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FLEXIBLE PROFILE COMPACT THERMAL MODELS FOR PRACTICAL GEOMETRIES

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ABSTRACT

Recent advances in Compact Thermal Models (CTM) have led to the emergence of a new concept allowing models to be created at any desired order of accuracy. Traditionally, increasing precision was attained by increasing the number of nodes. This strategy faces many problems. In particular, it fails in the case of multiple heat sources (MCM) and/or stacked dies, because different operating conditions will lead to different temperature and heat flux profiles that will require different node partitioning in order to be matched. Besides, classical approaches face a difficulty in selecting appropriate node size and position, as well as the inability to provide an a priori estimate of the number of nodes needed. The new concept is based on the use of a flexible profile to account for different possible uses of the model. In particular it can deal with different patterns of heat generation encountered in MCM and stacked dies, and hence it is truly Boundary Conditions Independent (BCI). Moreover, the new approach gives access to the tangential temperature gradient. This valuable information for designers in order to assess reliability can not be predicted by classical compact model approaches.

The concept was presented earlier for a simple rectangular 2D structure with surface heating [1]. In this paper, the concept will be generalized to 3D parallelepiped boxes with both surface and/or volumetric heating. The second achievement is possibility to deal with geometries that can be decomposed into boxes.

1. INTRODUCTION

Design of electronic systems involves more and more effects having different physical origins as well as at different levels of abstraction. It has been recognized since long that designing an electronic system can never be done in all its details in a single shot or by a single team. Compact models represent the enabling methodology for top down design approach. It allows different teams working on different physical aspects or at different levels of abstraction to exchange information with other teams and be able to correctly take into

consideration effects that were studied in details by other teams.

Compact models can very be useful, provided they succeed in describing the modeled object intrinsic behavior, regardless of the conditions of its usage. This constitutes the basis of the *Boundary and Initial Conditions Independence* (BICI) requirement [2]. Failing to meet this requirement is THE source of errors in CTMs, which were considered so far as inevitable.

The Flexible Profile approach is a new technology allowing to solve this dilemma, which is constructing a compact model that totally satisfies BICI conditions and hence give a minimal (or at least a controllable) error. This technology was proposed earlier [1] for very simple geometries, which is a constraint that will be removed in this paper by examining geometries of practical interest.

In this paper, the relation between BICI condition and errors will first be explained by reviewing the “compacting” process, i.e. the passage from detailed to compact models. The new technology based on flexible profiles proposed earlier, will briefly be overviewed, before ending with new developments ensuring the generality of the approach to practical geometries.

2. RELATION BETWEEN BICI REQUIREMENT AND COMPACT MODEL ERRORS

The detailed model taking into consideration only classical conduction effects (i.e. no quantum effects) at steady state is governed by the following Poisson’s equation:

$$\nabla \cdot (\lambda \nabla T(\mathbf{r})) = -q_v(\mathbf{r}) \quad \mathbf{r} \in \Omega \quad (2.1a)$$

$$\begin{aligned} T|_{\partial\Omega_D} &= T_s; \quad \lambda \mathbf{n} \cdot \nabla T|_{\partial\Omega_N} = q_s; \\ \lambda \mathbf{n} \cdot \nabla T|_{\partial\Omega_R} &= h(T_\infty - T|_{\partial\Omega_R}) \end{aligned} \quad (2.1b)$$

where T is the temperature field, q_v is a volumetric heat generation term, λ is the thermal conductivity and \mathbf{r} the coordinate. Equation (2.1a) describes the relation between temperature and heat fluxes *within* the domain Ω . Since this is a partial differential equation of the elliptic type, it must be supplied with boundary conditions (2.1b) describing *interactions* between the modeled object and

outside world. They can be either of the Dirichlet type on the sub-domain $\partial\Omega_D$ of the boundaries or Neumann on $\partial\Omega_N$ or finally of Robin type on $\partial\Omega_R$, involving the heat transfer coefficient h . Equations (2.1) can be simplified by assuming that λ is constant that may have different values over different sub-domains Ω_i of Ω . Two adjacent domains of different λ can be matched using the following conditions at the interface:

