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# Numerical modeling of 1-D transient poroelastic waves in the low-frequency range

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## Abstract

Propagation of transient mechanical waves in porous media is numerically investigated in 1D. The framework is the linear Biot's model with frequency-independent coefficients. The coexistence of a propagating fast wave and a diffusive slow wave makes numerical modeling tricky. A method combining three numerical tools is proposed: a fourth-order ADER scheme with time-splitting to deal with the time-marching, a space-time mesh refinement to account for the small-scale evolution of the slow wave, and an interface method to incorporate the jump conditions at interfaces. Comparisons with analytical solutions confirm the validity of this approach.

*Key words:* Elastic waves; Porous media; Biot's model; Time-splitting; ADER schemes; Space-time mesh refinement; Immersed interface method.

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## 1 Introduction

The propagation of mechanical waves in porous media is of interest in many areas in applied mechanics, including industrial foams, spongy bones and petroleum rocks. The most-widely-used model describing the evolution of small mechanical perturbations in a saturated porous medium is that proposed by Biot in 1956. Two regimes were distinguished by Biot: one corresponding to a low-frequency range [3], and one to a high-frequency range, where some of the physical parameters depend on the frequency [4]. We focus on transient mechanical waves whose frequency content lies in the low-frequency range.

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Up to the 95's, Biot's equations were mainly studied in the harmonic regime. Various time-domain methods have been proposed since, based on finite-differences [9,22], finite-elements [10,23], boundary-elements [19], and spectral methods [6]. Since non-realistic values of the physical parameters were used, the real difficulties arising when performing time-domain simulations were often overlooked [7,11]. These difficulties are induced by the coexistence of two solutions with radically different dynamics: a propagating "fast wave" and a diffusive "slow wave" [8]. The latter is highly dispersed and attenuated, and remains localized near sources and interfaces.

The aim of the present study is to develop an efficient numerical method to compute the solution made up of these two waves. A time-splitting is used together with a fourth-order ADER scheme [17]. A flux-conserving space-time mesh refinement is implemented at the places where the diffusive mode is localized [2]. Lastly, an immersed interface method gives a subcell resolution of the interfaces and accurately models the jump conditions between various materials [18]. The numerical tools used are described and tested in 1D.

The paper is organized as follows. The Biot's model is briefly recalled in section 2. The numerical tools used are described in section 3. Section 4 presents numerical experiments confirming the validity of this approach. In section 5, conclusions are drawn and some future perspectives are suggested.

## 2 Problem statement

### 2.1 Biot's model

Biot's theory describes the propagation of mechanical waves in a porous medium consisting of a solid matrix saturated with fluids circulating freely through the pores. The underlying hypotheses in the low-frequency range are as follows:

- the wavelength of the perturbations is large in comparison with the diameter of the pores, as well as with the representative macroscopic volumes;
- the amplitudes of the perturbations are small;
- the elastic and isotropic matrix is fully saturated by a single fluid phase.

Biot's model is based on 10 physical parameters: the density  $\rho_f$  and the dynamic viscosity  $\eta$  of the fluid; the density  $\rho_s$  and the shear modulus  $\mu$  of the solid; the porosity  $0 < \phi < 1$ , the tortuosity  $a \geq 1$ , the absolute permeability  $\kappa$ , the Lamé coefficient  $\lambda_f$ , and the two Biot coefficients  $\beta$  and  $m$  of the saturated matrix. The following notations are introduced:  $\rho_w = \frac{a}{\phi} \rho_f$ ,  $\rho = \phi \rho_f + (1 - \phi) \rho_s$ , and  $\chi = \rho \rho_w - \rho_f^2 > 0$ . From the previous parameters,

a critical frequency  $f_c$  is defined

$$f_c = \frac{\eta \phi}{2 \pi a \kappa \rho_f}. \quad (1)$$

With a harmonic wave of frequency  $f$ , the viscous effects are greater than the inertial effects if  $f < f_c$ . In the low-frequency range  $f < 0.15 f_c$  [5], the dynamic permeability  $\kappa/\eta$  does not depend on  $f$ . Lastly, the unknowns are the elastic velocity  $v_s$ , the elastic stress  $\sigma$ , the filtration velocity  $w = \phi(v_f - v_s)$ , where  $v_f$  denotes the fluid velocity, and the acoustic pressure  $p$ .

