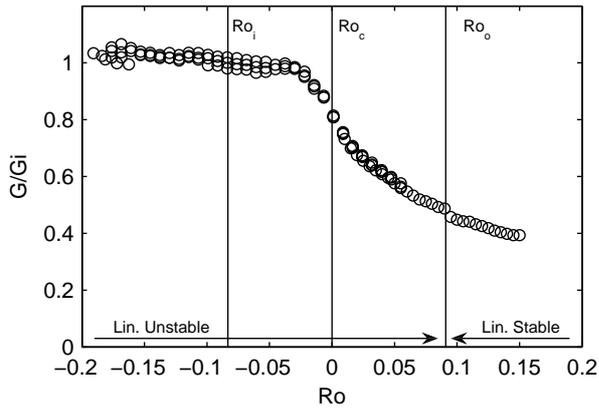


FIG. 3: Dimensionless torque G normalised by $G_i = G(Ro = \eta - 1)$ vs. Ro at a constant Reynolds number $Re = 1.4 \times 10^4$.

($Ro_o = (1 - \eta)/\eta$) is approximately 50% of it. These values compare well with the few available data compiled by Dubrulle *et al* [15]. The curve shows a plateau of constant torque from $Ro = -0.2$, *i.e.* when both cylinders rotate in the same direction, the inner cylinder rotating faster than the outer cylinder, to $Ro \simeq -0.035$, *i.e.* with a little bit of counterrotation, the inner cylinder still rotating faster than the outer cylinder. The torque then monotonically decreases when accelerating the outer cylinder. One can also notice an inflexion at $Ro_c = 0$. The transition is continuous and smooth and the effect of rotating the outer cylinder starts to be seen on the torque very close to perfect counterrotation.

We now are going to look at the changes in the mean flow. To extract quantitative data from the PIV measurements, we use the following model for the stream function: $\Psi = \sin(\pi(r - r_i)/d) \times (A_1 \sin(\pi(z - z_0)/l) + A_3 \sin(3\pi(z - z_0)/l))$, with free parameters A_1 , A_3 , l and z_0 . This corresponds to alternate rolls, with a wavelength of $2l$ and a typical velocity given by $\pi(A_1/l + 3A_3/l)$. It is supposed implicitly that flow is integrated sufficiently to restore the axisymmetry, which is *a posteriori* checked. We have first tried a single mode in axial direction, but it appears that using also the third harmonic does improve the matching between the model and the actual velocity fields, especially close to $Ro_i = \eta - 1$ (see Fig. 4).

We perform several measurements at different Ro , with a constant shear-based Reynolds number $Re = 1.4 \times 10^4$, corresponding to a velocity of 2Hz for inner cylinder rotating alone. We first give an example of a PIV data set analysis with a fixed outer cylinder and an inner cylinder started from rest and accelerated to 2Hz in 20s. A sequence of 4000 PIV images at a data rate of 3.7Hz is taken and 20 consecutive PIV images, *i.e.* approximately 11 cylinder revolutions are sufficient to obtain a good estimate of the mean flow. The length scale of the vortices for $Ro_i = \eta - 1$ is about 1.2 gap width, which is consistent with data from Bilson *et al.* [14]. On this peculiar example, one can notice that strong vortices grow very fast,

reach a first value at a strength of 0.08 m.s^{-1} , and then decay to get stabilised at a value around 0.074 m.s^{-1} after 400 seconds. It is known that for the first transitions, the observed flow state can depend on the starting conditions [6]. In the present case, for a slower acceleration than the given example, the first vortices are less strong and have a bigger length scale, before reaching the same final state. Transients are thus also very long in turbulent Taylor-Vortex flows for inner cylinder rotating alone.

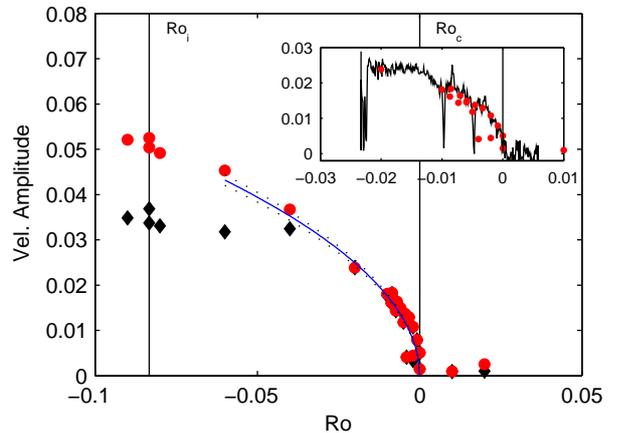


FIG. 4: Secondary flow amplitude *vs.* Rotation number, at constant shear. Black (\diamond): model with fundamental mode only and red (\circ): complete model with third harmonic. Solid line is a fit of the form $A = a(-Ro)^{1/2}$. Inset: zoom close to counterrotation, superimposed with a continuous experiment (see text).