$$T_1 = T_2; \lambda_1 \mathbf{n}_1 \cdot \nabla T_1 + \lambda_2 \mathbf{n}_2 \cdot \nabla T_2 = 0 \quad (2.2)$$

where \mathbf{n}_i is the unit outward normal to domain Ω_i . System (2.1), can be rewritten in a dimensionless form in each sub-domain of constant λ as:

$$\nabla^2 T(\mathbf{r}) = -q_v(\mathbf{r}) \quad \mathbf{r} \in \Omega \quad (2.3a)$$

$$T|_{\partial\Omega_D} = T_s; \quad \lambda \mathbf{n} \cdot \nabla T|_{\partial\Omega_N} = q_s$$

$$\mathbf{n} \cdot \nabla T|_{\partial\Omega_R} = Nu (T_\infty - T|_{\partial\Omega_R}) \quad (2.3b)$$

Where Nu is the Nusselt number ha/λ , a being a characteristic length defined as the biggest dimension or the cubic root of the modeled volume or any other convenient scale. Characteristic temperature difference $[T]$ and heat flux $[Q]$ (in Watts) can assume any set of values selected from boundary conditions (2.1b) provided that the following relation is respected:

$$[Q] = \lambda a [T] \quad (2.4)$$

There are different methodologies that can be used to construct compact models reviewed in [2,3], based on either the structural approach [4-13] or the behavioral approach [14-23]. In both cases, a compact model is issued describing the above distributive detailed model, involving temperatures and heat fluxes everywhere inside Ω and at its boundaries, to a ‘‘lumped’’ representation involving a small number of Degrees Of Freedom (DOFs). This relation can be cast in the form of a set of ‘‘thermal’’ resistances or in the form of a matrix relating temperatures to heat fluxes having a size of the order of 10. The former, resistance network, can always be rewritten in the form of a matrix.

Compact Model state variables, for which the matrix gives interrelations, are temperatures and heat fluxes over compact model ‘‘nodes’’. The latter are defined as zones of finite extent either on the boundary or inside the domain for which heat is being exchanged with the surroundings or due to internal generation. These lumped state variables, T_i or q_i over node i , are single numbers over each node approximating there distributed analogous quantities which are temperature or heat flux profiles over each node. Hence, by construction, CTMs in classical approaches are indifferent to T or q profiles. In other words, if a classical CTM was used to predict the behavior of an object under the influence of two different sets of boundary conditions having different profiles but with the same average, then the CTM will predict the same behavior for both cases, despite the fact that in reality, both cases behave differently. One can always

adjust CTM parameters to give correct results for one of the cases, but never both. The link between the departure from the BICI constraint and CTM errors has thus been established. The Delphi approach consists of letting the CTM approximate as good as possible a set of ‘‘relevant’’ boundary conditions. Hence, BICI constraint is considered as an ideal condition that can only be approximated, but never totally achieved.

The Flexible Profile methodology, to be described in the next section, was designed to overcome these problems.

3. THE FLEXIBLE PROFILE TECHNOLOGY

In this section, a brief review will be given of the Flexible Profile technology proposed earlier [1]. A modified Green’s function technique [24] is used to ‘‘compact’’ the detailed model (2.3). In fact, a Green’s function is used satisfying the following conditions:

$$\nabla^2 G(\mathbf{r}, \mathbf{r}') = -\delta(\mathbf{r} - \mathbf{r}') \quad \mathbf{r}, \mathbf{r}' \in \Omega \quad (3.1a)$$

$$\mathbf{n} \cdot \nabla G(\mathbf{r}, \mathbf{r}') = \begin{cases} -1 & \mathbf{r} \in \partial\Omega_1 \\ 0 & \mathbf{r} \notin \partial\Omega_1 \end{cases} \quad (3.1b)$$

$$\int_{\mathbf{r} \in \partial\Omega_1} G d\mathbf{r} = 0 \quad (3.1c)$$

where $\partial\Omega_1$ is the outside surface sub-domain corresponding to an arbitrarily chosen ‘‘reference’’ node.

