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A Novel Approach for Generating Dynamic Compact Models of Thermal Networks Having Large Numbers of Power Sources

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Abstract — A novel approach is presented for generating compact dynamic thermal networks having large numbers of power sources. The achievable accuracy of the compact model is controllable. The number of elements of the compact model linearly increases with the number of power sources.

1 Introduction

Various techniques are reported in literature for generating compact models whose responses approximate the responses of large thermal networks having multiple power sources [1–5]. With such techniques, a compact model can be determined by modeling the thermal problem as a whole. Usually, very accurate compact models are generated in this manner. However they have a number of components quadratically increasing with the number n of power sources in the thermal network. That is because distinct components are introduced in the compact models in order to represent each interaction among the n power sources. Thus the resulting compact models are actually compact only for small n . These techniques cannot be applied to generate compact models of thermal networks having large numbers of power sources. For instance, these techniques cannot be applied for generating compact models, at the transistor level, for electronic circuits with many electrical devices and, at the package levels, for modules with many dies.

However, a compact model of a thermal network having a large number of power sources can be obtained as proposed in [2, 4]. Firstly the spatial domain of the thermal problem is partitioned into sub-domains in such a way that only few power sources are in each sub-domain. Secondly the thermal problem in each of these sub-domains is represented by a boundary conditions independent compact model. Lastly, by interconnecting the obtained boundary conditions independent compact models, a compact model of the whole thermal network is obtained. Since each boundary condition independent compact model has a number of components independent from n , and since the number of the boundary condition independent compact models is proportional to n , the composite model has a number of components proportional to n . However, this com-

compact model is not assured to be accurate, since the boundary conditions accurately modeled by each boundary condition independent compact model, in general cannot model the actual boundary conditions of the thermal problem.

In this paper a novel technique is proposed for generating dynamic compact models of discretized thermal networks having large numbers of power sources. A compact model generated in this manner assures a desired accuracy. Moreover it has a number of components proportional to the number n of the power sources. Such a compact model is the interconnection of boundary condition independent compact thermal networks. Each boundary condition independent compact thermal network is generated in such a way that a desired accuracy is assured. This is achieved by exploiting the projection vectors of the Multipoint Moment Matching Method. In this sense the novel method is a hybrid of the approach based on boundary condition independent models and of the Multipoint Moment Matching approach.

The rest of this paper is organized as follows. In section 2, dynamic thermal networks are introduced. The Multipoint Moment Matching method and the use of boundary condition independent models are revised in sections 3 and 4. The novel hybrid method is presented in section 5. The application of this technique to a power DMOS is presented in section 6.

2 Dynamic Thermal Networks

We refer to a generic heat diffusion problem in the bounded spatial region Ω . The relation between the power density $F(\mathbf{r}, t)$ and the temperature rise $x(\mathbf{r}, t)$ with respect to the ambient temperature is modeled by the heat diffusion equation

$$c(\mathbf{r})\frac{\partial x}{\partial t}(\mathbf{r}, t) + \nabla \cdot (-k(\mathbf{r})\nabla x(\mathbf{r}, t)) = F(\mathbf{r}, t) \quad (1)$$

in which $k(\mathbf{r})$ is the thermal conductivity and $c(\mathbf{r})$ is the volumetric heat capacity. Boundary conditions are

$$-k(\mathbf{r})\frac{\partial x}{\partial \nu}(\mathbf{r}, t) = h(\mathbf{r})x(\mathbf{r}, t) \quad (2)$$

in which $h(\mathbf{r})$ is the heat exchange coefficient and $\boldsymbol{\nu}(\mathbf{r})$ is the unit vector outward normal to the boundary $\partial\Omega$.

A dynamic thermal network \mathcal{N} is introduced as follows. The power density is limited to the form

$$F(\mathbf{r}, t) = \sum_1^n f_i(\mathbf{r})P_i(t) \quad (3)$$

in which $P_i(t)$, i -th element of the $n \times 1$ vector $\mathbf{P}(t)$, is the power of the i -th power source measured at the i -th port of the thermal network and $f_i(\mathbf{r})$ is a volumetric density function of support Ξ_i . As in [5] it is assumed

$$T_i(t) = \int_{\Omega} f_i(\mathbf{r})x(\mathbf{r}, t) d\mathbf{r}, \quad (4)$$

in which $T_i(t)$, i -th element of the $n \times 1$ vector $\mathbf{T}(t)$, is the temperature rise of the i -th power source measured at the i -th port of the thermal network. The only assumption on the $f_i(\mathbf{r})$ volumetric density functions is that they are *linearly independent*. In this manner the dynamic thermal network has the generalized form introduced in [6, 7]. This dynamic thermal network preserves the main physical properties of the heat diffusion problem, in particular passivity.