When dealing with heterogeneous media, all the parameters are assumed to be piecewise constant and discontinuous across the interfaces. Taking  $x = \alpha$  to denote the location of an interface, the jump conditions are [12]:

$$[v_s(\alpha, t)] = 0, \quad [w(\alpha, t)] = 0, \quad [\sigma(\alpha, t)] = 0, \quad [p(\alpha, t)] = -\frac{1}{\xi} w(\alpha^-, t), \quad (2)$$

where  $\xi$  is the hydraulic permeability of the interface. If  $\xi \rightarrow +\infty$ , we obtain the classical open-pore conditions, with a perfect hydraulic contact [5]. If  $\xi \rightarrow 0^+$ , no motion of the fluid relative to the matrix occurs: the pores are closed.

## 2.2 Initial boundary-value problem

Setting  $\mathbf{U} = {}^T(v_s, w, \sigma, p)$ ,

$$\mathbf{A} = \begin{pmatrix} 0 & 0 & -\rho_w/\chi & -\rho_f/\chi \\ 0 & 0 & \rho_f/\chi & \rho/\chi \\ -(\lambda_f + 2\mu) & -\beta m & 0 & 0 \\ \beta m & m & 0 & 0 \end{pmatrix}, \quad \mathbf{S} = \frac{\eta \rho}{\kappa \chi} \begin{pmatrix} 0 & -\rho_f/\rho & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad (3)$$

the Biot's evolution equations [3] are written together with the jump conditions (2) in the form of a first-order linear system with a source term

$$\begin{cases} \frac{\partial}{\partial t} \mathbf{U} + \frac{\partial}{\partial x} \mathbf{A} \mathbf{U} = -\mathbf{S} \mathbf{U} & \text{if } x \neq \alpha, t \geq 0, \\ [\mathbf{C} \mathbf{U}(\alpha, t)] = \mathbf{0}, \\ \mathbf{U}(x, 0) = \mathbf{U}_0(x), \end{cases} \quad (4)$$

where the  $4 \times 4$  matrices  $\mathbf{C}$  are deduced from (2). A detailed mathematical analysis of the solution to (4) in a homogeneous medium can be found in [10].

Here we restrict the discussion to basic properties that are directly useful for numerical modeling purposes [5]. The spectral radius of  $\mathbf{S}$  is  $R(\mathbf{S}) = \frac{\eta}{\kappa} \frac{\rho}{\chi}$ . A dispersion analysis of (4) shows the existence of two waves with phase velocities  $c_1(f) \leq c_1$  and  $c_2(f) \leq c_2$ , where  $\pm c_1$  and  $\pm c_2$  are the eigenvalues of  $\mathbf{A}$  ( $c_1 > c_2 > 0$ ). These waves are called the fast wave and the slow wave, respectively. If  $\eta = 0$ , which is physically irrelevant with usual media, the waves are purely propagated and the mechanical energy is constant. If  $\eta \neq 0$ , the fast wave is almost non-dispersive and non-diffusive. On the contrary, the slow wave becomes highly dispersive and diffusive. If  $f \ll f_c$ , then  $c_2(f) \ll c_2$ : the slow wave tends towards a non-propagating mode [8]. The direct contribution of the slow wave to the overall wave propagation processes is therefore negligible. However, the accurate computation of the fast wave depends crucially on the effects of the slow wave on the balance equations at interfaces [5].

