



HAL
open science

Influence of losses on microcantilever-based chemical sensors design

Frédéric Lochon, Isabelle Dufour, Dominique Rebiere

► **To cite this version:**

Frédéric Lochon, Isabelle Dufour, Dominique Rebiere. Influence of losses on microcantilever-based chemical sensors design. Eurosensors XIX, Sep 2005, Barcelone, Spain. hal-00183272

HAL Id: hal-00183272

<https://hal.science/hal-00183272>

Submitted on 15 Jan 2008

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

INFLUENCE OF LOSSES ON MICROCANTILEVER-BASED CHEMICAL SENSORS DESIGN

Frédéric Lochon^{*1}, Isabelle Dufour¹, Dominique Rebière¹

¹Laboratoire IXL – CNRS UMR 5818 – ENSEIRB – Université Bordeaux 1, 351 cours de la libération
33405 Talence - FRANCE

lochon@ixl.fr, Tel: +33 5 4000 2881, Fax: +33 5 56 37 15 45

Abstract: The current trend in increasing microcantilever resonant frequencies leads to relatively low quality factors for microcantilevers resonating in a gas medium. In order to increase the quality factor of such high-frequency microcantilevers, an approach that takes into account the different types of losses is provided to specify appropriate values of microcantilever aspect ratios (length-to-thickness, width-to-thickness). Conventional values of aspect ratios result in low quality factors, while those proposed here lead to slightly higher values of quality factor.

Keywords: Microcantilever, quality factor, aspect ratio

INTRODUCTION

In order to design a very sensitive microcantilever-based chemical sensor, it is required to use high-frequency devices. Indeed, the sensitivity of such sensors is proportional to the resonant frequency. Moreover, to achieve a minimal limit of detection, high quality factors are preferable because they (a) increase the sharpness of the resonance peak and, thus, the frequency shift measurement accuracy, and (b) result in lower short-term noise in oscillator configurations.

A theoretical approach is proposed to show the importance of taking into account different losses when designing a microcantilever-based sensor. The importance of increasing the resonant frequency is first presented and the four dominant losses that occur in a viscous medium are then detailed and analytical forms of equivalent quality factors are given. The relevance of taking into account each loss is detailed and illustrated. A conclusion is then made about aspect ratio values.

ASSUMPTIONS

This study is limited to uniform parallelepiped shaped microcantilevers with a negligible sensitive coating Young's modulus.

The microcantilever vibration amplitude is considered far smaller than any dimension to neglect non linear effects.

SENSOR SENSITIVITY

The resonant frequency f_n of a parallelepiped shaped microcantilever with a sensitive coating deposited on its top (Figure 1) is given by [1]:

$$f_n = \frac{1}{4\pi} \left(\frac{\lambda_n}{L_1} \right)^2 \sqrt{\frac{\hat{E}_1 h_1^3}{3(\rho_1 h_1 + \rho_2 h_2)}} \quad (1)$$

Where λ_n is a parameter depending on the mode order, L_1 is the microcantilever length, h_1 is the microcantilever thickness, \hat{E}_1 is the effective microcantilever Young's modulus, and ρ_1 and ρ_2 are the microcantilever and sensitive coating mass densities, respectively. From equation (1) it can be seen that the resonant frequency is inversely proportional to the dimensions if we do a geometric contraction by keeping constant the aspect ratios (L_1/h_1 , L_1/b_1 , h_2/h_1).

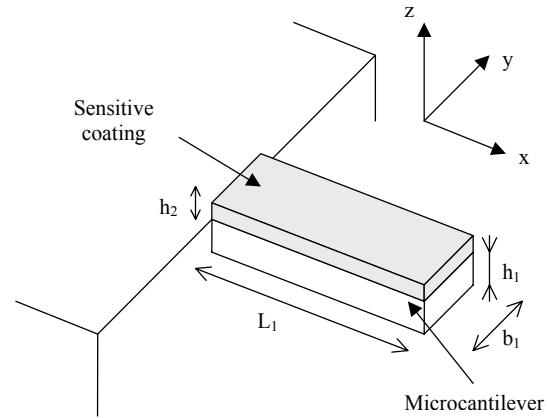


Figure 1: Geometry of the microcantilever and its sensitive coating

The sensitivity S of a sensor exposed to a gas concentration C_g is given by:

$$S = \frac{\Delta f_n}{C_g} = \frac{1}{2} f_n K \frac{r}{\rho_1 + \rho_2 r} \quad (2)$$

where Δf_n is the frequency shift due to gas sorption, K the partition coefficient of the sensitive coating – gas couple and $r = h_2/h_1$.

