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EFFECT OF TEMPERATURE ON VIBRATIONS OF PHYSICALLY NONLINEAR PIEZOELECTRIC RODS

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The paper is concerned with a physically nonlinear behavior of piezoelectric rods. Such rods are often employed as transducers. We evaluate the effect of a uniformly distributed temperature on vibrations of a piezoelectric rod polarized in the axial direction and subjected to a harmonic (periodic-in-time) electric field. We conclude that temperature can significantly affect vibrations of a physically nonlinear transducer in a broad spectrum of frequencies. It is also shown that physical nonlinearity has a strong effect on the dynamic response of piezoelectric rods to alternating electric fields, especially in the vicinity of the resonance frequency and at high power fields.

1. INTRODUCTION

Physically linear relationships between the tensors of stress and strain, on one hand, and the vectors of electric field and electric displacement, on the other, can be employed only if the electric field is weak and, as a result of that, the nonlinear effects in piezoelectric materials may not be considered. While geometric nonlinearity can be nevertheless partially incorporated into the linear equations by an appropriate choice of the strain-displacement relationships, neither the geometric nonlinearity, nor physical nonlinearity of the piezoelectric material could be consistently accounted for in the linear constitutive equations.

Constitutive equations containing nonlinear products of the components of the strain tensor (geometrically nonlinear effect) and physically nonlinear terms have been obtained by Maugin et al. [1]. However, the complexity of these equations, as well as the lack of experimental data for the coefficients in the nonlinear terms, prevented wide acceptance of nonlinear constitutive equations in practical design efforts and in practical applications. This circumstance makes it important to elucidate a relative contribution of geometrically and physically nonlinear terms to the dynamic response of piezoelectric materials subject to relatively strong electric fields. There is an obvious incentive to assess the need in considering nonlinear terms in the dynamic analysis of structural elements made of such materials. While the comprehensive review of studies of the effect of physical nonlinearity is outside the scope of this paper, the reader is referred to the forthcoming papers [2], [3] for a reference on this subject.

The problem of the mechanical behavior of piezoelectric transducers driven by voltage, which is applied along the direction of the wave propagation, has been considered since the mid-sixties (see, for example, the monograph by Mason, [4]). Physically nonlinear effects in piezoelectric transducers were considered in recent papers of Tan and Tong [5], [6], Wagner [7], [8] and Wagner and Hagedorn [9]. The recent papers by the authors [2], [3] also address this

problem. However, thermal effects were not included in these formulations.

The present paper contains a comprehensive formulation of equations of motion of a piezoelectric rod driven by a dynamic voltage in the direction of the wave propagation (axial direction) as shown in Fig. 1. The emphasis is on the assessment of the effect of temperature.

$$\begin{aligned} \Psi = & \frac{1}{2}C_{\alpha\beta}\varepsilon_\alpha\varepsilon_\beta - e_{m\alpha}E_m\varepsilon_\alpha - \frac{1}{2}e_{m\alpha\beta}E_m\varepsilon_\alpha\varepsilon_\beta - \frac{1}{2}\varepsilon_{mn}E_mE_n - \frac{1}{6}\varepsilon_{mnp}E_mE_nE_p - \frac{1}{2}l_{mn\alpha}E_mE_n\varepsilon_\alpha \\ & - \lambda_\alpha\varepsilon_\alpha T - \delta_\alpha\varepsilon_\alpha T^2 - p_m E_m T - \frac{1}{2}\rho_{\alpha\beta}\varepsilon_\alpha\varepsilon_\beta T - \eta_m E_m T^2 - \frac{1}{2}\chi_{mn}E_mE_n T - \kappa_{m\alpha}E_m\varepsilon_\alpha T \end{aligned} \quad (1)$$

Here $C_{\alpha\beta}$ are elastic stiffness constants; ε_α are strains; $e_{m\alpha}$ are piezoelectric constants; E_m are components of electric field in the corresponding direction; $e_{m\alpha\beta}$ are electroelastic constants; ε_{mn} and ε_{mnp} are dielectric coefficients (permittivity and third-order dielectric coefficients, respectively); $l_{mn\alpha}$ are electrostrictive coefficients; T is temperature in excess of the reference value; λ_α are thermoelastic coefficients; δ_α are second order thermoelastic coefficients; p_m are pyroelectric coefficients; $\rho_{\alpha\beta}$, η_m , χ_{mn} , $\kappa_{m\alpha}$ are other coefficients accounting for the interaction of the tensor of strains, vector of electric field and temperature.