### 3 Numerical tools

#### 3.1 Numerical scheme

This subsection deals with the numerical resolution of (4) far from  $\alpha$ . A uniform grid is considered here, with the spatial mesh size  $\Delta x$  and the time step  $\Delta t$ . An approximation  $\mathbf{U}_i^n$  of  $\mathbf{U}(x_i = i \Delta x, t_n = n \Delta t)$  is sought. The numerical methods recalled in section 1 usually consist in simultaneously discretizing the propagating part and the source term in (4). If  $\eta \neq 0$ , a Von-Neumann analysis of stability typically yields

$$\Delta t \leq \min \left( \frac{\Delta x}{c_1}, \frac{2}{R(\mathbf{S})} \right), \quad (5)$$

which is highly restrictive since  $R(\mathbf{S})$  may be large. For instance,  $\text{CFL} = c_1 \Delta t / \Delta x \approx 10^{-2}$  with the media considered in section (4.2), and  $\text{CFL} \approx 10^{-12}$  can be reached with highly dissipative fluids such as bitumen. A much more efficient approach is to split (4) and to successively solve the propagating part and the source term part by Strang's splitting:

$$\begin{cases} \frac{\partial}{\partial t} \mathbf{U} + \frac{\partial}{\partial x} \mathbf{A} \mathbf{U} = \mathbf{0}, \\ \frac{\partial}{\partial t} \mathbf{U} = -\mathbf{S} \mathbf{U}. \end{cases} \quad (6)$$

The first equation in (6) can be solved by applying any explicit two time step spatially-centered flux-conserving scheme which can be written abstractly

$$\bar{\mathbf{U}}_i = \mathbf{H} \left( \mathbf{U}_{i-s}^n, \dots, \mathbf{U}_{i+s}^n \right), \quad (7)$$

where  $s$  is the width of the stencil. In the present numerical experiments, a temporally and spatially fourth-order accurate ADER scheme was used [17,20], with  $s = 2$ . This scheme accounts for waves over long distances with small dispersion and diffusion errors, even on coarse grids. The second equation in (6) is solved exactly:  $p$  and  $\sigma$  are unchanged, whereas the velocities become

$$v_i^{n+1} = \bar{v}_i + \frac{\rho f}{\rho} \left(1 - e^{-\frac{\eta}{\kappa} \frac{\rho}{x} \frac{\Delta t}{2}}\right) \bar{w}_i, \quad w_i^{n+1} = e^{-\frac{\eta}{\kappa} \frac{\rho}{x} \frac{\Delta t}{2}} \bar{w}_i. \quad (8)$$

The splitting (6) with (8) yields the optimal stability condition  $\text{CFL} \leq 1$ , but it decreases the theoretical order of convergence to 2. In practice, the convergence rate measured was close to 3 in our experiments.

### 3.2 Mesh refinement

If  $\eta \neq 0$  and  $f \ll f_c$ , the slow wave has much smaller spatial scales of evolution than the wavelength of the fast wave. A very fine grid is consequently required to account for its evolution. Since the use of a fine uniform grid on the whole computational domain is out of reach in view of 2-D simulations, the grid refinement procedure provides a good alternative. In addition, since the slow wave remains localized near the sources and the interfaces, grid refinement is necessary only around these places.

To limit the numerical dispersion on the coarse grid, it is preferable to also perform temporal refinement using a local CFL stability condition. Many algorithms for space-time mesh refinement have been recently developed in a variational framework, which ensure the stability of the coupling between grids by conserving the discrete energy [16]. Here we adopt another approach based on flux conservation [2], which is more naturally coupled to the flux-conserving scheme (7). The extrapolated values required to couple coarse and fine grids are obtained by performing linear interpolation in space and time on the numerical values at the surrounding nodes. In the case of the Lax-Wendroff scheme applied to the advection equation, the stability of the coupling has been proved in [1], whatever the refinement factor.

### 3.3 Interface method

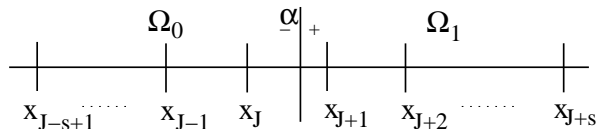


Fig. 1. Interface between media  $\Omega_0$  and  $\Omega_1$  and the irregular nodes around  $\alpha$ .

