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PI OBSERVER DESIGN FOR DISCRETE-TIME DECOUPLED MULTIPLE MODELS

**Rodolfo Orjuela, Benoît Marx,
José Ragot and Didier Maquin**

*Centre de Recherche en Automatique de Nancy
UMR 7039, Nancy-Université, CNRS
2, Avenue de la Forêt de Haye, 54 516
Vandœuvre-lès-Nancy, France*

Abstract: This paper deals with the state estimation of nonlinear systems. Our approach is based on the representation of a nonlinear system with the help of a decoupled multiple model. This kind of multiple model constitutes an alternative to the classic Takagi-Sugeno multiple model. Indeed, in opposition to this last, in the decoupled multiple model the dimension (i.e. the number of states) of the submodels may be different. In order to cope with the state estimation problem a Proportional-Integral observer (PIO) is designed. Indeed, in contrast to the classic Proportional observer the PIO is known by its robustness properties. Sufficient conditions for ensuring the estimation error convergence are given in LMI form.

Keywords: multiple model, nonlinear systems, state estimation, PI Observer.

1. INTRODUCTION

The knowledge of the state (i.e. internal variables) of a system is often necessary in order to design a control law or set up a FDI strategy. Unfortunately, as a general rule, the direct measurement of the state variables of a system is not possible owing to the inaccessibility of the variable, the sensor cost, the technological limitations, etc. Therefore, in several engineering problems the state estimation is a fundamental point.

A common solution, in order to provide a state estimation of a system, is to use an observer. An observer yields an estimation of the state using the available model of the system, the inputs and the measurable outputs of the system. The classic observer, used in the linear systems theory, is the famous Luenberger observer (Luenberger, 1971), also named Proportional observer (PO). However, it is well known that the precision of the estimation provided by this observer is directly

affected by the uncertainties of the model (i.e. knowledge of the parameters) and also by the quality of the employed signals which is affected by noise, perturbations, etc.

In order to provide a better estimation under system perturbations a Proportional-Integral observer (PIO) may be used. Indeed, the PI observer thanks to its integral action introduces a robustness degree in the state estimation (Weinmann, 1991). The purpose of this note is to extend the principle of the PIO observer used in the linear system framework (Beale and Shafai, 1989) to nonlinear systems. To this end, the multiple model approach can be employed.

In this modelling framework, the dynamic behaviour of a nonlinear system is described by taking into consideration the contribution of a collection of linear submodels. Each submodel is valid in a particular operating zone and the global model is obtained via an interpolation mechanism. Hence,

a multiple model is able to characterize accurately a complex system by increasing the number of submodels. Besides, the available tools for linear systems may partially be extended to nonlinear systems represented by a multiple model. Nowadays, the multiple model approach represents a powerful tool in order to cope with several control problems.

Two basic structures of multiple model can be distinguished for interconnecting the submodels between them (Filev, 1991). In the first structure, the submodels have the same state vector (*Takagi-Sugeno multiple model*); in the second one, the submodels are decoupled and their state vectors are different (*decoupled multiple model*).

The Takagi-Sugeno multiple model is commonly used in the multiple model framework. Several works deal with the modelling, the control and the diagnosis of nonlinear systems based on this multiple model. Besides, in order to tackle the state estimation problem, extensions of the popular proportional observer (Tanaka and Sugeno, 1992; Guerra *et al.*, 2006), sliding mode observer (Bergstern and Driankov, 2002) and unknown inputs observer (Marx *et al.*, 2007) have been successfully proposed.

The decoupled multiple model has been unfortunately less investigated in the literature. However, it represents an increasing alternative to Takagi-Sugeno multiple model. Indeed, the usefulness of this multiple model has been clearly established for the control (Gawthrop, 1995; Gatzke and Doyle III, 1999; Gregorcic and Lightbody, 2000) and the modelling (Venkat *et al.*, 2003; Thiaw *et al.*, 2007) of nonlinear systems. More recently, the state estimation problem has been investigated in (Orjuela *et al.*, 2007) and the design of a proportional observer has been proposed.