The discretization of Eqs. (1), (2) by any proper discretization technique has the form

$$\mathbf{C} \frac{d\mathbf{x}}{dt}(t) + \mathbf{K} \mathbf{x}(t) = \mathbf{f}(t) \quad (5)$$

in which $\mathbf{x}(t)$ is the $M \times 1$ vector of freedom degrees, $\mathbf{f}(t)$ is the $M \times 1$ source vector, \mathbf{C} and \mathbf{K} are $M \times M$ matrices. The discretized forms of Eqs. (3) and (4) are

$$\mathbf{f}(t) = \mathbf{F}\mathbf{P}(t), \quad (6)$$

$$\mathbf{T}(t) = \mathbf{F}^T \mathbf{x}(t) \quad (7)$$

in which \mathbf{F} is an $M \times n$ matrix. Eqs. (5)-(7) define a discretized dynamic thermal network. Also this discretized dynamic thermal network preserves the main physical properties of the heat diffusion problem, in particular passivity.

A compact dynamic thermal network is a lumped network that is simpler than a discretized dynamic thermal network but is able to accurately approximate the relation between the powers and the temperature rises at the ports of a dynamic thermal network. Hereafter the problem of generating a compact dynamic thermal network from a discretized dynamic thermal network is tackled.

3 Compact Dynamic Thermal Networks by Multipoint Moment Matching

A very accurate and efficient technique for generating a compact dynamic thermal network is the multipoint moment matching method [5], hereafter revised. In this technique, $\mathbf{x}(t)$ is approximated by

$$\mathbf{x}(t) \approx \sum_1^{\hat{m}} \mathbf{v}_j \hat{x}_j(t) \quad (8)$$

in which \mathbf{v}_j are basis vectors and $\hat{x}_j(t)$ are freedom degrees, with $j = 1, \dots, \hat{m}$. Equivalently

$$\mathbf{x}(t) \approx \mathbf{V}\hat{\mathbf{x}}(t) \quad (9)$$

in which

$$\mathbf{V} = [\mathbf{v}_1, \dots, \mathbf{v}_{\hat{m}}],$$

$$\hat{\mathbf{x}}(t) = \begin{bmatrix} \hat{x}_1 \\ \vdots \\ \hat{x}_{\hat{m}} \end{bmatrix}.$$

Firstly the basis vectors \mathbf{v}_j , with $j = 1, \dots, \hat{m}$, are determined. To this aim the q problems

$$(\alpha_r \mathbf{C} + \mathbf{K})\mathbf{V}_r = \mathbf{F}$$

are solved for the $M \times n$ rectangular matrices \mathbf{V}_r , with $r = 1, \dots, q$. Here the α_r matching points are

$$\alpha_r = \lambda_M \operatorname{dn} \left[\frac{2r-1}{2l} K, k \right] \quad (10)$$

in which K is the complete elliptic integral of first kind of modulus k , dn is the elliptic dn function of modulus k . Besides

$$k = \sqrt{1 - k'^2}$$

$$k' = \lambda_1 / \lambda_M,$$

in which λ_1 and λ_M are the minimum and maximum eigenvalues of matrix $\mathbf{C}^{-1}\mathbf{K}$. It is set

$$\mathbf{V} = [\mathbf{V}_1, \dots, \mathbf{V}_q]$$

having $\hat{m} = nq$ columns.