Note that the terms, which are dependent on temperature only, are excluded from (1), since these terms do not affect the

$$\sigma_\alpha = \frac{C_{\alpha\beta}\varepsilon_\beta - e_{m\alpha}E_m - e_{m\alpha\beta}E_m\varepsilon_\beta - \frac{1}{2}l_{mn\alpha}E_mE_n - \lambda_\alpha T - \delta_\alpha T^2 - \rho_{\alpha\beta}\varepsilon_\beta T - \kappa_{m\alpha}E_m T}{2} \quad (3)$$

2. ANALYSIS

The energy density is obtained without considering the geometric nonlinearities as an extension of the density expression obtained by Maugin et al. [1] for the case when the effect of temperature was not accounted for:

dynamic problem under consideration. Additional higher-order terms could be included in (1), but this is not advisable, because their effect is relatively small and the information about some of the coefficients in (1) is not available, even for this “truncated” formulation.

The constitutive relationships for the stresses and electric displacements are obtained as

$$\sigma_\alpha = \frac{\partial\Psi}{\partial\varepsilon_\alpha}, \quad D_m = -\frac{\partial\Psi}{\partial E_m} \quad (2)$$

This results in the following expressions for the components of the stress tensor and the electric displacements:

$$D_m = \underline{e_{m\alpha}\varepsilon_\alpha} + \frac{1}{2}e_{m\alpha\beta}\varepsilon_\alpha\varepsilon_\beta + \underline{\varepsilon_{mn}E_n} + \frac{1}{2}\varepsilon_{mnp}E_nE_p + l_{mn\alpha}E_n\varepsilon_\alpha + p_mT + \eta_mT^2 + \underline{\chi_{mn}E_nT} + \underline{\kappa_{m\alpha}\varepsilon_\alpha T} \quad (4)$$

The terms underlined in the right side of these equations are retained in the linear formulation. As indicated above, numerous authors implicitly account for the nonlinearity by using variable coefficients of these terms, dependent on stress, electric field or temperature (see discussion in [2]). It is observed that by retaining linear terms in strain, electric field and temperature one could reduce the eqns. (3, 4) to the equations derived using the thermodynamic electric Gibbs function [4]. Notably, the terms

quadratic in strain and/or electric field components and multiplied by temperature, i.e., $\varepsilon_\alpha\varepsilon_\beta T$, $\varepsilon_\alpha E_m T$, $E_m E_n T$, are excluded from (3) and (4) as a result of eliminating higher-order temperature dependent terms in (1).

Eqns. (3, 4) are further simplified reflecting the assumption that temperature is independent of time. Accordingly, retaining dynamic terms only yields

$$\sigma_\alpha = \underline{C_{\alpha\beta}\varepsilon_\beta} - e_{m\alpha}E_m - e_{m\alpha\beta}E_m\varepsilon_\beta - \frac{1}{2}l_{mn\alpha}E_mE_n - \underline{\rho_{\alpha\beta}\varepsilon_\beta T} - \underline{\kappa_{m\alpha}E_m T} \quad (5)$$

$$D_m = \underline{e_{m\alpha}\varepsilon_\alpha} + \frac{1}{2}e_{m\alpha\beta}\varepsilon_\alpha\varepsilon_\beta + \underline{\varepsilon_{mn}E_n} + \frac{1}{2}\varepsilon_{mnp}E_nE_p + l_{mn\alpha}E_n\varepsilon_\alpha + \underline{\chi_{mn}E_m T} + \underline{\kappa_{m\alpha}\varepsilon_\alpha T} \quad (6)$$