Our contribution in this note is to design a Proportional-Integral observer for nonlinear systems modelled by a decoupled multiple model. Indeed, to the best of the authors' knowledge, the design of this observer has not been reported previously. The outline of this paper is as follows. The two classic structures of multiple models are detailed in section 2. In section 3, the design of PIO is exposed and sufficient existence condition of the PIO is given in terms of LMI. Finally, in section 4, a simulation example illustrates the state estimation of a decoupled multiple model.

Notations. The following notations will be used all along this paper. $P > 0$ ($P < 0$) denotes a positive (negative) definite matrix P ; X^T denotes the transpose of matrix X and I is the identity matrix of appropriate dimension. We shall simply write $\mu_i(\xi(k)) = \mu_i(k)$.

2. STRUCTURES OF MULTIPLE MODELS

A multiple model is built by blending several linear submodels. Basically, two kinds of multiple models can be distinguished according to the coupling between the submodels.

Let us notice that several techniques for nonlinear modelling with a multiple model structure (i.e. the parameter estimation of the submodels) are available, see for instance (Murray-Smith and Johansen, 1997; Gasso *et al.*, 2001; Venkat *et al.*, 2003; Orjuela *et al.*, 2006) and the references therein for further information about these techniques.

2.1 Takagi-Sugeno multiple model

The Takagi-Sugeno multiple model structure is given by (Murray-Smith and Johansen, 1997):

$$\begin{aligned} x(k+1) &= \sum_{i=1}^L \mu_i(\xi(k)) \{A_i x(k) + B_i u(k)\} \quad , (1) \\ y(k) &= \sum_{i=1}^L \mu_i(\xi(k)) C_i x(k) \quad , \end{aligned}$$

where $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}^m$ the control input, $y \in \mathbb{R}^p$ the output and $A_i \in \mathbb{R}^{n \times n}$, $B_i \in \mathbb{R}^{n \times m}$ and $C_i \in \mathbb{R}^{p \times n}$ are known and constants matrices of appropriate dimensions.

The decision variable $\xi(k)$ is a real-time accessible signal, for example, the control input and/or a measurable output of the system.

The $\mu_i(\xi(k))$ are the *weighting functions* that ensure the blending between the submodels. They satisfy the following constraints:

$$\sum_{i=1}^L \mu_i(\xi(k)) = 1 \quad , \quad \forall k \quad (2a)$$

$$0 \leq \mu_i(\xi(k)) \leq 1 \quad , \quad \forall i = 1 \dots L, \quad \forall k. \quad (2b)$$

Let us notice that the Takagi-Sugeno multiple model can be related to a Linear Parameter Varying system (LPV). Moreover, one should note that an only one state vector appears in the dynamic equation of this multiple model.

2.2 Decoupled multiple model

The structure of the decoupled multiple model is given by (Filev, 1991):

$$\begin{aligned} x_i(k+1) &= A_i x_i(k) + B_i u(k) \quad , \\ y_i(k) &= C_i x_i(k) \quad , \\ y(k) &= \sum_{i=1}^L \mu_i(\xi(k)) y_i(k) \quad , \end{aligned} \quad (3)$$

where $x_i \in \mathbb{R}^{n_i}$ and $y_i \in \mathbb{R}^p$ are respectively the state vector and the output of the i^{th} submodel; $y \in \mathbb{R}^p$ is the output of the multiple model. The known and constants matrices $A_i \in \mathbb{R}^{n_i \times n_i}$, $B_i \in \mathbb{R}^{n_i \times m}$ and $C_i \in \mathbb{R}^{p \times n_i}$ are of appropriate dimensions.

Let us notice that in this multiple model no blend between the parameters of the submodels is performed. Indeed, the dynamics of the submodels are completely independent and the submodel contributions are taken into account via a weighted sum between the submodel outputs. Hence, the main feature of this multiple model is that the dimension of the submodels, i.e. the numbers of the states, can be different (obviously the submodel outputs must be of compatible dimensions).