Thus, high resonant frequency devices will have high sensitivity. But in order to have a low

limit of detection, a high sensitivity is not sufficient: the noise has to be taken into account.

ABOUT LOSSES

The quality factor, a key parameter for the accuracy of the measurement, expresses the losses influence : low-loss microcantilevers will have a high quality factor.

For most microcantilevers resonating in vacuum, the quality factor is determined by thermoelastic losses and support losses. But when used in air, these cantilevers can have a much lower quality factor because of the viscous losses due to the surrounding fluid and because an acoustic radiation may occur for wide cantilevers.

To evaluate the global quality factor, each loss is first studied separately. Then the global quality factor is calculated using:

$$\frac{1}{Q_{global}} = \frac{1}{Q_1} + \frac{1}{Q_2} + \dots + \frac{1}{Q_n} \quad (3)$$

This approach is fully valid for low loss microcantilevers and thus it can be applied to determine how to minimize losses.

VISCOUS LOSSES

When vibrating in a fluid, a microcantilever is perturbed by viscosity and mass density of the environment. A general theoretical model describing viscous losses has been established by Sader [3].

For microcantilevers with a rectangular cross section, the equivalent quality factor associated with viscous losses can be analytically expressed:

$$Q_{viscous} = \frac{4\rho_1 h_1 + \Gamma_r(\omega_n)}{\pi\rho_0 b_1 \Gamma_i(\omega_n)} \quad (4)$$

Where ω_n is the resonant pulsation, ρ_0 is the density of the fluid, and Γ_r and Γ_i are the real and imaginary components of the hydrodynamic function $\Gamma(\omega_n)$ of the microcantilever.

Sader [3] has also proposed an approximate analytical expression of the hydrodynamic function for a wide range of Reynolds number.

Using equation (4), it can be seen that viscous losses are important for high values of L_1/h_1 or for very narrow microcantilevers.

SUPPORT LOSSES

To obtain equation (1), it has been supposed an infinitely rigid support. In fact, the vibration of the microcantilever produces a mechanical work on the support.

This mechanical work generates an elastic wave into the support. The generated wave power is lost by propagation.

Hao *et al.* [4] have proposed an analytical approach to quantify these losses.

The associated equivalent quality factor for a silicon based microcantilever is given by:

$$Q_{support} = 3.97 \frac{1}{(\chi_n \lambda_n)^2} \left(\frac{L_1}{h_1} \right)^3 \quad (5)$$

With:

$$\chi_n = \frac{\sin(\lambda_n) - \sinh(\lambda_n)}{\cos(\lambda_n) + \cosh(\lambda_n)} \quad (6)$$

It can be seen from equation (5) that support losses are important for low values of L_1/h_1 .

ACOUSTIC LOSSES

The vibration of the microcantilever in the fluid creates an acoustic pressure which propagates into the fluid.

The evaluation of this acoustic radiation in the case of a microcantilever with a rectangular cross section is not trivial.

By approximating the rectangular cross section with an elliptical cross section, Blake [5] has obtained an approximate expression for the radiated power:

$$P_\omega = \frac{\pi\rho_0 c_0 (k_0 b_1)^4}{512} \int_{\theta=0}^{\pi} \sin^3 \theta \left| \overline{U}_\omega(k_0 \cos \theta) \right|^2 d\theta \quad (7)$$

With:

$$\overline{U}_\omega(v) = \int_0^{L_1} U_\omega(x) e^{-jk_0 v} dx \quad (8)$$

Where c_0 is the speed of sound in the fluid, $k_0 = \omega/c_0$ is the wavenumber in the fluid and with $U_\omega(x)$ the microcantilever velocity along the z axis at pulsation ω .