Using the above Green’s function, as well as the Green’s theorem, equation (2.3a) can be transformed into:

$$T(\mathbf{r}) - T_{av1} = \sum_{j=1}^N \int_{\mathbf{r}' \in \Omega_j} G(\mathbf{r}, \mathbf{r}') q(\mathbf{r}') d\mathbf{r}' \quad (3.2)$$

where T_{av1} is the average temperature over node 1 and Ω_j is the j^{th} sub-domain at which heat is exchanged, i.e. the sub-domain belonging to compact model ‘‘node’’ j ranging from 1 to N . This sub-domain can be either a volume or a surface depending on the nature of heat source. In the sequel, T_{av1} will disappear, since we will consider that all temperatures are relative to that reference. Equation (3.2) has many important features rendering it a good starting point towards the construction of a high precision compact model:

- It relates T and q at model nodes only, hence it treats the object as a black box reacting to external solicitations,
- It has been obtained analytically with no approximating assumption whatsoever,
- It has not made use at any level of boundary conditions (2.3b): neither the nature (Dirichlet, Neumann or Robin) nor the value nor even the profile shape (uniform, linearly varying, sinusoidal, ...)

The last property deserves a bit more of attention. In order to obtain (3.2) an ‘‘auxiliary’’ function G was used satisfying conditions (3.1), which imply conditions on G , but have nothing to do with conditions on the temperature field T . In other words, expression (3.2) can still *exactly*

satisfy any arbitrarily given set of boundary conditions (2.3a), with the *same* auxiliary function G .

The only difference between (3.2) and a CTM lies in the fact that it involves the whole T and q profile at each node and not just one value as in classical CTMs.

Using any convenient orthonormal set $\phi_i^u(\mathbf{r})$ over each node i such that:

$$\int_{\mathbf{r} \in \Omega_i} \phi_i^u(\mathbf{r}) \phi_i^v(\mathbf{r}) d\mathbf{r} = \delta_{uv} \quad (3.3)$$

We can express T and q profiles as:

$$T(\mathbf{r})|_{\mathbf{r} \in \Omega_i} = \sum_{u=0}^{\infty} T_i^u \phi_i^u(\mathbf{r}) \quad (3.4a)$$

$$q(\mathbf{r})|_{\mathbf{r} \in \Omega_i} = \sum_{u=0}^{\infty} q_i^u \phi_i^u(\mathbf{r}) \quad (3.4b)$$

Hence, by substituting in (3.2) we get after multiplying both sides by $\phi_i^u(\mathbf{r})$, integrating using (3.3) and truncating the series after U terms:

$$T_i^u = \sum_{j=1}^N \sum_{v=0}^U R_{ij}^{uv} q_j^v \quad i \in [1, N], u \in [0, U] \quad (3.5a)$$

$$R_{ij}^{uv} = \int_{\mathbf{r} \in \partial\Omega_i} \int_{\mathbf{r}' \in \partial\Omega_j} \phi_i^u(\mathbf{r}) G(\mathbf{r}, \mathbf{r}') \phi_j^v(\mathbf{r}') d\mathbf{r}' d\mathbf{r} \quad (3.5b)$$

This is the Flexible Profile Compact Model involving as state variables at each node, not only one single value, but rather the coefficients of the expansion of T and q profiles over a given complete set. Taking a sufficient order of precision over each node, typically 0 to 4, one can approximate reasonably well any T or q profile with one and the same model. It is worth noting that classical approaches are explicitly or implicitly equivalent to the proposed approach with the restriction of the number of terms of the series (3.4) to only one term. The proposed flexible profile approach is thus more general and capable of self adapting itself to any arbitrarily given profile to any desired degree of accuracy depending on the number of terms retained in the series (3.4). Although theoretically speaking we need to take an infinite number of terms to get the analytical solution, in practice, due to the diffusive nature of the governing equation (2.3a), profiles are relatively smooth and hence very few terms can give surprisingly good results.