Let us remark that the outputs $y_i(k)$ of the submodels are virtual outputs. Indeed, these outputs are artificial modelling signals only used in order to provide an approximation of the output of the real physic system. Therefore the outputs $y_i(k)$ cannot be employed as accessible signals to drive an observer.

3. STATE ESTIMATION

In the present note, the concept of the PI observer proposed in (Beale and Shafai, 1989) will be extended in order to provide a state estimation of a nonlinear system modelled by a multiple model. Conditions for ensuring the exponential convergence towards zero of the state estimation error are established in LMIs terms (Boyd *et al.*, 1994).

The decoupled multiple model (3) is rewritten in the following compact form in order to simplify the mathematic manipulations:

$$\begin{aligned} x(k+1) &= \tilde{A}x(k) + \tilde{B}u(k) , \\ y(k) &= \tilde{C}(k)x(k) , \end{aligned} \quad (4)$$

where:

$$\tilde{A} = \begin{bmatrix} A_1 & 0 & 0 & 0 & 0 \\ 0 & \ddots & 0 & 0 & 0 \\ 0 & 0 & A_i & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & A_L \end{bmatrix}, \quad \tilde{B} = \begin{bmatrix} B_1 \\ \vdots \\ B_i \\ \vdots \\ B_L \end{bmatrix},$$

$$\tilde{C}(k) = [\mu_1(k)C_1 \cdots \mu_i(k)C_i \cdots \mu_L(k)C_L],$$

$$x(k) = [x_1^T(k) \cdots x_i^T(k) \cdots x_L^T(k)]^T \in \mathbb{R}^n, n = \sum_{i=1}^L n_i.$$

Notice that \tilde{A} and \tilde{B} are constant matrices and consequently the dynamic equation of the multiple model takes a linear form. On the other hand,

$\tilde{C}(k)$ is a time variable matrix. Indeed, the blend between the submodels is carried out in the output equation.

3.1 Proportional-Integral Observer structure

In the continuous-time case, a PIO is characterized by the use of an integral term of the estimation error via a supplementary variable $z(t)$. Hence, thanks to this extra variable a robustness degree of the state estimation with respect to the plant perturbation is achieved (Weinmann, 1991).

In order to built a discrete-time PIO, the decoupled multiple model (4) is modified as follows:

$$\begin{aligned} x(k+1) &= \tilde{A}x(k) + \tilde{B}u(k) , \\ z(k+1) &= \tilde{C}(k)x(k) , \\ y(k) &= \tilde{C}(k)x(k) . \end{aligned} \quad (5)$$

The above equations can be rewritten in the following augmented form:

$$\begin{aligned} x_a(k+1) &= \tilde{A}_1(k)x_a(k) + \tilde{C}_1\tilde{B}u(k) , \\ y(k) &= \tilde{C}(k)\tilde{C}_1^T x_a(k) , \\ z(k) &= \tilde{C}_2^T x_a(k) , \end{aligned} \quad (6)$$

where

$$\begin{aligned} x_a(k) &= \begin{bmatrix} x(k) \\ z(k) \end{bmatrix}, \quad \tilde{A}_1(k) = \begin{bmatrix} \tilde{A} & 0 \\ \tilde{C}(k) & 0 \end{bmatrix}, \\ \tilde{C}_1 &= \begin{bmatrix} \mathbf{I} \\ 0 \end{bmatrix}, \quad \tilde{C}_2 = \begin{bmatrix} 0 \\ \mathbf{I} \end{bmatrix}. \end{aligned}$$

The state estimation of the decoupled multiple model (6) is achieved with the help of the following Proportional-Integral observer:

$$\begin{aligned} \hat{x}_a(k+1) &= \tilde{A}_1(k)\hat{x}_a(k) + \tilde{C}_1\tilde{B}u(k) \\ &\quad + K_P(y(k) - \hat{y}(k)) \\ &\quad + K_I(z(k) - \hat{z}(k)), \\ \hat{y}(k) &= \tilde{C}(k)\tilde{C}_1^T \hat{x}_a(k) , \\ \hat{z}(k) &= \tilde{C}_2^T \hat{x}_a(k) . \end{aligned} \quad (7)$$

3.2 Observer design

Our approach for designing the observer is similar to the approach proposed in (Hua and Guan, 2005) used in the synchronization of a chaotic system.

