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Degree of polarization of thermal light emitted by gratings supporting surface waves

F. Marquier, C. Arnold, M. Laroche, J.J. Greffet
École Centrale Paris, Laboratoire d'Énergétique Moléculaire et Macroscopique;
Combustion, Centre National de la Recherche Scientifique, Grande Voie des
Vignes, 92295 Châtenay-Malabry Cedex, France
francois.marquier@em2c.ecp.fr

Y. Chen

École Normale Supérieure de Paris, Laboratoire de microfluidique,
organisation chimique et nanotechnologies, Centre National de la Recherche
Scientifique, 45, rue d'Ulm, F-75230 Paris cedex 05

Abstract

Absorption and emission of light due to the resonant excitation of surface waves on a grating is a well-known phenomenon. We report the first complete study of the influence of the role of angle and polarization on thermal emission by lamellar gratings. We derive the emitted Stokes vectors in any direction. We find that a source can be quasi isotropic from the point of view of the intensity but strongly anisotropic for polarized light. It follows that the degree of polarization can vary between 0 and 1, depending on directions.

1 Introduction

The possibility of engineering the thermal emission properties of materials by taking advantage of surface resonances has been demonstrated in the early nineties using doped silicon gratings [1]. The authors reported important modifications of the emissivity depending on the surface of the material and also on the polarization of the detected wave. Further experiments taking advantage of surface plasmons have been performed on a gold grating for which thermal emission has been shown to be directional at a given wavelength [2]. Actually, it has been known since the seventies that 100% absorption can be obtained with a metallic grating [3, 4] for given angle of incidence and wavelength corresponding to the resonant excitation of a surface plasmon. According to Kirchhoff's law [5], the emissivity is then 100%: this has been observed in ref [2, 6, 7]. To be more general, total absorption or emission can be obtained when coupling a surface wave to a propagating wave in free space. Such a surface wave can be either a surface plasmon-polariton for metals and doped semiconductors or a surface phonon-polariton for polar materials, or a surface wave along a truncated photonic crystal [8]. Experiments using infrared surface phonon polaritons propagating along SiC gratings were reported in ref. [9–11]. The authors showed that a SiC grating can emit very narrow peaks for a given wavelength: the grating behaves like a thermal antenna. It was stressed that this property is a consequence of the spatial coherence of the fields in the source due to the

presence of the surface wave. It has been shown that directional emissivity is related to the propagation length of the surface wave which appears as a spatial coherence length. Therefore this effect has been called coherent thermal emission [10, 11].

Many other sources have been proposed to achieve coherent thermal emission using periodic structures, cavities [12, 13] or geometrical resonances [14]. It has been shown that a SiC wafer surrounded by a planar multilayer structure exhibits strong directional emission peaks [15, 16]. In such a system, evanescent waves exist in the stop band of the photonic crystal, which can be coupled to surface phonon polariton. Hence, it is possible to couple surface waves to propagating waves and generate directional thermal emission. Multilayers structures can also be used as Bragg's mirrors to create a resonant cavity and directional emission [12]. Battula and Chen working with both cavity and surface plasmons modes also obtain very directional thermal emission [13]. Waveguide modes can also produce directional emission as demonstrated in ref. [17]. The application to spectrally selective surfaces has been considered by some authors [18–22]. In the previous papers, only one-dimensional structures (1D-photonic crystal or lamellar gratings) were used [9–11, 23]. Two dimensional periodic structures have also been used to couple propagating waves to a resonant mode [22, 24–26]. The emission properties of 3D-photonic crystals have been studied since 1999 by several authors [7, 27–32].

A key feature of thermal emission assisted by surface waves is that it is p-polarized in the plane perpendicular to the lines of the grating. Few studies have dealt with the polarization of the thermal emission. Yet, it has been shown that by modifying the local orientation of the grating for instance, it is possible to control the polarization of the emitted wave [33, 34]. More recently [35], we have shown that surface wave assisted light emission could be also s-polarized out of the plane perpendicular to the grooves. In this paper we focus on the SiC sources of Ref. [10, 11] to analyze the polarization of the thermal emission emitted in any direction. In particular, we report a study of the degree of polarization.