To calculate the microcantilever velocity we can approximate the microcantilever mode shape in presence of low losses to the microcantilever mode shape without losses. It then comes:

$$U_{\omega_n} = j\omega_n w(x, \omega_n) \quad (9)$$

With:

$$w(x, \omega_n) = A\Phi_{vac}(x) \quad (10)$$

Where A is the oscillation amplitude and $\Phi_{vac}(x)$ the mode shape of the microcantilever given by:

$$\Phi_{vac}(x) = \cos\left(\frac{\lambda_n x}{L_1}\right) - \cosh\left(\frac{\lambda_n x}{L_1}\right) + \frac{\cos(\lambda_n) + \cosh(\lambda_n)}{\sin(\lambda_n) + \sinh(\lambda_n)} \left[\sinh\left(\frac{\lambda_n x}{L_1}\right) - \sin\left(\frac{\lambda_n x}{L_1}\right) \right] \quad (11)$$

Using (7) and (8) it then comes:

$$P_{\omega_n} = \frac{\pi \rho_0 c_0 (k_0 b_1)^4}{512} \int_{\theta=0}^{\pi} \sin^3 \theta \left| I_{k_0 \cos \theta} \right|^2 d\theta \quad (12)$$

With:

$$I_v = \int_{x=0}^{L_1} w(x, \omega_n) e^{-j\nu x} dx \quad (13)$$

The radiated energy during one period is given by:

$$W_{radiated} = \frac{P_{\omega_n}}{2\pi\omega_n} \quad (14)$$

The equivalent quality factor associated with acoustic radiation can be calculated using [4]:

$$Q_{rad} = 2\pi \frac{W_{cantilever}}{W_{radiated}} \quad (15)$$

With:

$$W_{cantilever} = \frac{1}{2} \mu_{12} \omega_n^2 \int_{x=0}^{L_1} w^2(x, \omega_n) dx \quad (16)$$

Where $\mu_{12} = b_1(\rho_1 h_1 + \rho_2 h_2)$ is the linear mass of the microcantilever with the sensitive coating.

Using equations (9) to (16), it then comes:

$$Q_{rad} = \frac{1024\pi\mu_{12}\omega_n^2 \int_0^{L_1} \Phi_{vac}^2 dx}{\rho_0 c_0 k_0^4 b_1^4 \int_0^{\pi} \sin^3 \theta \left| \int_0^{L_1} \Phi_{vac}^2 e^{-jxk_0 \cos \theta} dx \right|^2 d\theta} \quad (17)$$

By using numerical calculations of (17) it can be seen that acoustic radiation becomes important for wide microcantilevers and for "acoustically long microcantilevers".

The term "acoustically long microcantilevers" refers to cantilevers for which the structural wavelength (L_1/λ_n) is comparable to the acoustic wavelength ($\lambda = 2\pi/k_0$) in the fluid.

THERMOELASTIC LOSSES

Thermoelastic losses are caused by the microcantilever material heating due to vibration.

Lifshitz *et al.* [6] have expressed analytically the equivalent quality factor associated with thermoelastic losses:

$$Q_{thermo} = \frac{C_{p_1} \rho_1}{6\tilde{E}_1 \alpha_1^2 T} \frac{\zeta^2}{1 - \frac{1}{\zeta} \frac{\sinh(\zeta) + \sin(\zeta)}{\cosh(\zeta) + \cos(\zeta)}} \quad (18)$$

With:

$$\zeta = h_1 \sqrt{\frac{\omega_n \rho_1 C_{p_1}}{2\kappa}} \quad (19)$$

Where C_{p_1} , α_1 , T , κ , are the heat capacity, the thermal expansion coefficient, the temperature, and the thermal conductivity of the microcantilever material, respectively, and ω_n is the microcantilever resonant pulsation.

From equation (18) it can be seen that thermoelastic losses are maximum for $\zeta \cong 2.2246$. At this maximum loss, the thermoelastic quality factor for a silicon made microcantilever at room temperature is bigger than 10^4 .

TOTAL QUALITY FACTOR

The combination of all the losses shows that there is no particular ideal shape to minimize all losses. A compromise has to be made.

In fact, when combining the losses, it comes that optimal aspect ratios can be found for a given length (Figure 2).

