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Interval Arithmetic for the Design of Electrical Engineering Systems

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Abstract. Electrical engineering system design requires more and more light models like analytical ones. Using interval arithmetic calculation on algebraic formulas brings several benefits like stability analysis, tolerance design or global optimization. Software based on these techniques have been developed to bring these functionalities to designer hands, especially for preliminary stages of design.

Keywords: Interval arithmetic, formulas stability, tolerance design, global optimization.

I. INTRODUCTION

Analytical models in the field of electrical engineering have been developed since years. These models are more and more used especially for system design needs, multiphysics needs, quick exploration of inverse problem solution space, etc. Based on algebraic formulas, interval arithmetic computes interval instead of real number. For instance, the equation $y=a.x+b$ can be computed as follow: $a=[0.1;0.2]$, $b=[-1;1]$ and $x=[10; 20]$ (*interval=[inferior bound ; superior bound]*)

Then $y = [0.1;0.2] \times [10; 20] + [-1;1] = [1; 4] + [-1;1] = [0; 5] = y$

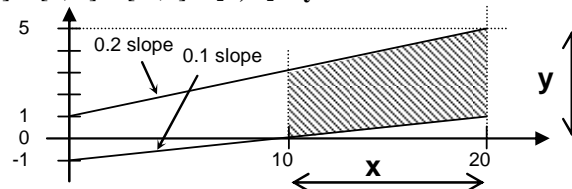


Fig. 1 – Example of interval arithmetic on $y=a.x+b$ equation.

Intervals are important information for preliminary design studies as it will be shown in this paper. As detailed in [1], interval analysis cannot be used for accurate modelling, but it is very useful for high level design with behavioural model.

II SOFTWARE ARCHITECTURE

Our design framework is based on software component architecture. A component represents a part of the system to design or all the system thanks to composition software [2]. Two kind of tools are then available, generators who create components (Fig.2-a) and tools which bring services to components (Fig.2-c) (*like optimization framework*).

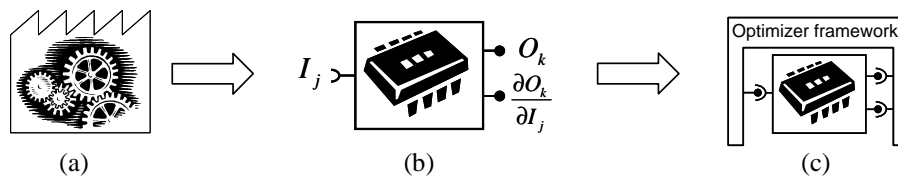


Fig. 2 – (a) Generators create (b) components / (c) Services use (b) components.

An interval arithmetic generator has been developed using BIAS library [3] (*Basic Interval Arithmetic Subroutines*). BIAS uses code differentiation to produce interval gradients (Fig.2-b), necessary for our optimization needs. The generator translates algebraic equations of the device model to a component dedicated to optimization (*which computes equations propagating intervals as well as formally exact sensitivities*).

A calculator is available to compute a model instance or to plot parameters. An optimizer is also available with several algorithms like SQP (*sequential quadratic programming*), genetic, hybrid, global, or mixed discret-continuous.

Three uses of this generator tool for design needs are presented in the following three parts.

III. MODEL STABILITY

First of all, our generator is helpful to detect computation rounding errors. Indeed, computer rounds number when floating point values (*ANSI/IEEE Standard 754-1985, Standard for Binary Floating Point Arithmetic*) are propagated through the model. When modeling a system with huge different value orders (*like electromagnetic compatibility study in electromechanical converters*), there can be instable formulae. Thanks to interval analysis, which adapts computer rounding method (*upper for superior bound and lower for inferior bound*), instability can be observed. Since it is possible to stabilize formulae by reordering numbers in the equations, interval computing is an interesting tool to debug models.

For example ¹, the classical formula (1) due to Heron of Alexandria which computes the area of a needle like triangle is unstable. It can be stabilized by writing equation (2).