In the subsequent analysis of forced vibrations generated by an alternating electric field we use the expression (5) for the stress. It is noteworthy that “material constants” in (5) are functions of the electric field and temperature:

$$\begin{aligned} C_{\alpha\beta} &= C_{\alpha\beta}^{E,T}, & e_{m\alpha} &= e_{m\alpha}^T, & e_{m\alpha\beta} &= e_{m\alpha\beta}^T, \\ l_{mn\alpha} &= l_{mn\alpha}^T, & \rho_{\alpha\beta} &= \rho_{\alpha\beta}^E \end{aligned} \quad (7)$$

Here the subscripts indicate that the corresponding term was measured at a

constant value of the vector of the electric field E or temperature T . Equations (5) and (6) represent a weak physically nonlinear formulation for a piezoelectric material, where the higher-order terms discussed above were not considered.

In the problem addressed in this paper, we are concerned with a dynamic response of a piezoelectric rod to the electric field applied in the axial direction, i.e. $E_3(t) = E_z(t)$. Accordingly, the nonzero elements of the tensor of stress given by (5) are

$$\begin{aligned} \begin{Bmatrix} \sigma_r \\ \sigma_\theta \\ \sigma_z \end{Bmatrix} &= \begin{bmatrix} C_{11} - e_{311}E_3 - \rho_{11}T & C_{12} - e_{312}E_3 - \rho_{12}T & C_{13} - e_{313}E_3 - \rho_{13}T \\ C_{12} - e_{321}E_3 - \rho_{21}T & C_{22} - e_{322}E_3 - \rho_{22}T & C_{23} - e_{323}E_3 - \rho_{23}T \\ C_{13} - e_{331}E_3 - \rho_{31}T & C_{23} - e_{332}E_3 - \rho_{32}T & C_{33} - e_{333}E_3 - \rho_{33}T \end{bmatrix} \begin{Bmatrix} \varepsilon_r \\ \varepsilon_\theta \\ \varepsilon_z \end{Bmatrix} - \quad (8) \\ \begin{Bmatrix} e_{31} - \kappa_{31}T \\ e_{32} - \kappa_{32}T \\ e_{33} - \kappa_{33}T \end{Bmatrix} E_3 - \frac{1}{2} \begin{Bmatrix} l_{331} \\ l_{332} \\ l_{333} \end{Bmatrix} E_3^2 \end{aligned}$$

The equation of motion of a polarized piezoelectric rod with arbitrary boundary conditions driven by an electric field in the axial direction can be derived from the Hamilton principle [2]. In this paper, such an equation was obtained from the generalized

Galerkin method that enables one to account for the static (stress) boundary conditions. In the present problem it is assumed that the rod is clamped at one end ($z=0$), while its other end is free. In such a case, the generalized Galerkin procedure implies that

$$\int_0^h (\sigma_z - m\ddot{w})Z_i(z)dz - \sigma_z(z=h)Z_i(h) = 0 \quad (9)$$

In (9), h is the height of the rod and $Z_i(z)$ are generalized coordinates that are used to characterize axial vibrations in terms of series that represent a sum of products of functions of time and coordinate:

$$w(z,t) = \sum_i Z_i(z)T_i(t) \quad (10)$$

The axial stress that appears in (9) can be derived from (8). Particularly, in a one-dimensional problem, the expression for this stress is

$$\sigma_z = (C_{33} - e_{333}E_3 - \rho_{33}T)\epsilon_z - (e_{33} - \kappa_{33}T)E_3 - \frac{1}{2}l_{333}E_3^2 \quad (11)$$

where $\epsilon_z = w_{,z}$.

