



**HAL**  
open science

# Weather Radar Signals with Alpha Stable Distributions

Mohand Lagha, Messaoud Bensebti

► **To cite this version:**

Mohand Lagha, Messaoud Bensebti. Weather Radar Signals with Alpha Stable Distributions. 2007.  
hal-00194153v2

**HAL Id: hal-00194153**

**<https://hal.science/hal-00194153v2>**

Preprint submitted on 15 Dec 2007

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Weather Radar signals with Alpha Stable Distributions

M LAGHA<sup>1</sup>, M BENSEBTI<sup>2</sup>

<sup>1</sup>Aeronautics Department, Engineering Science Faculty, SAAD DAHLAB University of BLIDA, B.P. 270 Road of SOUMAA – BLIDA, ALGERIA

<sup>2</sup>Electronics Department, Engineering Science Faculty, SAAD DAHLAB University of BLIDA, B.P. 270 Road of SOUMAA – BLIDA, ALGERIA

<sup>1</sup>E-mail: lmohand@yahoo.fr, Tel. /Fax: +213 25 43 36 36

***Abstract.*** This work, treats about a modeling of the weather precipitation echoes detected by a weather pulse Doppler radar. The methods used for the simulation of the Doppler spectrum, and  $I$  &  $Q$  signals representing the precipitations radar echoes, is based on a macroscopic models, i.e. random processes with assigned spectra. We propose global model, with a power spectrum having a range of statistical distributions ranging between *Cauchy* and *Gauss* called *alpha-stable* model. The results obtained shows that this new model represents correctly the Doppler spectrum of precipitations.

## I. INTRODUCTION

The computer simulation is mainly used for the generation and the analysis of the weather radar signals for low cost offered by simulations, [4], [9], [12].

The signals received at the weather radar's are a compound of a very great number of various particles of air (hydrometeors), or irregularities of the refraction index of the air, [2], [3], [4], [12]. In this fact these echoes are supposed to be modeled as a random process and where their power spectral density will obey to the Normal statistical distribution (*Gaussian* Law). This assumption is justified by the central limit theorem, [1].

In addition, we can find sometimes that this *Gaussian* model is not adequate [5], to represent of the spectral irregularities and also called spikes, which occur during the detection of weather echoes at low altitudes, from where we are interested to suggest the model of alpha stable distributions, for the representation of the Doppler spectrum of severe weather perturbations (winds, gusts of wind,...), and then to generate and simulate the radar signals  $I$  and  $Q$ .

The aim of this paper is to propose a global model which resolves the problem of the spectral spikes. In the second part a theoretical background about alpha stable distributions is done, in the third part a routine

generation of the  $I$  and  $Q$  radar data is discussed. Results and comments about the results obtained are presented in the fourth part.

## II. THEORETICAL BACKGROUND

We introduce in this section the use of *alpha stable* distribution  $S \alpha S$ , to model the radar echoes. The most aim of the introduction of this range of statistical distributions called *alpha-stable* law, for the modeling and the representation of the Doppler spectrum and signals  $I$  and  $Q$  of the precipitation radar echoes and/or of a severe weather perturbation, is that to have a good representation of a spectrum sullied with stationery echoes in the presence of the ground clutter effect, and the representation of multimodal aspects of the Doppler spectrum. This problem of spectral irregularities is mentioned by R.J. Keeler in the reference [5].

The range of the *alpha stable* distributions is defined as a direct generalization of the *Gaussian* law [6],[7], [10], [11], it includes in fact the *Gaussian* distribution density as particular case ( $\alpha=2$ ). The principal characteristic of a *stable* function (non *Gaussian*) with probability density function *pdf*, is that its spectrum are broader than those of the *Gaussian* density. It is one of the principal reasons for which the *alpha-stable* law (*Levy* laws) is considered adapted to model useful signals and noises with impulsive nature, and processes with heavy tail [7], [10]. This law defined by its characteristic function *pdf*, and given as:

$$F(w) = \exp(\delta.j.w - \gamma.|w|^\alpha) \quad (1)$$

The parameters,  $\alpha$ ,  $\gamma$  and  $\delta$  describes completely a *stable* distribution. The characteristics parameter  $\alpha$  (**alpha**) acts directly, over the width of the probability density function (*pdf*), and on impulsiveness of the *alpha stable* process  $S \alpha S$ . The parameter  $\alpha$  can take values in the interval  $[0,2]$ , a smaller value implies broader extents and heavy tails, and for  $\alpha=2$  (*Gaussian* case) the spectral width is narrow. The dispersion parameter  $\gamma$  ( $\gamma >0$ ) plays a part similar to the variance and refers to the width distribution. When  $\alpha=2$  (*Gaussian* case) the variance is equal to  $2\gamma$ . The dispersion parameter  $\delta$  is comparable with the mean of the distribution, where  $\delta$  can take values in the interval  $]-\infty, +\infty [$ , [6], [7].