This subsection deals with the time-marching at the grid nodes where the stencil of (7) crosses  $\alpha$ . The grid is assumed to be uniform around  $\alpha$ , and we define  $J$  by  $x_J \leq \alpha < x_{J+1}$  (figure 1). At the so-called irregular nodes  $x_{J-s+1}, \dots, x_{J+s}$ , the scheme (7) must not be applied naively, for three reasons. First, the spatial derivatives of the solution are not smooth across  $\alpha$ , hence the order of convergence of the scheme decreases. Secondly, the subcell position of  $\alpha$  inside the mesh is ignored, which leads to a  $O(\Delta x)$  error. Thirdly, the jump conditions (2) are not included in the scheme. The numerical waves diffracted by the interface may therefore not tend towards the exact waves.

At the irregular nodes, we adapt an immersed interface method [18]. This method requires knowing the jump conditions satisfied by the spatial derivatives of  $\mathbf{U}$ . We deduce these conditions from (4) for all  $m \geq 1$

$$\begin{aligned} \frac{\partial^m}{\partial t^m} [\mathbf{C}\mathbf{U}(\alpha, t)] &= \left[ (-1)^m \mathbf{C} \left( \mathbf{A} \frac{\partial}{\partial x} + \mathbf{S} \right)^m \mathbf{U}(\alpha, t) \right] = \mathbf{0}, \\ &\Rightarrow \left( \mathbf{U}, \dots, \frac{\partial^m}{\partial x^m} \mathbf{U} \right) (\alpha^+, t) = \mathbf{D}_m \left( \mathbf{U}, \dots, \frac{\partial^m}{\partial x^m} \mathbf{U} \right) (\alpha^-, t), \end{aligned} \quad (9)$$

where  $\mathbf{D}_m$  is a  $4(m+1) \times 4(m+1)$  matrix depending on the permeability  $\xi$  and on the physical parameters around  $\alpha$ . Let  $k$  be a positive integer. At the  $2k$  grid nodes surrounding  $\alpha$ ,  $2k$ -th order Taylor expansions of  $\mathbf{U}(x_i, t_n)$  on  $\alpha^\pm$ , together with the jump conditions (9), are written in the matrix form

$$\begin{pmatrix} \mathbf{U}(x_{J-k+1}, t_n) \\ \vdots \\ \mathbf{U}(x_{J+k}, t_n) \end{pmatrix} = \mathbf{M} \begin{pmatrix} \mathbf{U}(\alpha^-, t_n) \\ \vdots \\ \frac{\partial^{2k-1}}{\partial x^{2k-1}} \mathbf{U}(\alpha^-, t_n) \end{pmatrix} + \begin{pmatrix} \mathbf{O}(\Delta x^{2k}) \\ \vdots \\ \mathbf{O}(\Delta x^{2k}) \end{pmatrix}, \quad (10)$$

where  $\mathbf{M}$  is a  $8k \times 8k$  matrix. The Taylor rests are removed from (10), and the exact values are replaced by numerical ones. The values of the spatial derivatives obtained by performing inversion of (10) are used to build smooth extensions of the solution, called modified values, on the right of  $\alpha$ :

$$i = J+1, \dots, J+s, \quad \mathbf{U}_i^* = \left( \mathbf{I}_4, \dots, \frac{(x_i - \alpha)^{2k-1}}{(2k-1)!} \mathbf{I}_4 \right) \mathbf{M}^{-1} \begin{pmatrix} \mathbf{U}_{J-k+1}^n \\ \vdots \\ \mathbf{U}_{J+k}^n \end{pmatrix}, \quad (11)$$

where  $\mathbf{I}_4$  is the  $4 \times 4$  identity matrix. Modified values on the left of  $\alpha$  are defined

similarly. They are then injected into the scheme at the irregular nodes:

$$\begin{aligned} i = J - s + 1, \dots, J, \quad \bar{\mathbf{U}}_i &= \mathbf{H}_{\Omega_0} \left( \mathbf{U}_{i-s}^n, \dots, \mathbf{U}_J^n, \mathbf{U}_{J+1}^*, \dots, \mathbf{U}_{i+s}^* \right), \\ i = J + 1, \dots, J + s, \quad \bar{\mathbf{U}}_i &= \mathbf{H}_{\Omega_1} \left( \mathbf{U}_{i-s}^*, \dots, \mathbf{U}_J^*, \mathbf{U}_{J+1}^n, \dots, \mathbf{U}_{i+s}^n \right), \end{aligned} \quad (12)$$

where  $\mathbf{H}_{\Omega_0}$  and  $\mathbf{H}_{\Omega_1}$  denote the operator  $\mathbf{H}$  with physical parameters of media  $\Omega_0$  and  $\Omega_1$ , respectively. Some comments about the interface method:

- since the jump conditions are linear, the work is mainly carried out during a preprocessing step. At each time step, only small matrix-vector products (11) are required to compute the modified values. The computational cost is therefore negligible in comparison with that of time-marching;
- the matrix  $\mathbf{M}$  in (10) depends on the jump conditions and on the position of  $\alpha$  inside the mesh. Using the modified values (12) introduces into the scheme a subcell resolution of the interface, which removes the  $O(\Delta x)$  error [18];
- in the limit case where the parameters are the same on both sides of  $\alpha$  and the hydraulic contact is perfect,  $\mathbf{U}_i^* = \mathbf{U}_i^n$  if  $k \geq s$ . Consequently, we again obtain the scheme applied in homogeneous medium;
- with a  $r$ -th order accurate scheme, the local truncation error of (12) at irregular nodes is  $r$ -th order if  $2k - 1 \geq r$  [18]. The fourth-order ADER scheme therefore requires  $k = 3$ . As deduced from [13],  $k = 2$  suffices to ensure fourth-order overall accuracy if no splitting is required;
- GKS analysis [14] was performed in the case of inviscid fluids on a uniform grid to determine the stability of the hybrid scheme made up of (7) and (12). This analysis is based on the possible existence of discrete increasing modes emitted solely by the interface without any incident field [21]. A parametric study showed that the hybrid scheme was stable in tests 1 and 2 presented below, whatever the position of the interface, with  $k = 1, 2, 3$ .

## 4 Numerical experiments

A 400-m domain with an interface at  $\alpha = 200.67$  m is studied. Incident, reflected and transmitted waves are denoted by I, R and T. Fast and slow waves are denoted by F and S. Analytical and numerical solutions are shown in solid lines and circles. Vertical dotted lines denote the position of mesh refinement. If  $\eta = 0$  in both media, computing the analytical solution is straightforward. Otherwise, it follows from a Fourier analysis. The source is a  $C^4$  truncated sinusoid with a central frequency  $f = 30$  Hz, as shown in Figure 2-a. On the main grid,  $\Delta x = 1$  m, and the computations are performed with CFL = 0.9.

Two sets of physical parameters are used (table 1). In tests 1 and 2, they model sandstone saturated with water ( $\Omega_0$ ) and schist saturated with water



Parameters	$\Omega_0$	$\Omega_1$	$\Omega_0$	$\Omega_1$
$\rho_f$ (kg/m <sup>3</sup> )	1040	1040	1040	10
$\eta$ (Pa.s)	0	0	$10^{-3}$	$2.2 \cdot 10^{-5}$
$\rho_s$ (kg/m <sup>3</sup> )	2650	2211	2650	2650
$\mu$ (Pa)	$1.85 \cdot 10^9$	$3.54 \cdot 10^9$	$1.85 \cdot 10^9$	$1.85 \cdot 10^9$
$\phi$	0.3	0.01	0.3	0.3
$a$	2	2	2	2
$\kappa$ (m <sup>2</sup> )	$10^{-12}$	$10^{-16}$	$10^{-12}$	$10^{-12}$
$\lambda_f$ (Pa)	$8.40 \cdot 10^9$	$4.69 \cdot 10^9$	$8.40 \cdot 10^9$	$2.43 \cdot 10^9$
$\beta$	0.88	0.01	0.88	0.35
$m$ (Pa)	$7.05 \cdot 10^9$	$2.46 \cdot 10^{11}$	$7.05 \cdot 10^9$	$5.37 \cdot 10^7$
$c_1(f)$ (m/s)	2364.9	2314.1	2364.5	1817.8
$c_2(f)$ (m/s)	774.9	1087.7	38.1	30.3
$f_c$ (kHz)	0	0	23	52.5