The design of the observer (7) consist in finding matrices K_P and K_I such that the estimation error given by:

$$e_a(k) = x_a(k) - \hat{x}_a(k) , \quad (8)$$

tends toward zero for any initial conditions and for any blend between the submodel outputs.

From (8), (6) and (7), the following estimation error dynamics are obtained:

$$e_a(k+1) = A_{obs}(k)e_a(k) , \quad (9)$$

where $A_{obs}(k)$ is defined by:

$$A_{obs}(k) = \tilde{A}_1(k) - K_P \tilde{C}(k) \tilde{C}_1^T - K_I \tilde{C}_2^T . \quad (10)$$

Let us notice that the matrix $\tilde{C}(k)$ can be rewritten as follows:

$$\tilde{C}(k) = \sum_{i=1}^L \mu_i(\xi(k)) \tilde{C}_i , \quad (11)$$

where \tilde{C}_i is a constant bloc matrix given by:

$$\tilde{C}_i = [0 \cdots C_i \cdots 0] . \quad (12)$$

Hence, by taking into account the form (11) of $\tilde{C}(k)$, the matrix $\tilde{A}_1(k)$ becomes:

$$\tilde{A}_1(k) = \sum_{i=1}^L \mu_i(k) \bar{A}_i , \quad (13)$$

where

$$\bar{A}_i = \begin{bmatrix} \tilde{A} & 0 \\ \tilde{C}_i & 0 \end{bmatrix} . \quad (14)$$

Finally, by using (11) and (13), the matrix $A_{obs}(k)$ can be rewritten as follows:

$$A_{obs}(k) = \sum_{i=1}^L \mu_i(k) \Phi_i , \quad (15)$$

$$\Phi_i = \bar{A}_i - K_P \tilde{C}_i \tilde{C}_1^T - K_I \tilde{C}_2^T . \quad (16)$$

The following theorem provides sufficient conditions for ensuring the exponential convergence of the estimation error using the Proportional-Integral observer.

Theorem 1. Consider the decoupled multiple model (4) and the Proportional-Integral observer (6). The exponential convergence of the estimation error is guaranteed if there exists a symmetric, positive definite matrix P , matrices L_P and L_I such that:

$$\begin{bmatrix} (1-\gamma)P & \mathcal{A}_i^T \\ \mathcal{A}_i & P \end{bmatrix} > 0 , \quad i = 1 \dots L . \quad (17)$$

where

$$\mathcal{A}_i = P \bar{A}_i - L_P \tilde{C}_i \tilde{C}_1^T - L_I \tilde{C}_2^T ,$$

for a prescribed scalar $0 < \gamma < 1$. The observer gains are given by $K_P = P^{-1}L_P$ and $K_I = P^{-1}L_I$.

Proof. Let us consider the classic quadratic Lyapunov function:

$$V(k) = e_a^T(k) P e_a(k) , \quad P > 0 \quad P = P^T , \quad (18)$$

and its variation denoted by:

$$\Delta V(k) = V(k+1) - V(k) . \quad (19)$$

The exponential convergence of the estimation error is ensured if the following condition holds:

$$\exists P = P^T > 0 \rightarrow \Delta V(k) \leq -\gamma V(k) , \quad \forall k , \quad (20)$$

where γ is the so called *decay rate*.