The theoretical justification of the use of the *alpha-stable* distributions as a basic tool of statistical modeling comes from the generalized central limit theorem [1]. This theorem stipulates that the sum of independent random variables identically distributed (i.i.d) with or without variance converges towards a *stable* distribution, [7], [10], [11].

If a signal or a noise observed can be regarded as result of a great number of identically distributed independent variables, then the generalized central limit theorem suggests that a stable model can be well adapted to model it, [1].

But, an expression with compact form does not exist for the *stable* probability density function, except for the particular distributions *Gaussian* ( $\alpha=2$ ) and of *Cauchy* ( $\alpha=1$ ). In this fact to model the power spectrum of the weather precipitation, we can use a linear interpolation between the *Gaussian* distribution and that of *Cauchy* for  $\alpha$  ranging between 0 and 2, and it is shown, [7], [11], that the probability density *pdf* of these range distributions is given by the equation as: -

$$\hat{f}_\alpha(x) = (1-p) \cdot f_1(x) + p \cdot f_2(x) \quad (2)$$

Where, the parameter  $p$  lies between 0 and 1, for  $\alpha$  ranging between 0 and 2 ( $0 \leq \alpha \leq 2$ ).  $f_2(x)$ ,  $f_1(x)$  are respectively the probability densities functions of the Gaussian and Cauchy distributions.

From [7], we can use the approximation  $P = g(\alpha)$ :

$$g(\alpha) = 0.3484\alpha^2 - 0.0355\alpha - 0.3147 \quad (3-a)$$

or

$$p = (\alpha - 1)^{1.25} \quad (3-b)$$

Thus for our case of study, i.e. the modeling of the weather radar echo characterizing a severe perturbation and/or a precipitation, the probability density function of the weather Doppler spectrum is given by (2).

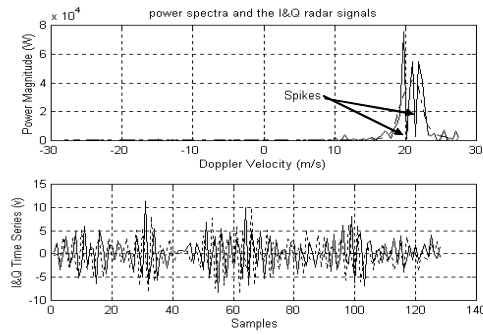
With:

- $f_1(x) = \left( (1/\pi) \cdot \left( 1 / \left[ 1 + (x - \theta)^2 \right] \right) \right)$ , Cauchy distribution with  $(x, \theta)$ , parameters.
- $f_2(x) = \left( 1 / \sqrt{2\pi\sigma} \right) \exp\left( - (x - \mu)^2 / 2\sigma^2 \right)$ , Gaussian distribution with the parameters  $(\mu, \sigma)$ , where  $\mu$  is the variance and  $\sigma^2$  is the width.

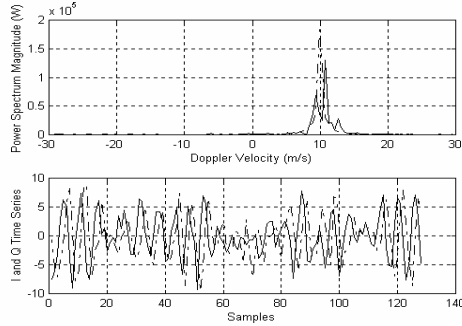
The equation (2) represents the numerical interpolation of the  $\alpha$ -stable distribution (*Levy* laws), for the parameter  $\alpha$  ranging between 0 and 2.