Table 1

Physical parameters in tests 1-2 (central column) and tests 3-4 (right column).

( $\Omega_1$ ), except that  $\eta = 0$  [9]. This is not physically realistic, but it sheds light on the ADER scheme and on the interface method:  $\mathbf{S} = \mathbf{0}$  means that no splitting (6) is required. Nor is mesh refinement required, since the slow wave propagates. In tests 3 and 4,  $\eta \neq 0$  and  $f \ll f_c$ : the slow wave is a static mode, which puts the focus on the mesh refinement. The parameters model sandstone saturated with water ( $\Omega_0$ ) and with gas ( $\Omega_1$ ).

#### 4.1 Inviscid media

First we consider identical media  $\Omega_0$  linked by an imperfect hydraulic contact  $\xi = 10^{-16} \text{ m.s}^{-1}.\text{Pa}^{-1}$ . Wave conversions are shown in Figure 2-b, and a good agreement is seen between the analytical and numerical values; on this scale, the reflected fast wave is not visible. Since no contrast of Biot's coefficients occurs, the unmodified scheme (7) without the interface method (12) would propagate the incident wave without diffraction.

Test 2 deals with media ( $\Omega_0, \Omega_1$ ) linked by a perfect hydraulic contact  $1/\xi = 0$ . Without using the interface method, GKS analysis and direct simulations show that the scheme is unstable and the solution blows up when crossing  $x = \alpha$ . Figure 3-a gives the analytical and numerical values obtained. The results of convergence studies are presented in Figure 3-b. As mentioned in section 3.3,

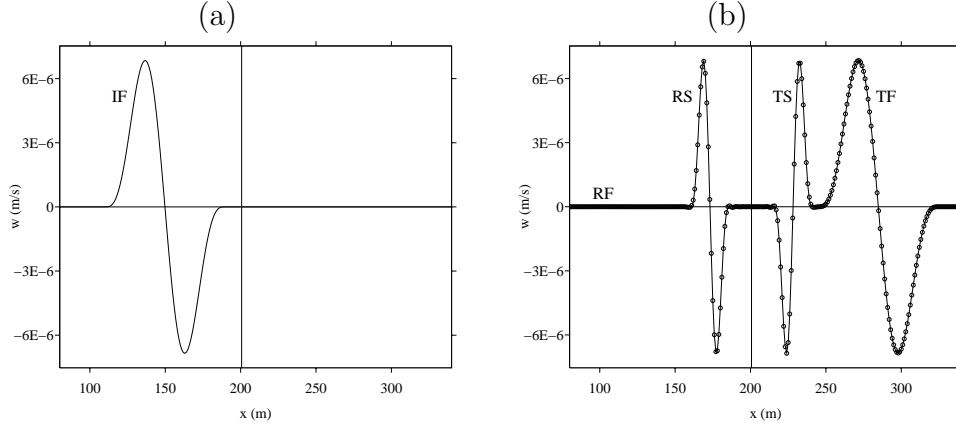


Fig. 2. Test 1: at initial instant (a) and after crossing the interface (b).