The axial displacements are chosen in the form of series satisfying the corresponding boundary conditions. However, it appears impossible to choose such series, while satisfying the conditions both at the “anchored” cross section $z = 0$ (where the axial displacement is equal to zero) and at the free end of the rod $z = h$ (where the axial stress is equal to zero). Therefore, the solution is sought in the form that satisfies only the kinematic condition at the clamped end ($z=0$), while the boundary

condition at $z=h$ is accounted for through the generalized Galerkin formulation (9):

$$w = \sum_i W_i(t)h \sin \frac{i\pi z}{2h} \quad (12)$$

The substitution of (11) and (12) into (9), where $Z_i = \sin \frac{i\pi z}{2h}$, leads to a system of uncoupled differential equations. In particular, the i -th equation of this system is

$$\frac{mh^2}{2}\ddot{W}_i + (C_{33} - e_{333}E_3 - \rho_{33}T)\frac{i^2\pi^2}{8}W_i = \left((e_{33} - \kappa_{33}T)E_3 + \frac{1}{2}l_{333}E_3^2 \right) \sin \frac{i\pi}{2} \quad (13)$$

It is assumed that the rod is driven by a periodic harmonic electric field, $E_3(t) = E \cos \omega t$. Then the equation of

motion (13) becomes a nonhomogeneous Mathieu equation:

$$\frac{mh^2}{2} \ddot{W}_i + (C_{33} - \rho_{33}T - e_{333}E \cos \omega t) \frac{i^2 \pi^2}{8} W_i = \left(\frac{1}{4} l_{333} E^2 + (e_{33} - \kappa_{33}T) E \cos \omega t + \frac{1}{4} l_{333} E^2 \cos 2\omega t \right) \sin \frac{i\pi}{2} \quad (14)$$

The coefficient of thermal expansion does not enter the equation of motion, since the thermal expansion does not affect the motion of the rod, which is unconstrained in the axial direction in a geometrically linear formulation.

The coefficients in the constitutive equations and in the equation of motion, i.e. C_{33}, e_{33}, e_{333} and l_{333} , depend on temperature. In addition, the explicit effect of temperature is present through the terms proportional to T . Accordingly, available information on the effects of temperature on the stiffness and

“material constants” based on a comparison of empirical data with linear constitutive equations reflects two basic phenomena: (a) effect of temperature on the material constants, and (b) terms explicitly dependent on temperature that are not considered in a linear formulation.

The present formulation can be extended to the problem of axisymmetric axial-radial vibrations. In this case, extending the generalized Galerkin formulation implies [2], [3]:

$$\int_0^{2\pi} \int_0^h \int_0^a \left(\frac{\partial \sigma_r}{\partial r} + \frac{\sigma_r - \sigma_\theta}{r} - m\ddot{u} \right) r \sin \frac{i\pi r}{2a} dr dz d\theta - \int_0^{2\pi} \int_0^h \sigma_r(r=a) a \sin \frac{i\pi}{2} dz d\theta = 0$$

$$\int_0^{2\pi} \int_0^h \int_0^a \left(\frac{\partial \sigma_z}{\partial z} - m\ddot{w} \right) r \sin \frac{i\pi z}{2h} dr dz d\theta - \int_0^{2\pi} \int_0^h \sigma_z(z=h) r \sin \frac{i\pi}{2} dr d\theta = 0 \quad (15)$$

Here a is the radius of the rod, the axial motion is given by (12) and the radial motion is represented by

$$u = \sum_{i=1} U_i(t) \sin \frac{i\pi r}{2a} \quad (16)$$

The substitution of the stresses (8) and the expression for the harmonic electric field in the axial direction as that considered above yield the system of two coupled nonhomogeneous Mathieu equations for each

value of an integer i that is omitted here for brevity.

3. NUMERICAL EXAMPLES

The numerical analysis is confined to the case of axial vibrations, i.e., the effect of radial motion is disregarded. Accordingly, the analysis relies on the solution of the nonhomogeneous Mathieu equation (14). This solution is discussed in detail in the paper by Birman [2]. The height of the rod does not

affect the following results. The amplitude of the electric field was chosen to reflect reported highest allowable fields that can be applied to currently used piezoelectric materials (less or equal to 2MV/m).