Thus by replacing the parameter  $\alpha=1$  in the equation (3-b), we obtains ( $p=0$ ) and this will induce a distribution of Cauchy on the equation (2), well after elimination of the second term by its multiplication by the term ( $p=0$ ). In the same manner by replacing  $\alpha=2$ , in the equation (3-b) we obtains ( $p=1$ ), which will imply the elimination of the first term (that of *Cauchy*) and thus there will be a Gaussian distribution in the equation (4). For the values  $\alpha$  ranging between 0 and 2 ( $0 \leq \alpha \leq 2$ ), there will be Doppler spectra with intermediate distributions.

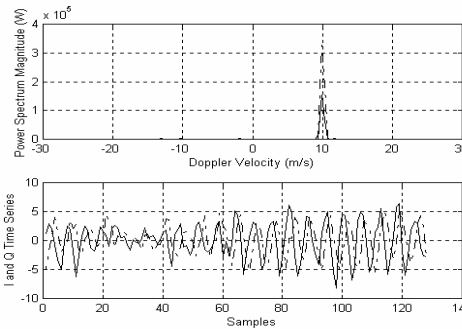
The modeling of the Doppler spectrum of the radar signals  $I$  and  $Q$  representing the radar echoes of a weather precipitation at low altitude, by the *Levy* law for various values of the parameter,  $\alpha$ , is illustrated in the figures 1, 2 and 3.



**Fig. 1.** Power spectrum of the weather echoes and the I, Q radar signals for  $\alpha = 1$



**Fig. 2.** Power spectrum of the weather echoes and the I, Q radar signals for  $\alpha = 1$



**Fig. 3.** Power spectrum of the weather echoes and the I, Q radar signals for  $\alpha = 2$

It is well noted that there is a visible effect of the noise on the Doppler power spectrum of the weather perturbation. This noise is materialized by secondary peaks power (spikes). These secondary peaks with impulsive nature, are all the more numerous for low values of the parameter  $\alpha$  ( $\alpha < 2$ ), which are adequate for the modeling of the radar echoes drowned in a background noise and particularly at low altitude.

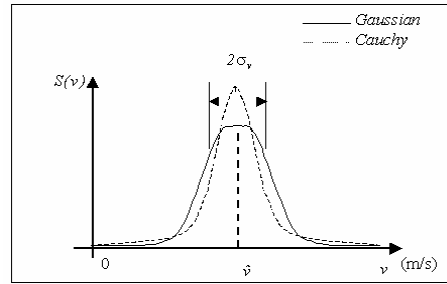
Let us note that the rate of the background noise is much more important at low altitudes, and in particular when the antenna sweeps nearing the ground, it can come from the reflections of the ground, or of the vehicles circulating in the agglomeration covered by the pulsed weather Doppler radar's [4], [5].

In practice these secondary peaks power (spikes) characterized by almost null velocities can harm the quality of the estimations made on the Doppler spectrum, which makes necessary the use of adaptive filters in order to eliminate them from the spectral contents of the radar echoes, [5].

### III. DATA'S GENERATION

#### *Work Hypothesis*

It is shown [3], [4], [5], [6], [7], [12] that the received signal of the weather targets is well represented by a random process with Gaussian (Normal) distributions. But, nevertheless some authors in this field mention that the impulsive character of the noises and some of the useful signals, are not well represented by this model. We propose a global model having statistical distributions ranging between the *Gaussian* law and *Cauchy* law according to the prevalence of the noise see figure 4.



**Fig. 4.** Weather Doppler spectrum, received Power according to the Doppler mean velocity

It is a direct consequence owing to the fact that;

- 1) The number of reflections in the illuminated volume by the emitted impulse radar is large ( $>10^6$ );
- 2) The illuminated volume by impulse is large compared with the transmitted wavelength;
- 3) The volume of impulse is filled with reflections, causing all of the random phases ranging between 0 and  $2\pi$ ;
- 4) The particles are moving the ones compared to the others. This movement is due to turbulence created by weather precipitations, and at their velocities which are also variable.

The superposition of the dispersed electric fields of such great number of particles (each one with random phase) gives place, by the central limit theorem limit, a signal with stable statistics (*alpha stable* law) [5], [11].

Since the particles are moving the ones compared to the others, there is also a Doppler diffusion, often indicated under the name of dissension of the Doppler spectrum or also of Doppler frequency.