There is another crucial advantage of the proposed approach, based on its ability to calculate profiles and not only single values over each node. This gives access to tangential temperature gradients, which are extremely important for thermo-mechanical analysis.

4. DEVELOPMENT FOR ARBITRARY GEOMETRIES

The main contribution in this work is in the generalization of the Flexible Profile approach presented earlier for very simple geometries to geometries pertaining to packages of MCM and stacked dies, including the possibility of having different materials with different thermal conductivities.

Geometries encountered in electronic systems usually involve outside surfaces that are parallel to one of the

Cartesian coordinate surfaces. Based on this remark, we can easily “break” an arbitrary body having these boundary surfaces into a small number of parallelepipeds that we will call boxes. Each box is a simple 3D parallelepiped that can be modeled by a Flexible Profile CTM using an analytically obtained Green’s function G . In case the reference was node 1 was the whole base area (corresponding to $z = 0$) of the box, the corresponding Green’s function is:

$$G(\mathbf{r}, \mathbf{r}') = \sum_{i,j=0}^{\infty} \psi_i(x, \alpha) \psi_i(x', \alpha) \psi_j(y, \beta) \psi_j(y', \beta) g_k(\gamma, z, z')$$

$$\psi_k(x, c) = \sqrt{2/(c(1 + \delta_{k0}))} \cos(k\pi x/c)$$

$$g_0(z, z') = \begin{cases} z & z < z' \\ z' & z > z' \end{cases} \quad (3.6)$$

$$g_{k>0}(z, z') = \frac{1}{k\pi \operatorname{sh}(k\pi\gamma)} \begin{cases} ch(k\pi(\gamma - z')) ch(k\pi z) & z < z' \\ ch(k\pi z') ch(k\pi(\gamma - z)) & z > z' \end{cases}$$

$$k^2 = i^2/\alpha^2 + j^2/\beta^2$$

Where α, β, γ are dimensions in x, y and z directions.

Similar expressions can be easily found for any other reference node, i.e. not covering the whole base area or lying at side walls.

A matrix relation can thus be obtained for any box describing object intrinsic behavior, i.e. regardless of externally applied conditions in terms of mutual interactions between different T and q profiles. Box nodes are composed of external nodes that were present in the initial body before decomposing it as well as newly created “internal” nodes at the interface between boxes. By joining two different boxes, state variables (T and q coefficients) at common internal nodes have to be equated and hence can be eliminated to get a CTM of the composed object containing, in fine after joining all boxes, only external nodes.

While joining two boxes, care must be made to the fact that the model describing the behavior of each box may have as a reference the average temperature over a node that is different from the reference node of the other box. Hence, CTM of each box has to be obtained with a reference which is the common interface node.

Suppose we have to join two boxes, the following steps need to be followed:

- We first need to construct the CTM for each individual box. Assume that the first box had N nodes, while the second had M nodes. Reference node for each model should be adjusted to be that of the common node.
- Second, we have to concatenate both matrices into one matrix of size $N+M$. The diagonal block going from 1 to N will contain the CTM of the first box, while the diagonal block ranging from $N+1$ to $N+M$ will contain the CTM of the second box. All other elements will be set to zero (nodes of the second box are renumbered).