The following examples illustrate the effect of temperature on two representative materials, i.e. PZT-5H and PZN-4.5%PT. The properties of these materials at room temperature are shown in Table 1 [2]. The information on the effect of temperature on material properties of piezoelectrics is limited, but it is known that this effect on the stiffness within the range of operating temperatures is small. Therefore, in the subsequent examples it is assumed that $(C_{33} - \rho_{33}T) \approx C_{33}$. The effect of temperature on the coefficient l_{333} is not known, but numerical analysis (Birman, 2005) showed that the effect of this coefficient is negligible. Although data on the effect of temperature on the piezoelectric coefficient e_{33} are not available, it is known that the piezoelectric constant d_{33} increases with temperature [10], [11]. Therefore, the former constant should also increase with temperature reflecting the relationship [4]

$$e_{33} = d_{33}C_{33}^E$$

where the stiffness is measured for a constant electric field. As reported, the variations of the coefficient d_{33} with temperature can be as high as hundreds percent, even if temperature varies between zero and 100°C (see for example, [11]). Accordingly, if the stiffness remains little affected by such variations of temperature, the coefficient $e_{33} - \kappa_{33}T$ may significantly increase within such limited range of temperature variations. This is reflected in the following examples (a direct correlation between temperature and the value of the above-mentioned coefficient is impossible due to the lack of experimental data).

Numerical results were obtained for a physically “reasonable” range of variations of the temperature adjusted piezoelectric coefficient $\bar{e}_{33} = e_{33} - \kappa_{33}T$ that accounts for both the effect of temperature on the piezoelectric constant, as well as the presence of the term explicitly dependent on temperature. The examples focus on the analysis of the effect of the adjusted piezoelectric coefficient \bar{e}_{33} on the response of the rod, assuming that other thermal effects, while explicitly shown in the theoretical solution, are less important.

Table 1. Material constant of representative piezoelectrics

Material	$e_{33} \left(\frac{C}{m^2} \right)$	$e_{333} \left(\frac{C}{m^2} \right)$	$C_{33} \text{ (GPa)}$	$l_{333} \left(\frac{F}{m} \right)$
PZT-5H	24.54 (or 31.02)	$5.7 \cdot 10^4$	108.0	$-21.21 \cdot 10^{-6}$
PZN-4.5%PT	12.1 (or 13.02)	$4.2 \cdot 10^4$	89.0	$-1.61 \cdot 10^{-6}$

The effect of physical nonlinearity in the constitutive equations has been investigated by Birman [2] and Birman and

Suhir [3]. It was shown that this nonlinearity has a significant effect on the amplitude of forced axial vibrations of the rod, particularly

in the vicinity of resonant frequencies. For example, the ratio of amplitude obtained by the linear theory to its nonlinear counterpart (R) is shown as a function of the ratio of the frequency of the electric field to the fundamental frequency of axial vibrations (F) in Fig. 2. As follows from this figure, physical nonlinearity cannot be neglected in the vicinity of the resonance conditions. Moreover, the width of the spectrum of driving frequencies, where neglecting physically nonlinear effects is unacceptable, becomes broader for higher amplitude of the electric field.

The effect of the adjusted piezoelectric coefficient (as explained above, this coefficient reflects thermal contributions) is shown in Figs. 3-5 for PZT-5H rods and three different amplitudes of the electric field. The results for PZN-4.5% rods are also shown in Fig. 6. In these figures, the amplitude of axial vibrations, i.e. W , is depicted as a function of the nondimensional driving frequency F . The results shown in Figs. 3-6 clearly illustrate that a higher temperature results in larger amplitude of forced axial vibrations of a rod (transducer). Predictably, the amplitude of motion increases with the amplitude of the electric field. However, a higher electric field does not change the qualitative effect of temperature.

4. CONCLUSIONS

The following conclusions can be drawn from the performed analysis.