¹ <http://www.cs.berkeley.edu/~wkahan/Triangle.pdf>

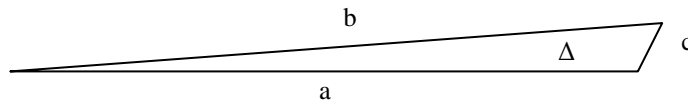


Fig. 3 – Needle like triangle.

$$\Delta_1 = \sqrt{s(s-a)(s-b)(s-c)} \quad \text{with} \quad s = \frac{1}{2}(a+b+c) \quad (1)$$

$$\Delta_2 = \frac{1}{4} \sqrt{(a+(b+c))(c-(a-b))(c+(a-b))(a+(b-c))} \quad (2)$$

$a=10^{14}$, $b=10^{14}$ and $c=1.1$ gives Δ_1 with $\pm 1.4\%$ interval radius error, against $\pm 7.10^{-14}\%$ for Δ_2 . It highlights possible instability for engineering models, but also the fact that variables order writing is important, as well as parenthesis! Full paper will show on engineering models some practical writing formula hints to decrease overestimation of interval.

IV. TOLERANCE DESIGN

Input parameters of a model are not accurate but attached to a precision then defined as intervals. Outputs are also intervals thought the intervals propagation mechanism, and can be constrained. As an example, a transformer optimization is presented here. Analytical model [4] was turned by our generator into an autonomous code which was plugged into the optimization tools and connected to an optimization algorithm (sequential quadratic programming). Several inputs were fixed with a precision (*left part of Table 1*) and others were optimizable (*right part of Table 1*).

TABLE 1: FIXED INPUTS WITH THEIR INCERTITUDE AND OPTIMIZABLE INPUTS WITH THEIR BOUNDS

	value	precision	unit		min	max	result	unit
frequency	50	5%	Hz	induction	0.5	1.9	1,69	T
apparent power	40	5%	MVA	current density	500	4500	2020	kA/mm ²
supply tension	60	5%	kV	winding turns (± 1)	100	800	573	-
				transformer height	1	10	3.43	m

Note that winding turns was also defined as an interval of radius 1 to express its integer type. Several outputs parameters were constrained (*total transformer cost, leakage reactance and total length*) as show optimization results in Fig 4.

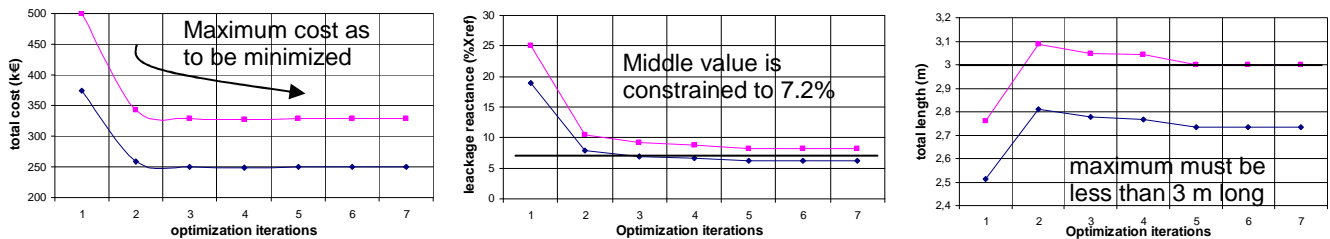


Fig. 4 – interval results (inf/sup) of optimization

V. GLOBAL OPTIMISATION

Third use of interval arithmetic is global optimization [5]. A simple algorithm using interval arithmetic was implemented into our framework. It is based on branch and bound technique using gradient sign of objective interval for its upper and lower bound. Efficient algorithms are available (*GlobSol*²...) and many literatures are also available³.

To illustrate rigorous global optimization of interval based algorithm, the Levy function [6] as been defined as a global minimization test (*780 local minima and 1 global minimum*). Interval algorithm converged successfully contrary to another “global” optimizer (DIRECT [7]) available in our framework, which is efficient but not rigorous as interval arithmetic.

VI. CONCLUSION

This paper shows how an interval arithmetic model generator (detailed in full paper), producing calculus component as well as sensitivity computation, can be used for the design of engineering systems. It is especially used for formula stability debugging, tolerated design, and global optimization (more detailed and illustrated in full paper).

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² http://interval.louisiana.edu/GLOBOSOL/what_is.html

³ <http://www.mat.univie.ac.at/~neum/glopt.html>