The SiC lamellar gratings are characterized by their period Λ , filling factor F and height h . These parameters are optimized to obtain 100% emissivity for a given wavelength. Two kinds of thermal light sources are considered : a quasi-isotropic source at a given wavelength or a directional thermal source. For both sources, we analyse the emission in planes which are not perpendicular to the grating lines. Notations for angles are summarized in Fig.1.

2 Surface-plasmon excitation in s-polarization

In this section we show how surface waves can be used to emit s-polarized radiation in free space. It has been already observed in a paper by Inagaki et al. [36] that reflectivity of a metallic grating can drop to zero when illuminated either in p or s-polarization in a direction which is not perpendicular to the lines of a grating. It has been shown that surface plasmons are involved in the

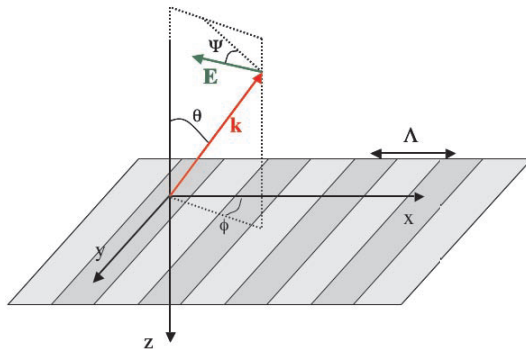


Figure 1: Emission of a grating in the direction (θ, ϕ) . The electric field \mathbf{E} makes an angle Ψ with the plane (\mathbf{z}, \mathbf{k}) . $\Psi = 0^\circ$ corresponds to the p-polarization and $\Psi = 90^\circ$ to the s-polarization.

absorption of s-polarized fields. There may be also polarization conversion upon reflection on a grating when exciting resonantly a surface plasmon [37]. This process can be viewed as the excitation of a surface plasmon by an p-polarized wave followed by the radiation of an s-polarized wave. All these processes clearly suggest that gratings supporting surface plasmons can radiate s-polarized light.

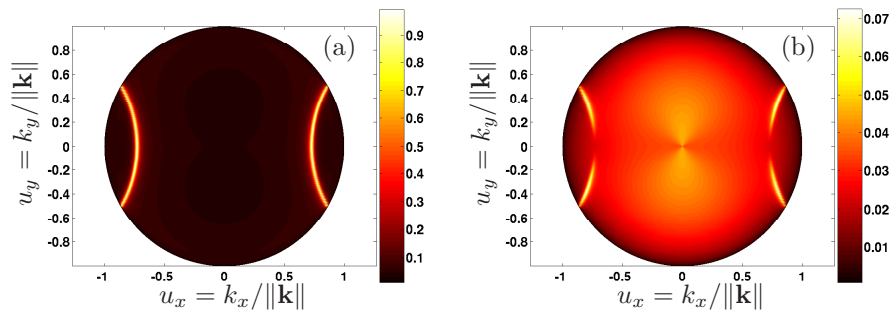


Figure 2: Polar representation of the emissivity at $\lambda = 11.36 \mu\text{m}$ for the directional source in both p-polarization (a) and s-polarization (b) (numerical simulations).