- The paper presents a physically nonlinear formulation for a piezoelectric material, accounting for thermal effects. The formulation is applied to the solution of the problem of forced vibrations of a piezoelectric rod (transducer) driven by an alternating electric field applied in the axial direction. The solution is obtained

by using the generalized Galerkin procedure, satisfying all the boundary conditions.

- The carried out numerical examples focus on the most prominent thermal contribution reflected in the adjusted piezoelectric coefficient $\bar{e}_{33} = e_{33} - \kappa_{33}T$ that accounts for both the explicit as well as implicit effects of temperature. The range of variations in this coefficient is chosen based on available experimental data. As follows from numerical examples, physical nonlinearity cannot be disregarded when predicting the response of a rod in the vicinity of resonant frequency. An elevated temperature can result in a noticeable increase of the amplitude of motion.

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Figures

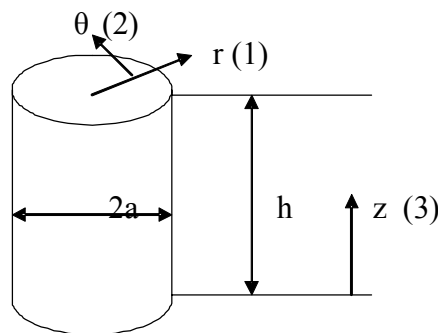


Fig. 1. Piezoelectric rod with the coordinate systems (r- θ -z and 1-2-3) used in the analysis. The end $z=0$ is assumed clamped, while the end $z=h$ is free.

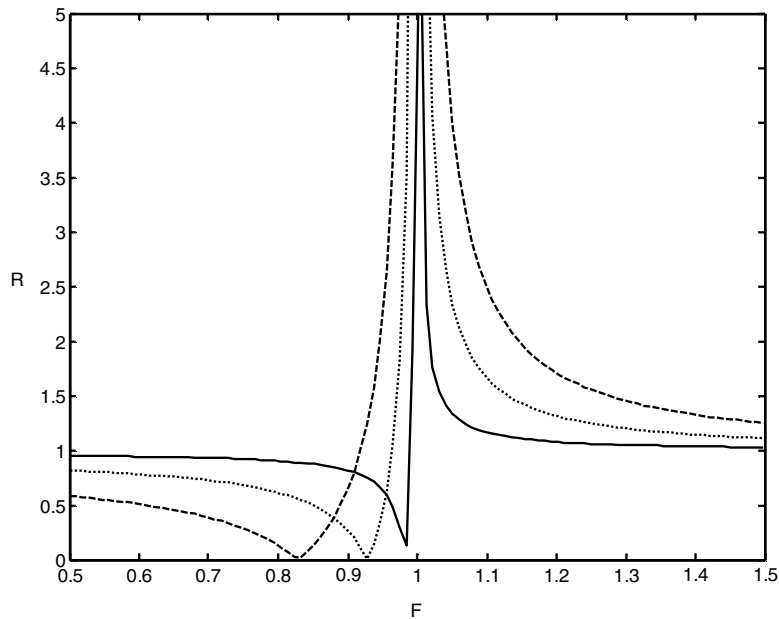


Fig. 2. Effect of voltage amplitude on the ratio of linear-to-nonlinear amplitudes of vibrations for a PZT-5H rod. Curves correspond to the amplitude voltage of 0.5MV/m (——), 1.0MV/m (.....), and 1.5MV/m (-----), respectively.