Now, conditions for ensuring the above inequality must be established. Using (18), it can be shown that $\Delta V(k)$ becomes:

$$\Delta V(k) = e_a^T(k+1) P e_a(k+1) - e_a^T(k) P e_a(k) , \quad (21)$$

and substituting (9) into (21):

$$\Delta V(k) = e_a^T(k) \{ A_{obs}^T(k) P A_{obs}(k) - P \} e_a(k) . \quad (22)$$

Now, by taking into consideration (18) and (22), (20) yields:

$$e_a^T(k) \{ A_{obs}^T(k) P A_{obs}(k) - P + \gamma P \} e_a(k) \leq 0 \quad (23)$$

that is a quadratic form in $e_a(k)$. Hence, the condition (20) for ensuring the exponential convergence of the estimation error is guaranteed if:

$$A_{obs}^T(k) P A_{obs}(k) - (1-\gamma)P < 0 . \quad (24)$$

Now, substituting (15) into (24) gives:

$$\sum_{i=1}^L \mu_i(k) \Phi_i^T P \sum_{j=1}^L \mu_j(k) \Phi_j - (1-\gamma)P < 0 , \quad (25)$$

and by using the Schur complement and the property (2a) of the weighting functions, it is possible to write:

$$\sum_{i=1}^L \mu_i(k) \begin{bmatrix} (1-\gamma)P & \Phi_i^T P \\ P \Phi_i & P \end{bmatrix} > 0 , \quad (26)$$

and according to (2b), the above inequality is also satisfied if:

$$\begin{bmatrix} (1-\gamma)P & \Phi_i^T P \\ P \Phi_i & P \end{bmatrix} > 0 , \quad i = 1 \dots L . \quad (27)$$

Let us notice, by considering the definition (16) of Φ_i , that this inequality is not a LMI in K_P ,

K_I and P . However, it becomes a strict LMI by setting $L_P = PK_P$ and $L_I = PK_I$:

$$P\Phi_i = P\bar{A}_i - L_P\tilde{C}_i\bar{C}_1^T - L_I\bar{C}_2^T. \quad (28)$$

Now, standard convex optimization algorithms can be used in order to find matrices P , L_P and L_I for a prescribed γ . This completes the proof. \square

Remark 1. The so called decay rate (γ) is a mean to impose a velocity convergence of the estimation error. Hence, ensure the exponential convergence can greatly improve the dynamic performances of the observer.

Remark 2. The asymptotic convergence of the estimation error is easily stabilised from theorem 1 by taking $\gamma = 0$.

4. SIMULATION EXAMPLE

The state estimation problem using the proposed Proportional-Integral observer is illustrated in this section. Considerer the discrete-time decoupled multiple with sample time 0.5 and $L = 2$ different dimension submodels:

$$\begin{aligned} A_1 &= \begin{bmatrix} 0.8 & 0 \\ 0.4 & 0.1 \end{bmatrix}, & A_2 &= \begin{bmatrix} -0.3 & -0.5 & 0.2 \\ 0.7 & -0.8 & 0 \\ -2.0 & 0.1 & 0.7 \end{bmatrix}, \\ B_1 &= [0.2 \ -0.4]^T, & B_2 &= [0.2 \ -0.5 \ 0.3]^T, \\ C_1 &= \begin{bmatrix} 0.7 & 0.4 \\ 0.5 & 0.2 \end{bmatrix}, & C_2 &= \begin{bmatrix} 0.5 & 0 & 0.8 \\ 0.7 & 0.4 & 0.1 \end{bmatrix}. \end{aligned}$$

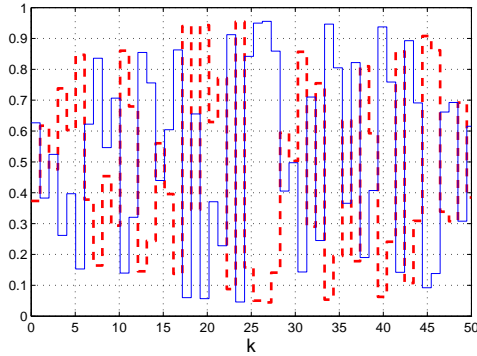


Fig. 1. Weighting functions, μ_1 (solid line) and μ_2 (dashed line)

Here, the decision variable $\xi(k)$ is the input signal $u(k) \in [0, 1]$. The weighting functions are obtained from normalised Gaussian functions:

$$\mu_i(\xi(k)) = \omega_i(\xi(k)) / \sum_{j=1}^L \omega_j(\xi(k)), \quad (29)$$

$$\omega_i(\xi(k)) = \exp\left(-(\xi(k) - c_i)^2 / \sigma^2\right), \quad (30)$$

with the standard deviation $\sigma = 0.4$ and the centres $c_1 = 0.25$ and $c_2 = 0.75$. Notice that the contributions of two submodels are taken into consideration at any time (see figure 1).