The first thermal source that we study in this paper is a directional thermal source. The grating is characterized by its period $\Lambda = 6.25 \mu\text{m}$, filling factor $F = 0.5$ and thickness $h = 288 \text{nm}$. One can show in Fig. 2 the emissivity pattern of such a grating calculated for both p- and s-polarization. The conditions

of coupling between surface waves and propagating waves have already been discussed in a previous paper [35]. According to this reference, there are two requirements in order to achieve an efficient coupling. The first one is obviously a phase matching condition, which is summarized in Fig. 3(a). This is the origin of the circle-like pattern that we can observe on Fig. 2 for both s and p-polarization. The second condition deals with the polarization of the emitted wave. Actually, the emitted wave must have an electric field component along the propagation direction of the surface wave. This is the origin of emissivity variations along the circles defined by the phase matching condition. To be more precise, a surface wave propagating along the interface with a wave vector \mathbf{k}_{SP} has two components of the electric field : one is parallel to \mathbf{k}_{SP} and the other is parallel to the normal of the surface. It follows that the surface current density associated with the surface plasmon in the material has a component parallel to \mathbf{k}_{SP} . An external electric field can excite a surface plasmon provided that it has a component parallel to \mathbf{k}_{SP} . This condition explains the ϕ dependence and the polarization dependence of the emissivity. Let us consider for example the absorption of a plane wave with an angle $\phi = 0$ so that the incident wave vector is along the x -axis ($k_x^{\text{inc}}, 0$). A plasmon can in principle be excited with a wavenumber ($k_x = k_{\text{SP}}, k_y = 0$) provided that $k_x^{\text{inc}} = k_{\text{SP}} - 2\pi/\Lambda$. If the incident plane wave is p-polarized, the incident field can excite a current density parallel to the x -axis and therefore excite the surface wave. Conversely, if the wave is s-polarized, the incident electric field is along the y -axis and therefore cannot excite a current density along the x -axis.

We note that the coupling in s-polarization is not efficient so that the corresponding emissivity is very low, below 10%. This is not due to fundamental reasons but instead to the fact that this grating had been optimized to emit in p-polarization [10, 11]. These low values of emissivity do not allow us to perform accurate measurements in s-polarization on such a source. We will see in the next section that an isotropic source instead of a directional one is more interesting to show both numerically and experimentally surface-wave assisted emission in s-polarization.

3 Isotropic source

The second thermal source is a quasi-isotropic source for a given wavelength. The grating is characterized by its period $\Lambda = 3.0\mu\text{m}$, filling factor $F = 0.4$ and thickness $h = 350\text{nm}$. The emission wavelength is $\lambda = 10.88\mu\text{m}$ and corresponds roughly to the asymptotic part of the dispersion relation of surface phonon polaritons, so that the emissivity is expected to reach one for this wavelength and to be very low for other wavelengths [10]. This can be viewed as a first step to the fabrication of efficient light sources which could emit monochromatic radiation in a very isotropic way. As in the case of directional sources, previous studies dealt only with the emission in a plane perpendicular to the grating lines. Figure 4 displays the emissivity pattern in both p- and s-polarization. One can see that the phase matching condition is always fulfilled (Fig. 3(b)).