We should produce the radar signals I and Q. The variables entry for our routine should include the signal noise ratio ( $SNR$ ), radial velocity  $V_r$ , a number data points (samples of data)  $ndata$ , ambiguous velocity  $V_a$ , the parameter  $\alpha$  and the spectral width  $\sigma_v$ , and the power of the emitted signal  $PP$  as well as the radar reflectivity  $dBz$ . The output data must be Doppler velocities, the Doppler spectrum and signals I and Q (time series).

#### IV. RESULTS AND COMMENTS

In this part we highlight the simulations results in the form of graphs. Initially we discuss the effect of each entry parameter on the Doppler spectrum velocities and on signals I and Q. the graphs will represent at the same time disturbed and non disturbed Doppler spectrum, and the time series I and Q. Then in the second time we comment the results and the incidence of each parameter entry on the Doppler spectra and signals I and Q generated during simulations.

##### 1. *The effect of the Signal Noise Ratio SNR on the Doppler spectrum and signals I & Q:*

###### **SNR=1dB (fig.5 and fig.6)**

It is noted that there are too many ripples on the Doppler spectrum (peak with  $3.10^5$  w), and the two signals  $I$  and  $Q$  overlap, i.e. that we cannot distinguish them at sight of eye.

This is obvious, because the useful signal merges with the noise signal ( $SNR=1$ ), from where the bad distinction enters the two signals, see figure 5. In practice this can occur when the useful signals radar and the interfering signals are of the same level, [4].

###### **SNR=10dB**

A level of noise of 10 dB, means that the signal has a level higher than the noise of ten times.

It is noted that there is less of ripples than the preceding case, and we distinguishes a quite apparent peak power on the Doppler spectrum disturbed from speeds to a magnitude of  $2,8.10^5$  w. And signals I and Q do not overlap see figure 6.

##### 2. *The effect of the number of data samples ndata on the Doppler spectrum and signals I and Q ; (fig.7 and 8)*

We can say that starting from 256 points we will have the same properties if one increases further this number (i.e. symmetry repetition), as we can see it on figures 7 and 8. This number is given in power of 2 ( $ndata=2^n$ ).

The number of points has a direct relation with the time and the distance cells on which the echoes are reflected. Thus to have a good simulation of the physical phenomenon and good resolutions of the radar signals I and Q and Doppler spectra correspondents to take a significant time as much, i.e.  $256T_s=256(1/2000)=0.1280$ , [8],[9].

Therefore, the number of samples of data  $n_{data}$  for a cell distance, does not affect in any manner the properties of the Doppler spectrum, except if the weather phenomenon is spread out over several seconds. In this case, it is necessary to increase the number of points (in power of 2), [9].

But, to exploit the data from air traffic radar's (airport radar's), we must take the number of data lower than 64, or lower than 32 samples [9].

### **3. *The effect of the peak power of the emitted signal on the Doppler spectrum and signals I/Q: (fig 9. and 10)***

The peak power of the emitted signal has a direct influence on the amplitude of the received Doppler spectrum. Thus if the peak power increases, the amplitude of the spectrum increases and the reverse is true, as we can see it on figures 9 and 10.

This is also explained, by a dilemma which exists between a higher or lower power of emitted radar signals; Because to be able to emit a signal with higher power, this one will require an expensive and cumbersome material, and to have a weak power signal can harm the quality of detection of precipitation's, and this is due to the attenuation's caused by the fixed obstacles (ground clutter), [3], [5].

The presence of a peak power on the received Doppler spectrum, means the presence of weather perturbation, and this same peak corresponds to the perturbation velocity that we can read directly on the velocities axis of the Doppler spectrum.

If figure10 is considered, we can see well the existence of a peak power of Doppler spectrum, which represents a weather perturbation with a velocity of 10 m/s.

### **4. *The effect of the spectral width (m/s) ( $\sigma_v$ ) on the Doppler spectrum and signals I and Q:***

$$\sigma_v=0.25 \text{ m/s}$$

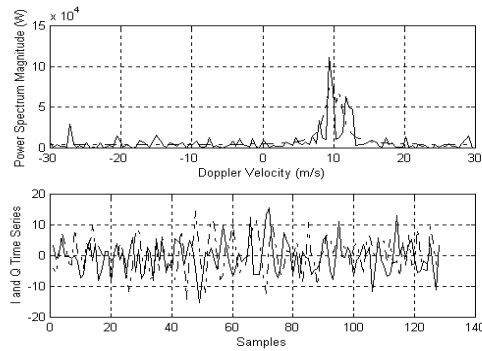
Doppler velocities, lie between (5 and 15 m/s), see figure 11.