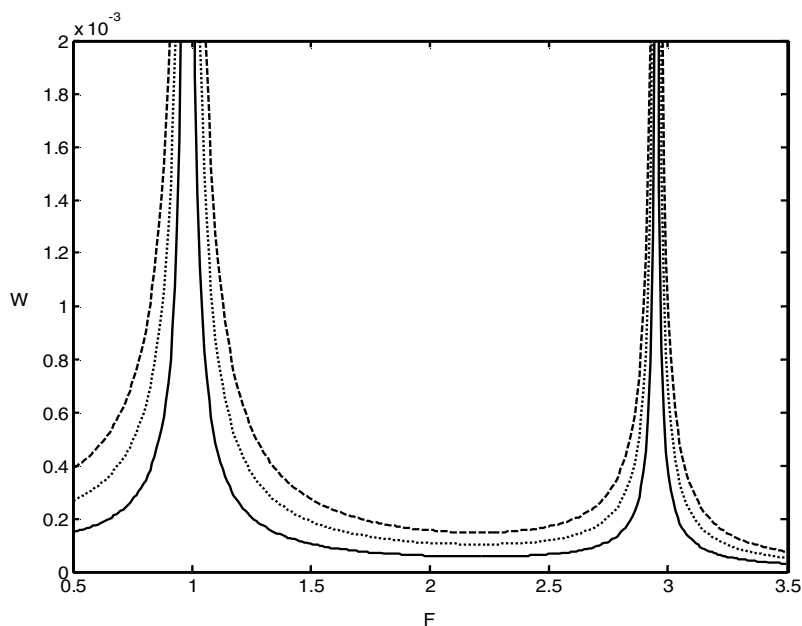


Fig. 3. Effect of the piezoelectric coefficient \bar{e}_{33} on the amplitude of physically nonlinear vibrations of a PZT-5H rod. The amplitude of the electric field is equal to 0.5MV/m. Curves correspond to $\bar{e}_{33}=24.5 \text{ C/m}^2$ (——), 44.5 C/m^2 (.....), and 64.5 C/m^2 (-----), respectively.

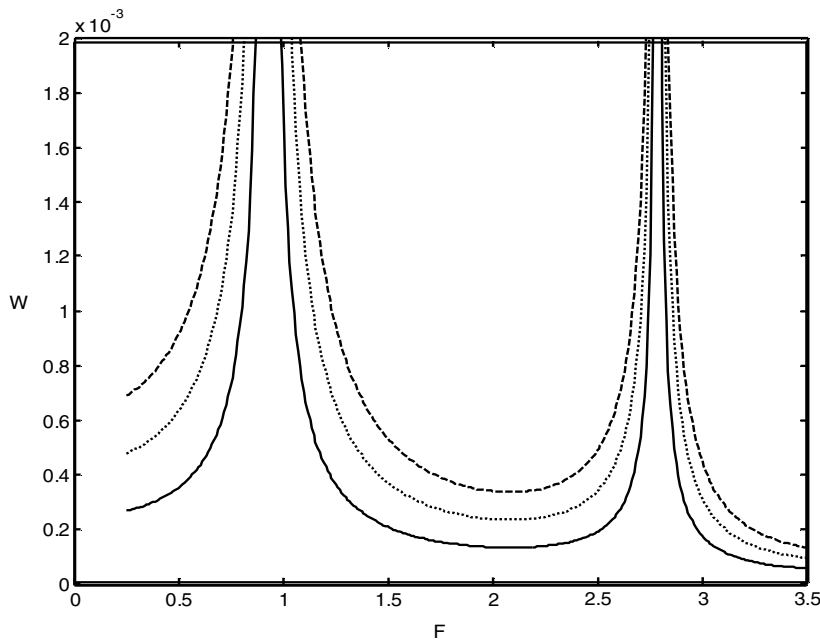


Fig. 4. Effect of the piezoelectric coefficient \bar{e}_{33} on the amplitude of physically nonlinear vibrations of a PZT-5H rod. The amplitude of the electric field is equal to 1.0MV/m. Curves correspond to $\bar{e}_{33}=24.5 \text{ C/m}^2$ (——), 44.5 C/m^2 (.....), and 64.5 C/m^2 (-----), respectively.