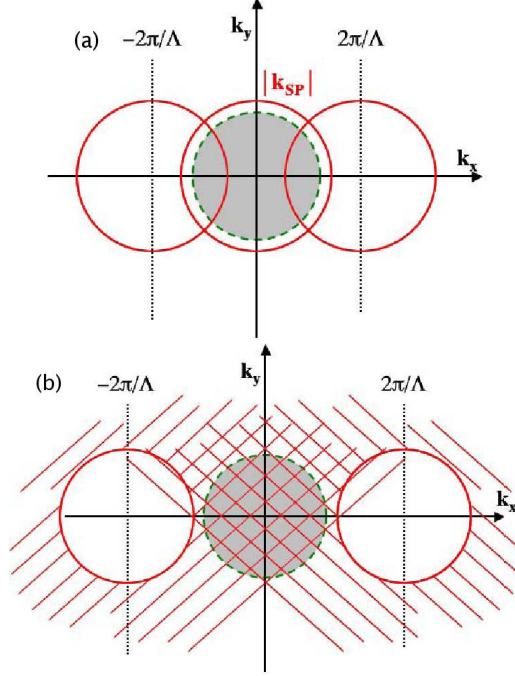


Figure 3: Dispersion relation of surface plasmon-polaritons in a (k_x, k_y) plane for two gratings with different period at a fixed frequency. In both figures, the radius of the dashed circle is the wave vector modulus in free space. Inside this circle (in gray), the emitted waves have a real z -component of \mathbf{k} , so that they are propagating waves. Outside the dashed circle, the waves are evanescent waves. In (a), the solid circle at the center has a radius equal to the modulus of the wave vector of the surface wave \mathbf{k}_{SP} . With a grating of period Λ , this solid circle is reproduced with a period $2\pi/\Lambda$: it represents the surface waves diffraction at the orders ± 1 . One can see in the first figure (a), corresponding to the case of section 2, that a part of the solid line is lying now in the light cone, so that the corresponding surface polaritons are coupled to propagating waves. In the second figure (b), corresponding to the case of section 3, the period Λ is lower so that the linear part of the dispersion relation is always outside the light cone. Only the surface waves lying on the asymptotic part of the dispersion relation can be coupled to propagating waves. This is represented by the parallel red lines which cover the whole gray area. This means that propagating waves can be coupled to surface waves in every directions (k_x, k_y) at the asymptotic wavelength.

The pattern given in Fig. 4 is due to the condition on the polarization of the emitted wave that we discussed above. By the way, in this case the emissivity in s-polarization goes to one in the direction parallel to the lines of the grating. In this direction, the emission is also isotropic.

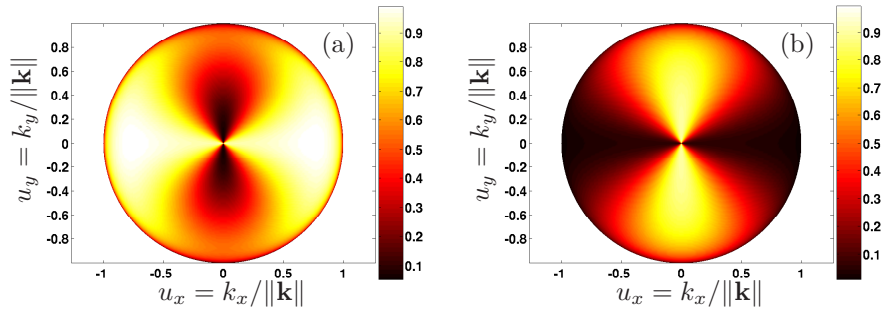


Figure 4: Polar representation of the emissivity at $\lambda = 10.88 \mu\text{m}$ for the quasi-isotropic source in both p-polarization (a) and s-polarization (b) (numerical simulations).

Observation of Figure 4 suggests that the emissivity pattern in p- and s-polarization are complementary. It is possible to calculate an average emissivity for this grating. The result is shown in Fig. 5(a). The emissivity reaches noticeably the same value of about 0.5 in every directions. This value is sufficiently large to be experimentally detected. Thus, we measured the emissivity of this source. The grating was heated at about 500°C . The experimental setup to measure and normalize the emitted flux to obtain emissivity is described in detail in ref. [10]. The measurement is shown in Fig. 5(b) for three different directions given by $\theta = 20^\circ$ and $\phi = 00^\circ, 45^\circ$ and 90° . We plotted the average emissivity in both p- and s-polarization. One can clearly see that the emissivity spectrum is the same for the three directions. It is important here to note that this effect is possible only at the wavelength corresponding to the asymptotic part of the dispersion relation of the surface wave. For this SiC grating, it corresponds to $11 \mu\text{m}$. For other wavelengths, the grating behaves almost like the plane interface, that is to say that between 10.5 and $12.5 \mu\text{m}$, the emissivity is very low, near 0.1. This grating allows the emissivity to reach 0.5 instead of 0.1 at $\lambda = 11 \mu\text{m}$.