Secondly the freedom degrees $\hat{x}_j(t)$ with $j = 1, \dots, \hat{m}$ are determined by means of Galerkin's method. To this aim Eqs. (5)-(7) are projected onto the space spanned by the vectors \mathbf{v}_j with $j = 1, \dots, \hat{m}$. It results in

$$\hat{\mathbf{C}} \frac{d\hat{\mathbf{x}}}{dt}(t) + \hat{\mathbf{K}}\hat{\mathbf{x}}(t) = \hat{\mathbf{f}}(t), \quad (11)$$

with

$$\hat{\mathbf{f}}(t) = \hat{\mathbf{F}}\mathbf{P}(t) \quad (12)$$

$$\mathbf{T}(t) = \hat{\mathbf{F}}^T \hat{\mathbf{x}}(t), \quad (13)$$

in which

$$\hat{\mathbf{C}} = \mathbf{V}^T \mathbf{C} \mathbf{V}, \quad (14)$$

$$\hat{\mathbf{K}} = \mathbf{V}^T \mathbf{K} \mathbf{V}, \quad (15)$$

$$\hat{\mathbf{F}} = \mathbf{V}^T \mathbf{F}. \quad (16)$$

Eqs. (11), (13) determine $\hat{\mathbf{x}}(t)$.

As proved in [8], with this choice of the basis vectors \mathbf{v}_j and of the freedom degrees \hat{x}_j , with $j = 1, \dots, \hat{m}$, the expansion given by Eq. (9) approximates $\mathbf{x}(t)$, with a relative error in the energy norm ε exponentially decreasing with q . Thus q is of order $O(-\log \varepsilon)$.

As a result multipoint moment matching can be interpreted as an effective method for generating basis vectors \mathbf{v}_j , with $j = 1, \dots, \hat{m}$, in such a way that a linear combination of such basis vectors can approximate $\mathbf{x}(t)$ with an error exponentially converging to zero with the number \hat{m} of basis vectors.

Eqs. (11), (13) accurately approximate also the relation between the port powers $P_i(t)$ and the port temperature rises $T_i(t)$, with $i = 1, \dots, n$. Thus they define a compact dynamic thermal network $\hat{\mathcal{N}}$ of state-space dimension \hat{m} . This compact dynamic thermal network preserves the main physical properties of the heat diffusion problem, in particular passivity. Also such compact dynamic thermal network can be synthesized by means of passive elements proceeding as follows. The generalized eigenvalue problem

$$\hat{\mathbf{K}} \hat{\mathbf{U}} = \hat{\mathbf{C}} \hat{\mathbf{U}} \hat{\mathbf{\Lambda}}, \quad (17)$$

$$\hat{\mathbf{U}}^T \hat{\mathbf{C}} \hat{\mathbf{U}} = \mathbf{1}, \quad (18)$$

is solved for the eigenvectors matrix $\hat{\mathbf{U}}$ and for the diagonal eigenvalue matrix $\hat{\mathbf{\Lambda}}$. The transformation of variables is then introducing

$$\hat{\mathbf{x}}(t) = \hat{\mathbf{U}} \hat{\mathbf{a}}(t),$$

by which Eqs. (11), (13) are transformed into the equivalent form

$$\frac{d\hat{\mathbf{a}}}{dt}(t) + \hat{\mathbf{\Lambda}} \hat{\mathbf{a}}(t) = \hat{\mathbf{F}} \mathbf{P}(t), \quad (19)$$

$$\mathbf{T}(t) = \hat{\mathbf{F}}^T \hat{\mathbf{a}}(t) \quad (20)$$

where

$$\hat{\mathbf{F}} = \hat{\mathbf{U}}^T \hat{\mathbf{F}}.$$

By interpreting Eqs. (19), (20) as the modified nodal analysis equations of a thermal network, the compact dynamic thermal network is synthesized by the canonical form [5] shown in Fig. 1. This canonical form has \hat{m} capacitors of positive capacitances, \hat{m} resistors of positive conductances and

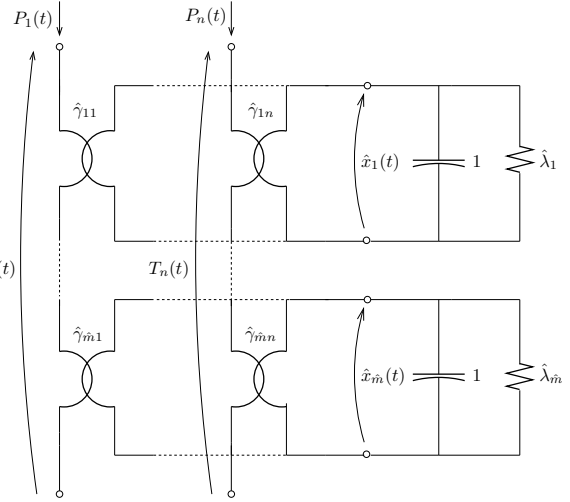


Figure 1: Equivalent network of an n -port compact dynamic thermal network $\hat{\mathcal{N}}$. Parameter $\hat{\gamma}_{ij}$ is the coefficient at the i -th row and j -th column of $\hat{\mathbf{F}}$.