In the following experiment, where we start from $\{Ro_i = \eta - 1; Re = 1.4 \times 10^4\}$ and vary the Rotation number step by step, keeping a constant shear, we therefore let the system spend 20 minutes in each state before acquiring PIV data sets. We then check that the fit parameters are stationary, and compute them on the average of the whole PIV data set for each Ro . The results are reported in Fig. 4. Please note that Ro has been varied both with increasing and decreasing values, to check a possible hysteresis.

All the points fall on a single curve: the transition is smooth and non-hysteretic. For $Ro \geq 0$, the fitted modes have zero amplitude, as expected [17]. One can notice then that as soon as $Ro < 0$, *i.e.* as soon as the inner cylinder starts to rotate faster than the outer cylinder, vortices start to grow. We plot in Fig. 4 the velocity strengths associated with the simple model (only one mode, \diamond) and with the complete model (modes 1 and 3, \circ). Very close to $Ro = 0$, the two models coincide: $A_3 \simeq 0$ and the secondary mean flow is well described by pure sinusoidal structures. For $Ro \lesssim -0.04$, the vortices starts to have elongated shapes, with large cores and small regions of large radial motions inbetween adjacent vortices: the third mode is then necessary to describe well the secondary flow. The first mode gets saturated in this

region. Finally, we give in Fig. 4 a fit of the amplitudes close to $Ro = 0$ of the form: $A = a(-Ro)^{1/2}$. The amplitude of the vortex behaves like the square root of the distance to $Ro = 0$, a situation reminiscent to a classical supercritical bifurcation, with A as order parameter, and Ro as control parameter.

We also performed a continuous experiment, varying the Rotation number from $Ro = 0.004$ to $Ro = -0.0250$ in 3000s. The experiment is thus quasistatic. The amplitude of the mean secondary flow, computed on sequences of 20 images is plotted in the inset of Fig. 4. The curve follows the single points experiments, but some downward peaks can be noticed: we have checked that they are not a fitting error. They indeed correspond to disappearance of the vortices from times to times. They could correspond to Eckhaus instability of the pattern wavelength. Still, the measurements are done at a fixed position in space. Though the very long time-averaged series lead to well stationary axisymmetric states, it is possible that the instantaneous whole flow consists of different regions. Further investigations including time-resolved single-point measurements or flow visualizations need to be done to check this possibility.

Conclusion The rotation has some effects on the torque scaling. Whereas the local exponent evolves in a smooth way for inner cylinder alone, the counterrotating case exhibits two sharp transitions, from $\alpha = 1$ to $\alpha \simeq 1.5$ and then to $\alpha \simeq 1.75$. We also notice that the second transition for counterrotation Re_{tc} is close to the threshold Re_{to} of turbulence onset for outer cylinder rotating alone. A way to analyse the flow is to decomposed it into two regions, dominated by each cylinder and separated by a neutral surface [16]. For $Ro_c = 0$, we could thus infer that at low Reynolds number, the inner zone dominates, and at higher Re , the outer zone dominates.

The Rotation number is thus a secondary control parameter. It is very tempting to use the classical formalism of bifurcations and instabilities to study the transition between featureless turbulence and Turbulent Taylor-Vortex Flow at constant Re , which seems to be supercritical: the threshold for the onset of coherent structures in the mean flow is $Ro_c = 0$. For anticyclonic flows ($Ro < 0$), the transport is dominated by large scale coherent structures, whereas for cyclonic flows ($Ro > 0$), it is dominated by correlated fluctuations reminiscent to thermal plumes, with no viscous contribution at all for $Ro \leq 0$ but a few 10% at Ro_o [17]. Counterrotation $Ro_c = 0$ corresponds to an inflexion point in the torque curve. The perfect counterrotating case thus seems to be a peculiar boundary for the turbulent Taylor-Couette flow. One last remark is that Rotation number $Ro = 0$ is also the threshold for linear instability for rotating Plane Couette Flow, which exactly corresponds to the limit of a Taylor-Couette flow with vanishing curvature [18].

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