4 Dependence on the polarization

A light source is characterized not only by its intensity but also by its polarization. We have seen that the isotropic source emits light whose polarization

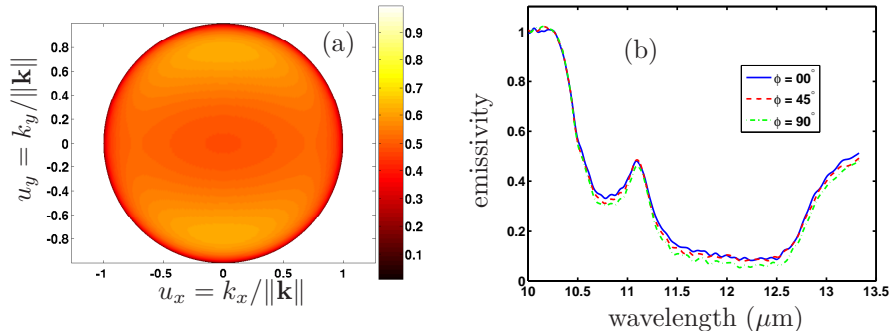


Figure 5: Polar representation of the average emissivity in both p- and s-polarization at $\lambda = 10.88 \mu\text{m}$ for the quasi-isotropic source (a) (numerical simulation). Average emissivity of the same source measured with three different azimuthal angles ϕ at $\theta = 20^\circ$ (b) (experimental measurements).

depends on the emission angle. In order to fully determine the polarization characteristics, one has to derive the Stokes parameters. We shall use them to derive the degree of polarization of the emitted field [38]. The four Stokes parameters are obtained considering different polarizations of the emitted radiation: on the one hand, s- and p-polarization and on the other hand two linear polarizations corresponding to $\psi = +45^\circ$ and -45° , respectively called a- and b-polarization here, and two circular polarizations, left-handed and right-handed, respectively called L- and R-polarization. Stokes's parameters are defined by the combination of emitted intensities in these polarizations. For a thermal source, the Stokes parameters are linearly related to the polarized directional emissivity. As a result we have:

$$\begin{aligned}
 S_0 &= \epsilon_s + \epsilon_p \\
 S_1 &= \epsilon_s - \epsilon_p \\
 S_2 &= \epsilon_a - \epsilon_b \\
 S_3 &= \epsilon_L - \epsilon_R
 \end{aligned}$$

It has been pointed out in two recent papers [39, 40] that a field whose intensity is uniform in a plane can have an interference structure. Indeed, the interference pattern produced by Young's experiments on a screen on which intensity (that is to say $S_0/2$) is uniform can present a maximum contrast for other Stokes's parameters. In an other way, Dahan et al. made some studies on the manipulation of polarization by SiO_2 gratings supporting surface phonon-polaritons [33]. They could use it for instance to encrypt thermal images [34]. Therefore, it appears that a polarization study of the light emitted by a SiC grating is important to fully characterize the thermal source. Figure 6 shows

the emissivity for the a-, b-, L- and R-polarization. We obtain these results by computing the absorptivity of polarized plane waves. It must be noticed that the average of the emissivity either in both a- and b-polarization or in L- and R-polarization gives the same result as in Fig. 5(a). This is not surprising considering the fact that we have in each case a basis of the polarization states. We can also observe that the emissivity patterns are very different from each other.