$\hat{m} \times n$ ideal transformers. Thus the number of elements in such a thermal network is $O(n^2)$, preventing its use for large n .

4 Compact Dynamic Thermal Networks by Boundary Condition Independent Models

As shown in [9] the dynamic thermal networks introduced in section 2 can be generated in such a way that they mimic boundary condition independent models. This is achieved by assuming that the $F(\mathbf{r}, t)$ function in Eq. (1) does not only model a volume power density but also models a *surface* power density over the boundary $\partial\Omega$ and by assuming that the heat exchange coefficient $h(\mathbf{r})$ in Eq. (2) is zero on the boundary $\partial\Omega$. In this way some of the $f_i(\mathbf{r})$ functions of Eqs. (3), (4) are assumed to be surface power density functions whose supports are on the boundary $\partial\Omega$. With a proper choice of the surface density functions such a dynamic thermal network mimics a boundary condition independent model. It is thus referred to as a boundary condition independent dynamic thermal network.

Such a boundary condition independent dynamic thermal network can still be discretized as shown in section 2. Besides the multipoint moment matching method can still be used with a slight modification for generating a boundary condition independent compact dynamic thermal network from the boundary condition independent discretized dynamic thermal network. Precisely the \mathbf{V} matrix is

assumed as

$$\mathbf{V} = [\mathbf{e}, \mathbf{V}_1, \dots, \mathbf{V}_q]$$

in which \mathbf{e} is an $M \times 1$ vector of ones. Moreover $k' = \lambda_2/\lambda_M$, being $\lambda_1 = 0$. This compact thermal network still admits the canonical form of section 3. One resistor has conductance $\hat{\lambda}_1 = 0$.

By generalizing the approach proposed in [2], such boundary condition independent compact dynamic thermal networks can be exploited for generating a compact model of the dynamic thermal network \mathcal{N} of Eqs. (1)-(4). Precisely

1. The Ω region is partitioned into the parts Ω_i , with $i = 1, \dots, v$. The non-empty intersections of these parts define the surfaces Σ_i , with $i = 1, \dots, f$, as shown in Fig. 2.
2. The volume power densities in Eqs. (3), (4) are introduced. Proper surface power densities having supports on the Σ_i surfaces, with $i = 1, \dots, f$, are also introduced.
3. The Ω_i parts, with $i = 1, \dots, v$, are modeled by boundary condition independent compact dynamic thermal networks $\tilde{\mathcal{N}}_i$, with $i = 1, \dots, v$. Each $\tilde{\mathcal{N}}_i$, with $i = 1, \dots, v$, has one port for each volume power density whose support is in Ω_i and has one port for each surface power density whose support is on the boundary $\partial\Omega_i$.
4. The ports which correspond to one surface power density are interconnected, as shown in Fig. 2.

Since the connection of passive thermal networks is passive the determined composite thermal network is passive and thus also *stable*.

If the partition is such that each of the Ω_i parts, with $i = 1, \dots, v$, contains only few of the supports Ξ_i , with $i = 1, \dots, n$, and has on the boundary only few of the surfaces Σ_i , with $i = 1, \dots, f$, then the generated compact dynamic thermal network has only $O(n)$ components and thus can be used also for large n . As it is well known, the drawback of this approach is that the achievable accuracy is not well controllable.

5 Compact Dynamic Thermal Network by Hybrid Approach

A compact dynamic thermal network composed of boundary condition independent models is accurate only if the restrictions of the heat flux density of Eqs. (1)-(4) to the Σ_i surfaces, with $i = 1, \dots, f$, are accurately approximated by linear combinations of the surface power density functions. This