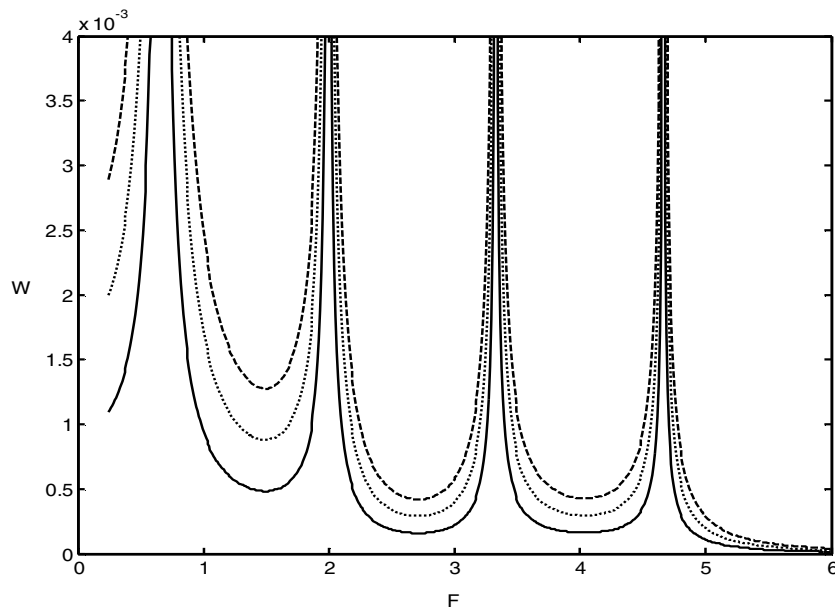


Fig. 5. Effect of the piezoelectric coefficient \bar{e}_{33} on the amplitude of physically nonlinear vibrations of a PZT-5H rod. The amplitude of the electric field is equal to 2.0MV/m. Curves correspond to $\bar{e}_{33}=24.5 \text{ C/m}^2$ (——), 44.5 C/m^2 (.....), and 64.5 C/m^2 (-----), respectively.

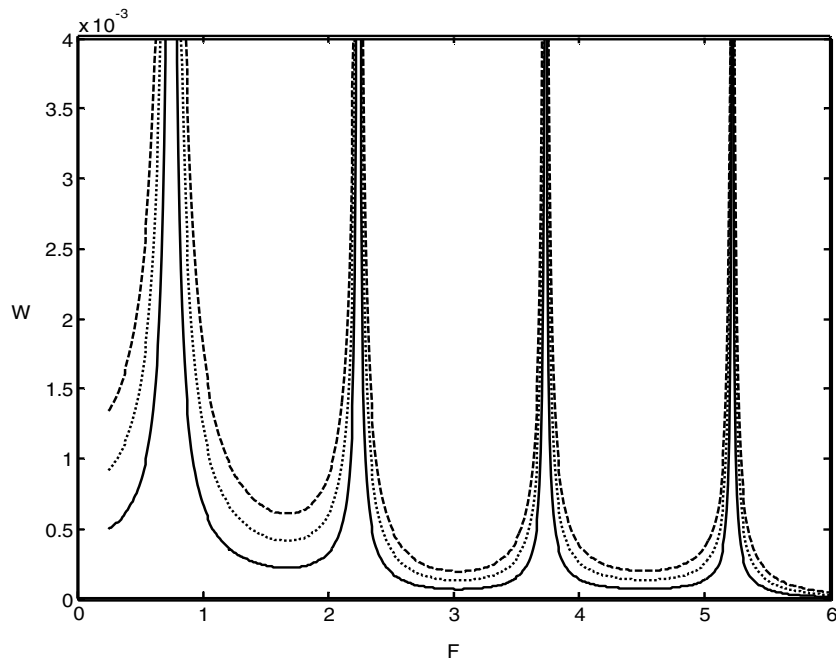


Fig. 6. Effect of the piezoelectric coefficient \bar{e}_{33} on the amplitude of physically nonlinear vibrations of a PZN-4.5% PT rod. The amplitude of the electric field is equal to 2.0MV/m. Curves correspond to $\bar{e}_{33}=12.1$ (—), 22.1 (.....), and 32.1 C/m² (-----), respectively.