$$\sigma_v=10 \text{ m/s}$$

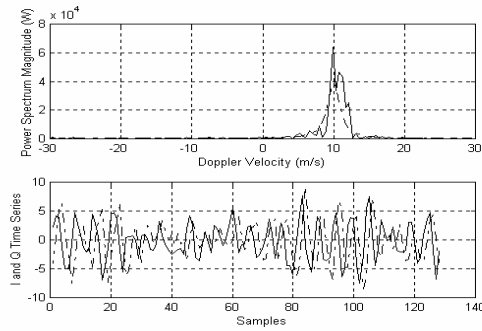
Doppler speeds lie between (- 10 and 30 m/s), see figure 12.



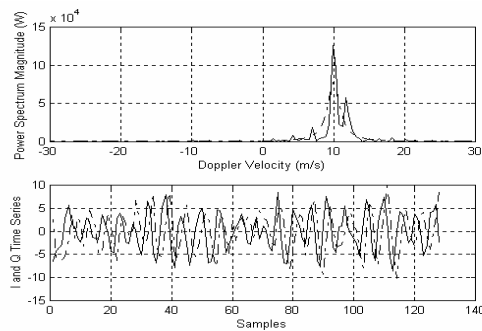
We can say that,  $\sigma_v$  has a direct influence on the width of the Doppler spectrum, [4], and [5]. Thus if we want to have a broader spectrum  $\sigma_v$  increases and this means that we will take in account a larger number of weather precipitation's phenomenon. And if we want to reduce the width of the weather spectrum, i.e. to select the range of the perturbations to be detected, we reverse the operation, i.e. and we reduces  $\sigma_v$ . This parameter  $\sigma_v$  (m/s) plays a very important part for the modeling of the Doppler spectra characterizing the weather perturbations; it means an average deviation of the Doppler velocities variations, [3], [4], [5].



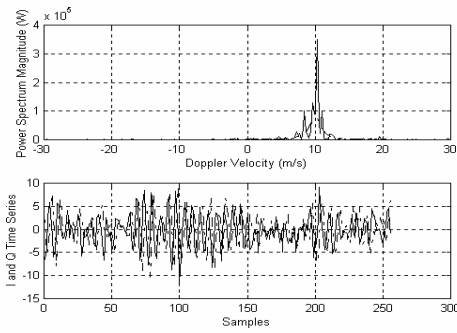
**Fig. 5.** Effect of the  $SNR$  on the Doppler spectrum and I, Q signals ( $SNR=1$ )



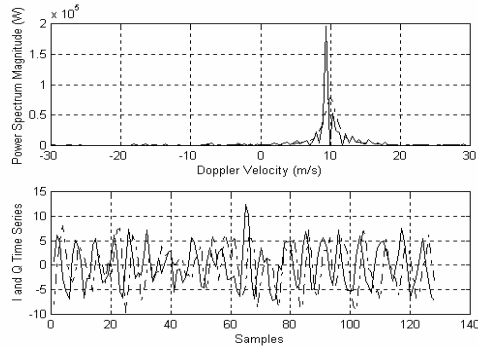
**Fig. 6.** Effect of the  $SNR$  on the Doppler spectrum and I, Q signals ( $SNR=10$ )



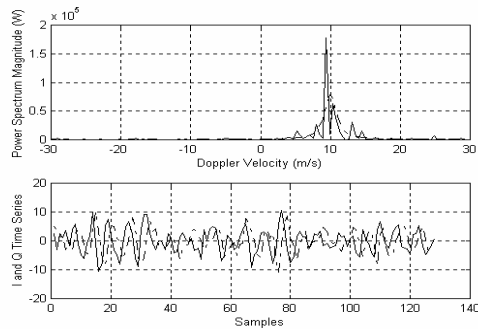
**Fig. 7.** Effect of data samples on the Doppler spectrum and I, Q signals ( $n_{data}=128$ )