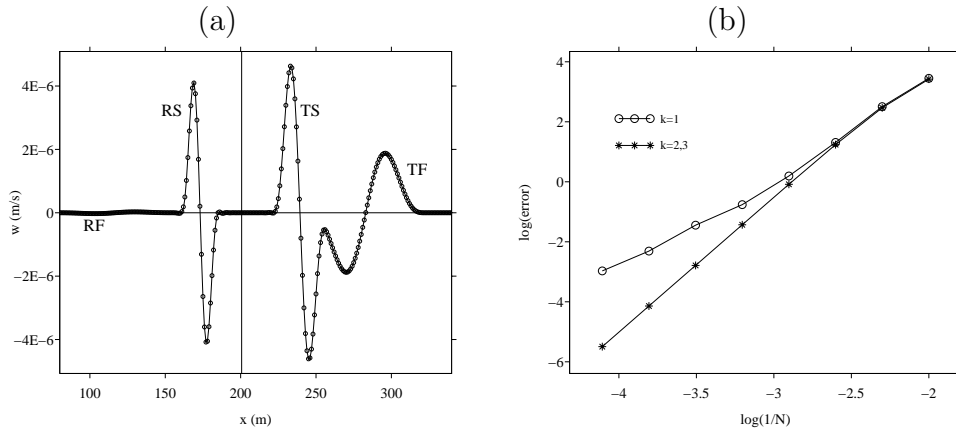


Fig. 3. Test 2: after crossing the interface (a) and convergence measurements (b).

$k = 2$  or  $k = 3$  maintain the fourth-order accuracy of the ADER scheme;  $k = 1$  does not suffice, since it leads to a rate of convergence of only 2.9.

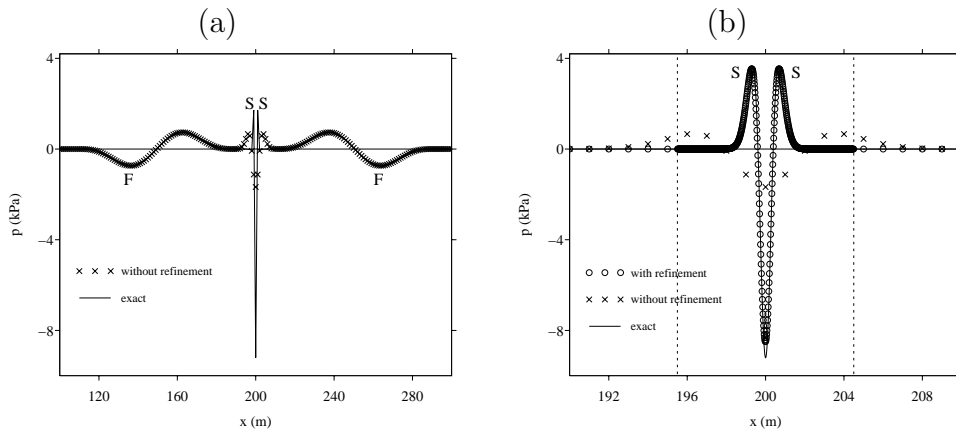


Fig. 4. Test 3: no refinement (a); zoom around  $x_s$ , with and without refinement (b).

## 4.2 Dissipative media

Test 3 focuses on the homogeneous dissipative medium  $\Omega_0$  excited by a ponctual stress source at  $x_s = 200$  m. A direct discretization with no splitting would give  $\text{CFL} = 0.03$  (5). A snapshot of  $p$  is shown in figure 4-a. Fast waves are advected rightwards and leftwards while the slow waves remain localized around  $x_s$ , and vary considerably on small spatial scales. Their numerical values are highly smeared. In figure 4-b, the crude solution is compared with a solution refined 64 times around  $x_s$ . Good agreement is seen between the exact and refined numerical values.