4.1 PI observer performances

A solution satisfying conditions of theorem 1 can be found by using, for example, YALMIP interface coupled to SeDuMi solver. Conditions of theorem 1 are fulfilled with:

$$K_P = \begin{bmatrix} 0.045 & 0.121 \\ -0.001 & 0.107 \\ 0.003 & 0.151 \\ 0.041 & 0.148 \\ 0.075 & 0.395 \\ 0.535 & 0.458 \\ 0.270 & 0.193 \end{bmatrix}, K_I = \begin{bmatrix} 0.136 & -0.097 \\ 0.148 & -0.092 \\ 0.295 & -0.192 \\ 0.225 & -0.276 \\ 0.668 & -0.119 \\ 0.023 & -0.347 \\ 0.068 & -0.257 \end{bmatrix},$$

for a $\gamma = 0.35$.

It can be seen from figure 2 that the proposed PI observer provides a good state estimation. In this figure the elements of the estimation error vector (9) are noted e_i . Let us notice that the initial conditions of the multiple model are: $x(0) = [0.4 \ 0.8 \ -0.2 \ 0.5 \ -0.4]^T$, and the initial conditions of the observers are null.

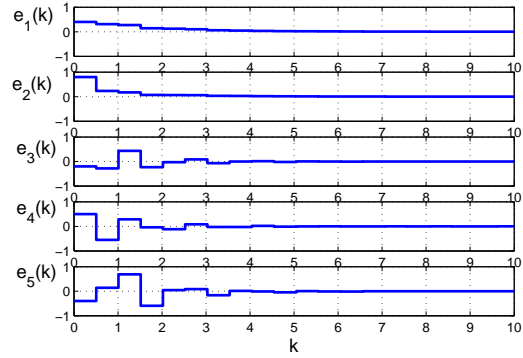


Fig. 2. State estimation errors

4.2 Comparison between P and PI observers

A Proportional observer, based on the decoupled multiple model, is designed by using the procedure proposed in (Orjuela *et al.*, 2007). The obtained gain is given by:

$$K = \begin{bmatrix} 0.031 & -0.008 & 0.334 & 0.271 & 0.699 \\ 0.132 & 0.149 & -0.717 & -0.324 & -1.65 \end{bmatrix}^T,$$

for a $\gamma = 0.35$.

For the sake of performances comparison between both observers, a constant perturbation:

$$w(k) = [0 \ -0.3]^T, \forall k, \quad (31)$$

is added to the output at $k = 20$. This perturbation can be due, for example, to a sensor fault. The estimation errors provided by both observers are plotted in figures 3. As clearly seen from these pictures, the PI observer provides the best state estimation under the considered perturbation.

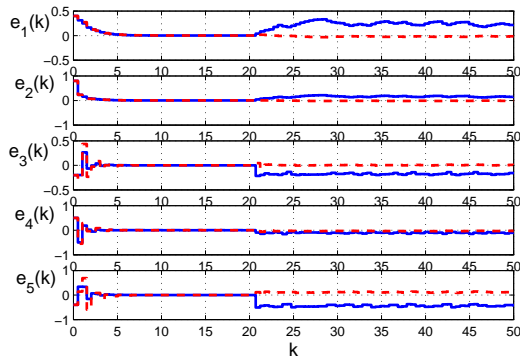


Fig. 3. State estimation errors under perturbation. P Observer (solid line), PI Observer (dashed line)

5. CONCLUSION

This paper has proposed the design of a new Proportional-Integral observer, based on a decoupled multiple model approach, to estimate the state of nonlinear system. In this multiple model each submodel has a different state vector in opposition to the well known Takagi-Sugeno model where the submodels have the same state vector. The effectiveness of the proposed approach and a comparison between Proportional and Proportional-Integral observers are illustrated via a simulation example.

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