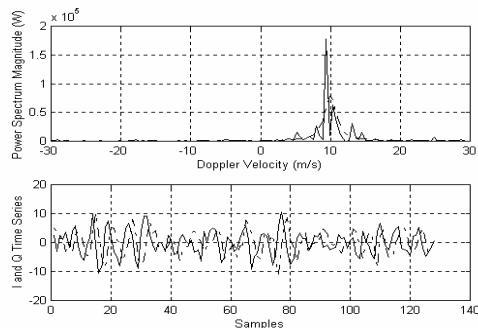
**Fig. 8.** Effect of the data samples on the Doppler spectrum and I, Q signals ( $n_{data}=256$ )



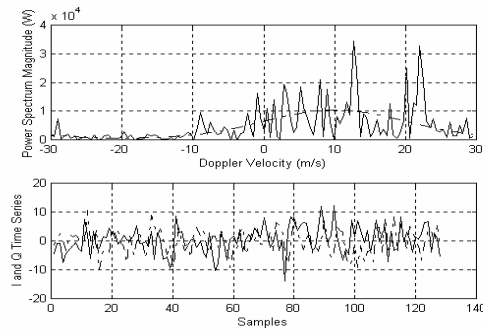
**Fig. 9.** Effect of the peak power ( $pp$ ) on the Doppler spectrum and I, Q signals ( $pp=125.10^3 w$ )



**Fig. 10.** Effect of the peak power ( $pp$ ) on the Doppler spectrum and I, Q signals ( $pp=250.10^3 w$ )



**Fig. 11.** Effect of the spectral width  $\sigma_v$  on the Doppler spectrum and the I, Q signals ( $sig=0.25 m/s$ )



**Fig. 12.** Effect of the spectral width  $\sigma_v$  on the Doppler spectrum and I, Q signals (sig=10 m/s)

## V. CONCLUSION

In this paper we have discussed a model simulation of weather radar signals of precipitation's, was described. In order to have a good simulation of the ground clutter effects on the Doppler spectrum, we have proposed the *alpha stable* distribution which proves to be adequate, with the representation of the spectral spikes. The distribution proposed is adequate and can also represent the Doppler spectrum of the weather perturbation even with a reduced number of sequences (lower than 64 points), which is used in the air traffic control radar's [9].

## REFERENCES

- [1] Papoulis A. 1991, Probability, Random Variables, and Stochastic Processes. *McGraw-Hill*, New York
- [2] M Lagha, M Bensebti, 2007, "Performances comparison of pulse-pair and 2-step prediction algorithms for the Doppler spectrum," [Multidimensional Systems and Signal Processing](https://doi.org/10.1007/s11045-007-0030-3), doi : 10.1007/s11045-007-0030-3.
- [3] Skolnik, M. I. 1990, *Radar Handbook*, McGraw-Hill Book Company, 2<sup>nd</sup> edition.
- [4] Doviak RJ, Zrnić D.S. 1993, Doppler radar and weather observation, *Academic Press*.
- [5] R.J. Keeler and R. E. Passarelli 1990, Signal Processing for Atmospheric Radars in Meteorology, Boston, MA, American Meteorological Society, p. 199-229.
- [6] Ilow J, and Hatzinakos D, "Analytic alpha-stable noise modeling in a Poisson field of interferers or scatterers", *IEEE Trans. On Signal Processing*, Vol. 46, Issue 6, pp. 1601-1611.
- [7] Nikias CL, Shao M 1995, "Signal Processing with Alpha-Stable Distributions and Applications", *Wiley, New York*.
- [8] D. S. Zrnić 1975, "Simulation of weatherlike Doppler spectra and signals,". *Journal of Applied. Meteorology.*, 14 pp. 619-620.
- [9] G. Galati and G.. Pavan. 1995, "Computer simulation of weather radar signals", *Simulation Practice and Theory*. 3, 17-44.
- [10] Tsihrintzis G.A., Nikias C.L. 1995, "Performance of optimum and suboptimum receivers in the presence of impulsive noise modeled as an alpha-stable process," *Transaction on signal processing*, Vol. 43, Issue 234, pp. 904 – 914.
- [11] Tsihrintzis G.A, Nikias C.L. 1995, "Fast estimation of the parameters of alpha-stable impulsive interference," *Transactions on Acoustics, Speech, and Signal Processing*, 44, 6, pp. 1492–1503.
- [12] V.N. Bringi, and V. Chandrasekar, 2001, "Polarimetric Doppler Weather Radar: Principles and Applications," Cambridge University Press.