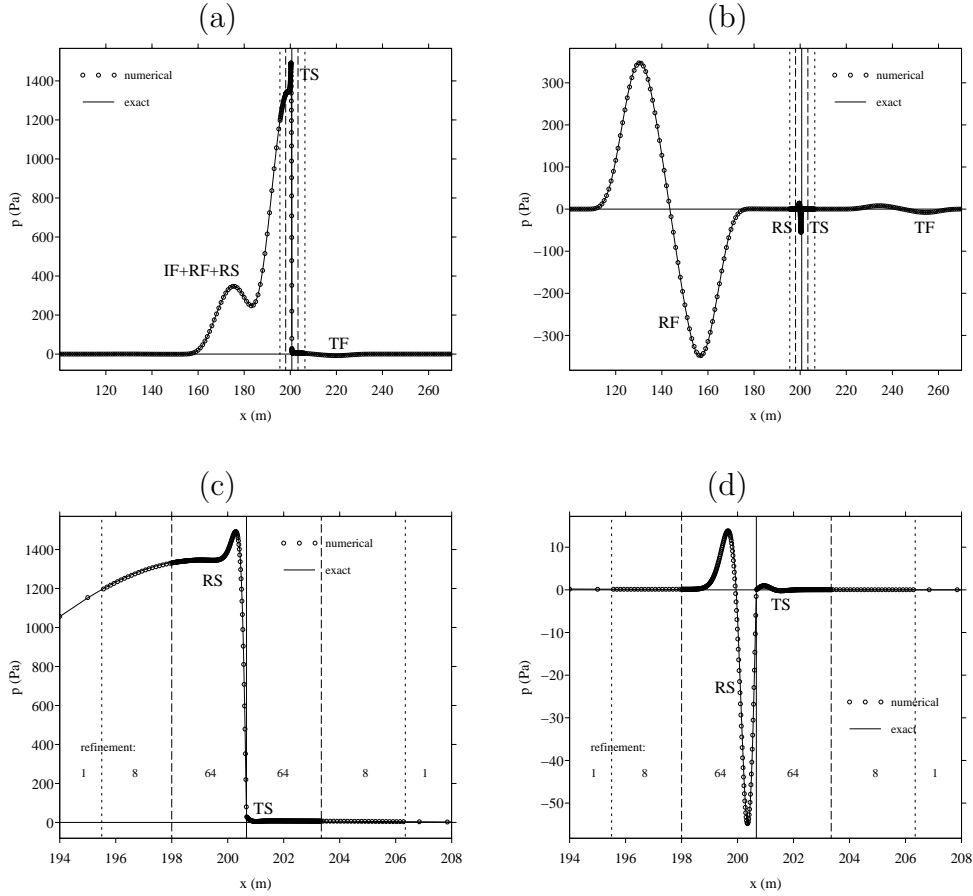


Fig. 5. Test 4: snapshot of  $p$  at time  $t_1$  (a-c) and  $t_2$  (b-d). Zoom around  $\alpha$  (c-d).

Test 4 is performed on  $(\Omega_0, \Omega_1)$ , with  $1/\xi = 0$ . Two successive mesh refinements with factors 8 and 64 are used around  $\alpha$ . Snapshots of  $p$  at  $t_1$  and  $t_2 > t_1$  are shown in figure 5. During the interaction of the incident fast wave with the interface (a-c), since the slow waves have a greater amplitude than that of fast waves, they play a crucial role in the balance of momentum and mass: a poor assessment of the slow waves would invalidate that of the other ones. At  $t_2$  (b-d), the slow waves are greatly attenuated and remain localized near  $\alpha$ .

## 5 Conclusion

Numerical modeling of 1-D transient Biot's equations was addressed here with frequency-independent coefficients. Three numerical tools were combined to obtain a method able to describe accurately the full process of wave propagation: a fourth-order scheme with time-splitting; a space-time mesh refinement; and an immersed interface method. This method is required to account for the properties of the slow wave whatever the frequency. Obviously, the application of the method is limited to the validity domain of the Biot's model in the low-frequency range. Further research is suggested:

- studying the high-frequency range [4], where  $\kappa/\eta$  is proportional to  $f^{1/2}$ . Fractional derivatives are therefore involved in the time-domain [15];
- accounting for dissipative effects in the solid skeleton;
- coupling Biot's model with nonlinear boundary conditions, to model seismic rupture;
- extending the method to two-dimensional configurations. The validity of each of the numerical tools has already been established in 2D.

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