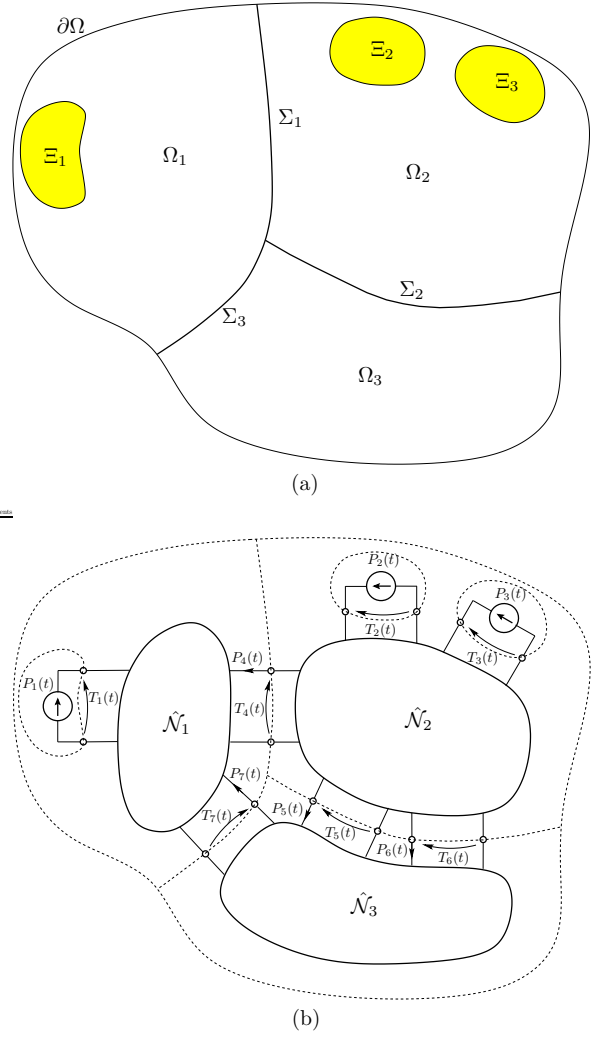


Figure 2: (a) Partition of the Ω region. (b) Compact Dynamic Thermal Network Composed of Boundary Condition Independent Models.

is assured if the surface power density functions are the restrictions to the Σ_i surfaces, with $i = 1, \dots, f$, of the heat flux densities due to the temperature rises \mathbf{v}_j , with $j = 1, \dots, nq$, determined by means of the multipoint moment matching method for Eqs. (1)-(4).

Let \mathbf{Q}_k be the matrix whose columns are the restriction of such heat flux densities to the Σ_k surface, with $k = 1, \dots, f$. The question of approximating the linear combinations of the nq columns of \mathbf{Q}_k by linear combinations of $p_k \ll nq$ vectors is here considered. It is well known [10] that by determining the Singular Value Decomposition of \mathbf{Q}_k ,

$$\mathbf{Q}_k = \mathbf{U}_k \Sigma_k \mathbf{W}_k^T,$$

and by choosing the p_k vectors as the first p_k

columns of \mathbf{U}_k , the relative error in the energy norm reaches its minimum given by the $p_k + 1$ -th singular value. The following procedure for choosing the surface power density functions is thus deduced. The first p_k step of the Singular Value Decomposition of \mathbf{Q}_k are performed, stopping when the $p_k + 1$ -th singular value is smaller than the desired accuracy ε . The surface power density functions are then chosen equal to the first p_k columns of \mathbf{U}_k . The Multipole Algorithm [11] suggests that for a heat diffusion problem the singular values exponentially decrease with p_k . As a result only $O(-\log \varepsilon)$ surface power density functions are introduced in this manner for each Σ_k surface, with $k = 1, \dots, f$.

Similar considerations can be made for determining the projection vectors for generating the boundary condition independent compact dynamic thermal networks. Let \mathbf{V}_k be the matrix whose columns are the restriction to the Ω_k region, with $k = 1, \dots, v$, of the temperature rises \mathbf{v}_j , with $j = 1, \dots, nq$, determined by means of the multipoint moment matching method for Eqs. (1)-(4). The question of approximating the linear combinations of the nq columns of \mathbf{V}_k by linear combinations of $r_k \ll nq$ vectors is considered. Again by determining the Singular Value Decomposition of \mathbf{V}_k ,