- Assume the node n has to be joined with node m . Rewrite CTM relations (3.5) in terms of T_i and q_i which are vectors containing expansion coefficients of T and q respectively on node i as follows:

$$T_{i \neq n, m} = \sum_{j \neq n, m} R_{ij} q_j + R_{in} q_n + R_{im} q_m$$

$$T_n = \sum_{j \neq n, m} R_{nj} q_j + R_{nn} q_n + R_{nm} q_m$$

$$T_m = \sum_{j \neq n, m} R_{mj} q_j + R_{mn} q_n + R_{mm} q_m$$

Using the obvious relations:

$$T_n = T_m; q_n + q_m = 0, \text{ we get:}$$

$$T_{i \neq n, m} = \sum_{j \neq n, m} \bar{R}_{ij} q_j$$

$$\bar{R}_{ij} = (R_{in} - R_{im}) Y (R_{nj} - R_{mj})$$

$$Y = (R_{nn} + R_{mm} - R_{nm} - R_{mn})^{-1}$$

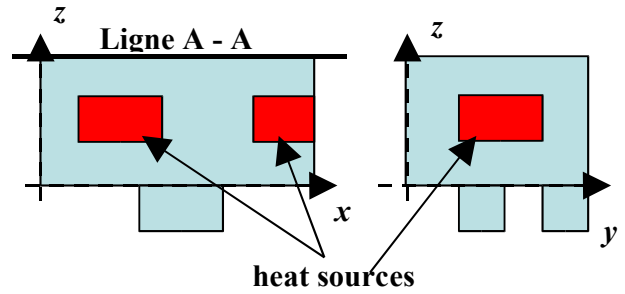


Figure 1 – Model geometry

The above sketched procedure has been applied to obtain the compact model for the composite body shown in figure 1. The obtained flexible profile model was used to predict the behavior of the object when both left and right modules are differentially heated (left module dissipates more energy than right module) and a Robin boundary condition is applied at the upper surface. All model nodes were approximated at order 0, except the upper surface which was approximated at order 4, since it has a wider horizontal extent and may exhibit different maxima for different heating conditions of both modules. Results are given in figure 2 showing temperature profile over the line AA for both the flexible profile CTM and the detailed model solved by a finite volume method. Note that the flexible profile CTM (with 29 degrees of freedom) has predicted the temperature profile almost as good as a detailed model, with as much as 4864 degrees of freedom

5. CONCLUSION

The Flexible Profile methodology has been now generalized to cases that are relevant to practical electronic systems. This includes 3D boxes as well as objects composed of any number of such boxes, including cases where different thermal conductivities are involved. It includes also volumetric as well as surface heating in addition to all different boundary condition types (Diriclet, Neumann and Robin). This allows us to benefit from the advantages of the flexible profile methodology, i.e.:

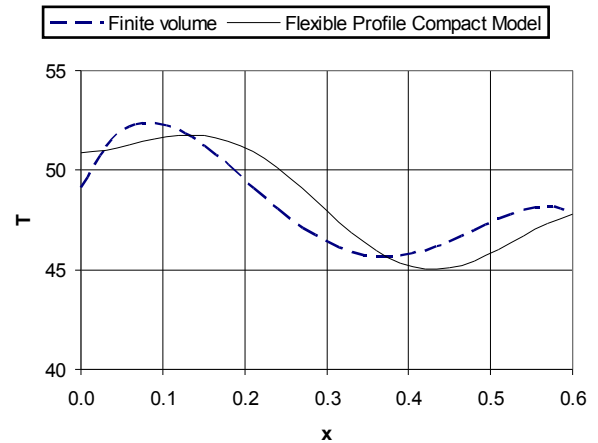


Figure 2 – Flexible profile compact model predictions

- It has a high flexibility to deal with different or variable external or internal heat flux patterns including the cases of MCM and stacked dies.
- The precision is perfectly controllable. It can be increased at will by increasing the number of terms on critical nodes. In all cases, an a priori error estimate is available, based on the rate of decay of matrix elements (3.5).
- It gives the temperature profile variations and not only the average. This furnishes data unavailable before like the maximum temperature and the tangential gradient, which is responsible for thermo-mechanical stresses.

- It does not presuppose any type of boundary conditions (Dirichlet, Neumann or Robin) for the temperature field. A case with linear Robin BC has been worked out showing high performance. The non-linear treatment of the non-linear case has been sketched.

It is to be noted that the new flexible profile approach can be used by other compact modeling methodologies to enhance precision creating thus a new generation of CTM.

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