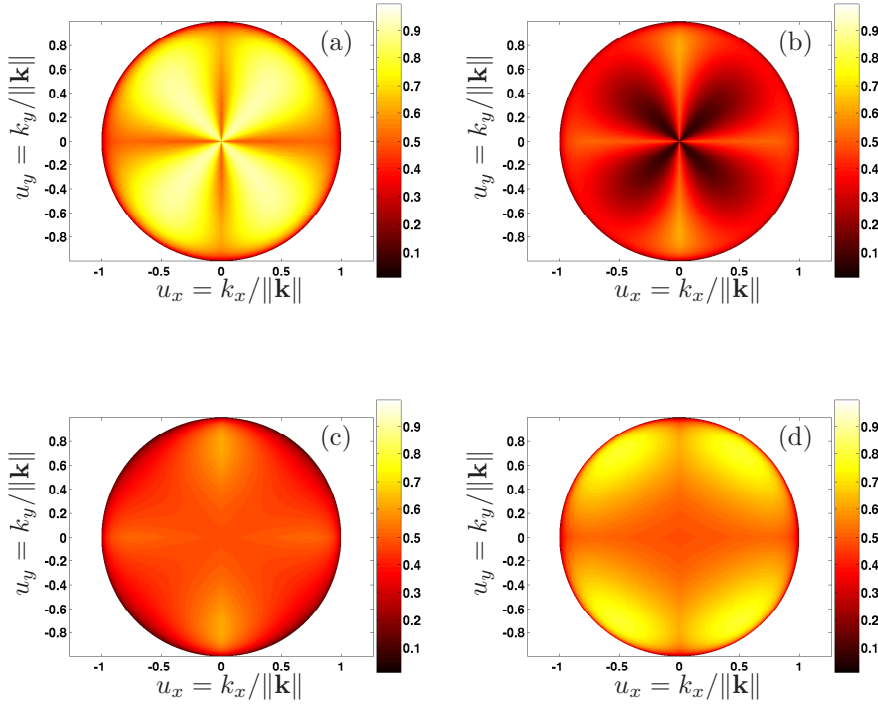


Figure 6: Polar representation of the emissivity in different polarization at $\lambda = 10.88 \mu\text{m}$ for the quasi-isotropic source: linear polarization $\psi = +45^\circ$ (a), linear polarization $\psi = -45^\circ$ (b), left-hand circular polarization (c) and right-hand circular polarization (d).

The degree of polarization is defined from Stokes's parameters [38]:

$$P = \frac{(S_1^2 + S_2^2 + S_3^2)^{1/2}}{S_0}$$

Using Fig. 4 and Fig. 6, it is possible to calculate the degree of polarization of our thermal isotropic source. The result is shown in Fig. 7. Firstly, one can see

that the degree of polarization of this source strongly depends on the emission direction. In directions characterized by $\phi = 00^\circ$, that is to say in the plane perpendicular to the lines of the grating, P is close to one. That means that the emitted radiation is fully polarized in this plane. On the contrary, P drops to zero when θ tends to 90° in the plane parallel to the lines of the grating. Thus the beam is completely unpolarized. Actually, in the plane perpendicular to the lines, the only polarization which can be couple to a surface wave is the p-polarization. Consequently, the radiation is emitted in a polarization defined by a basis vector: this explains the large value of the degree of polarization in this plane. In other directions, both p- and s-polarized waves can be coupled to surface waves, it is thus possible to have the same emissivity for both polarizations and therefore a zero-degree of polarization.

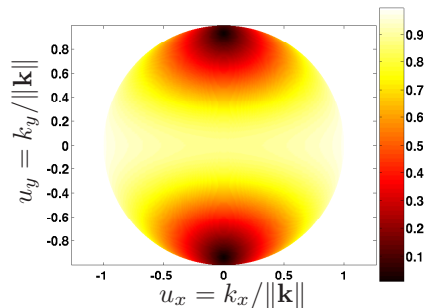


Figure 7: Polar representation of the degree of polarization P given by Stokes's parameters.

5 Conclusion

The polarization of thermal emission of SiC gratings has been studied. In particular, we focused on the emission in planes which are not perpendicular to the grooves of the gratings. It has been shown both numerically and experimentally that it was possible to couple surface phonons to s-polarized propagating waves. We have extended the scope of this work to a general study of the emitted radiation of such sources: Stokes's vectors and degree of polarization. We report the first study of polarized emission of a 1D grating in any direction. We have found that the overall emissivity does not depend on the polarization while the polarized emissivity strongly depends on the emission angle. It follows that the degree of polarization can vary between 0 and 1. Let us stress that that even if this paper deals with surface phonon on SiC, our conclusions are more generally valid for surface waves and can be applied to surface plasmon and metallic gratings [6, 35].

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