$$\mathbf{V}_k = \mathbf{U}_k \Sigma_k \mathbf{W}_k^T,$$

and by choosing the r_k vectors as the first r_k columns of \mathbf{U}_k , the relative error in the energy norm reaches its minimum given by the $r_k + 1$ -th singular value. The following procedure for choosing the projection vectors is thus deduced. The first r_k step of the Singular Value Decomposition of \mathbf{V}_k are performed, stopping when the $r_k + 1$ -th singular value is smaller than the desired accuracy ε . The projection vectors are chosen equal to the first r_k columns of \mathbf{U}_k . Again the Multipole Algorithm [11] suggests that for a heat diffusion problem the singular values exponentially decrease with r_k . As a result only $O(-\log \varepsilon)$ projection vectors are introduced in this manner for each Ω_k region, with $k = 1, \dots, v$. Since only $O(-\log \varepsilon)$ surface power density functions are introduced for each Σ_k surface, with $k = 1, \dots, f$, and only $O(-\log \varepsilon)$ projection vectors are introduced for each Ω_k region, with $k = 1, \dots, v$, the generated compact dynamic thermal network has only $O(n)$ components and thus can be used also for large n . Moreover the achievable accuracy ε is controllable.

6 Numerical Example

The heat diffusion problem for two fingers of a power DMOS is considered. All 96 channel re-

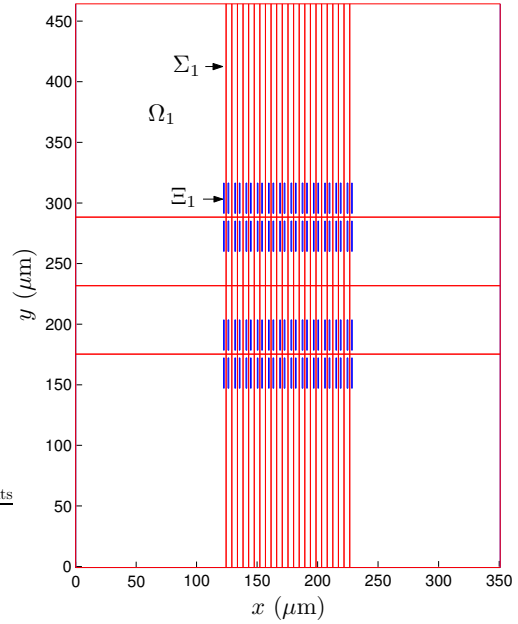


Figure 3: Partition of the Ω region.

gions are assumed as independent power sources. Firstly a compact dynamic thermal network is generated by means of the Multipoint Moment Matching method with $\varepsilon = 10^{-4}$. The relative error, both in the time and in the frequency domains, is below 0.1%. The number of elements in the thermal network is 112 896.

Secondly a compact dynamic thermal network is generated by means of a boundary condition independent model. The Ω region is partitioned into the parts Ω_k , with $k = 1, \dots, 96$ as shown in Fig. 3. A single uniform surface power density function is introduced for each Σ_k surface, with $k = 1, \dots, 164$. The relative error, both in the time and in the frequency domains, is above 15%, as shown in Figs. 4, 5. The number of elements in the thermal network is 33 024.

Lastly a compact dynamic thermal network is generated by means of the hybrid method with $\varepsilon = 10^{-4}$. The partition of Fig. 3 is still used. The relative error, both in the time and in the frequency domains, is below 0.1%, as shown in Figs. 6, 7. The number of elements in the thermal network is 12 832.

7 Conclusion

In this paper a novel approach has been presented for generating compact dynamic thermal networks modeling heat diffusion problems with large numbers of power sources. As in the multipoint moment

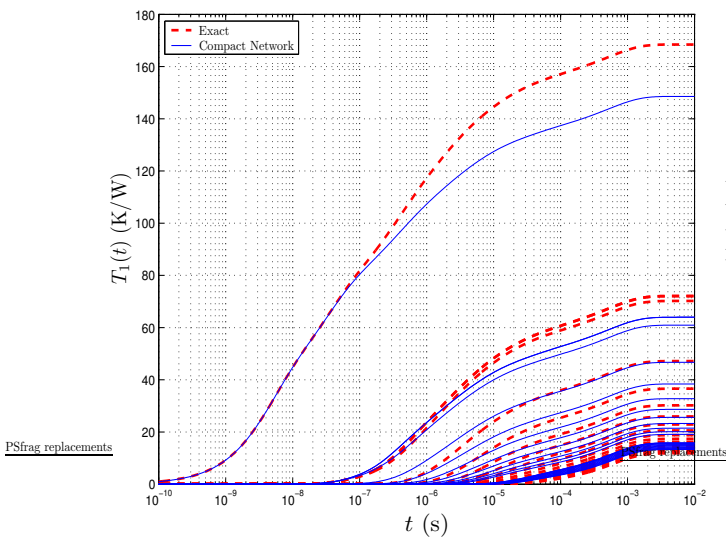


Figure 4: Power step thermal responses of the compact dynamic thermal network composed of boundary condition independent models.

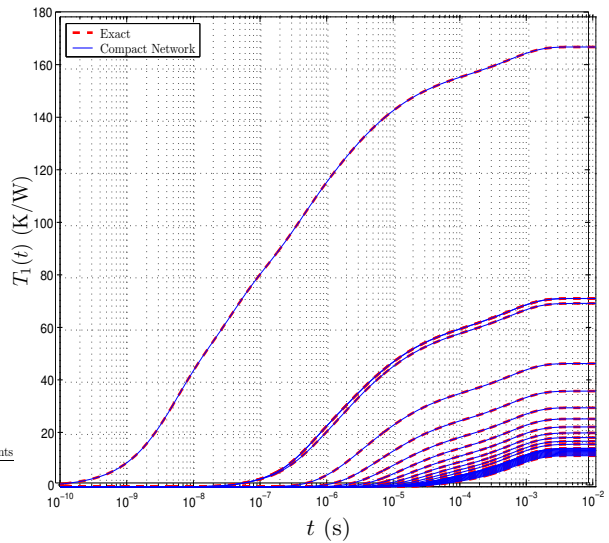


Figure 6: Power step thermal responses of the compact dynamic thermal network derived by the hybrid method.

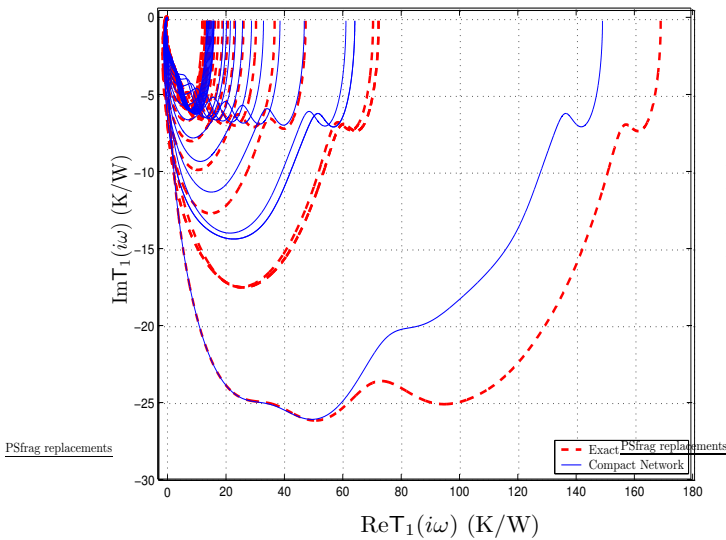


Figure 5: Nyquist plot of thermal impedances of the compact dynamic thermal network composed of boundary condition independent models.

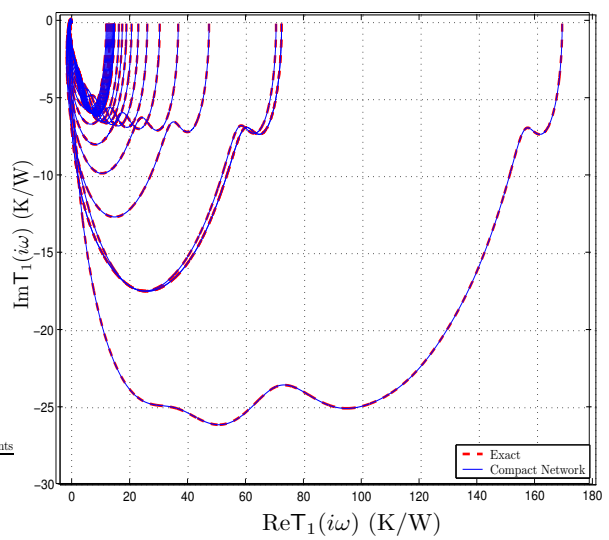


Figure 7: Nyquist plot of thermal impedances of the compact dynamic thermal network derived by the hybrid method.

matching approach, the achievable accuracy of the model is controllable. As in the approach based on boundary condition independent approach, the number of elements of the compact thermal network linearly increases with the number of power sources.

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