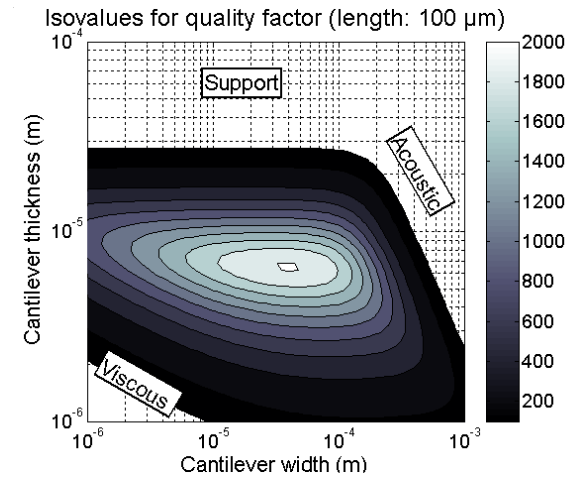


Figure 2: Total quality factor and dominant losses for a 100 μm long microcantilever as a function of the microcantilever width and thickness

Indeed, the thickness can be adjusted to *equilibrate* viscous losses and support losses (Figure 3), while the width can be adjusted to *equilibrate* viscous losses and acoustic losses (Figure 4).

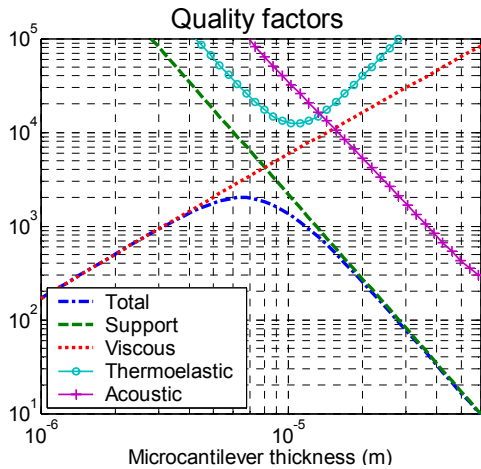


Figure 3: Quality factors (log scale) at the optimal width (42 μm) for a 100 μm long microcantilever as a function of the thickness (log scale)

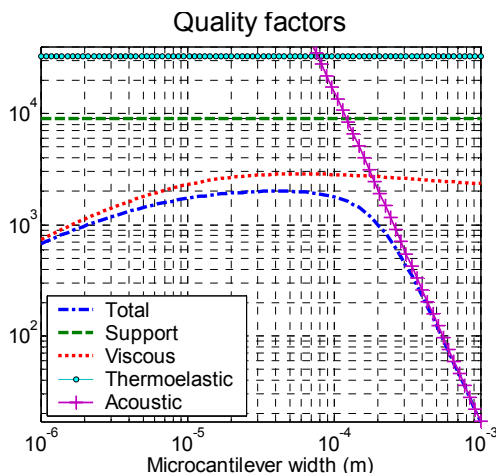


Figure 4: Quality factors (log scale) at the optimal thickness (6 μm) for a 100 μm long microcantilever as a function of the width (log scale)

Because of the approximations in the evaluation of the quality factor associated with acoustic losses, no general equations can be given to choose the ideal aspect ratios (L_1/h_1 , L_1/b_1). However, by determining the optimal width and thickness for a given microcantilever length, it is possible to observe that using $L_1/h_1 = 20$ and $L_1/b_1 = 3$ is a simple rule to obtain an almost optimal quality factor.

A quick comparison with conventional aspect ratios ($L_1/h_1 > 50$, $L_1/b_1 > 10$) is made in table 1. From figure 2, it can be seen that using these aspect ratios implies that viscous losses are dominant and thus sufficient to calculate the quality factor. But in that case, the quality

factor can be much lower while the resonant frequency remains low because of a high L_1/h_1 ratio.

Table 1: Comparison between different aspect ratios for a 100 μm long microcantilever

Microcantilever aspect ratios	Quality factor	Resonant frequency
$L_1/h_1 = 20$ $L_1/b_1 = 3$	1750	607 kHz
$L_1/h_1 = 50$ $L_1/b_1 = 10$	345	243 kHz
$L_1/h_1 = 100$ $L_1/b_1 = 20$	73.4	121 kHz

CONCLUSION

The new aspect ratios that have been proposed ($L_1/h_1 = 20$, $L_1/b_1 = 3$, in air) should be considered by designers to increase the sensor performance by increasing the resonant frequency while keeping high quality factors.

This work has not taken into account losses that can occur in the sensitive coating (for example viscoelastic losses when using polymers). These aspect ratios can be significantly modified when using high loss sensitive coating.

But if analytical expressions can be found for these losses, a similar work may be conducted to find new aspect ratios.

These aspect ratios are no longer valid in water, but similar work can be done since the equations can be applied on any fluid as long as viscosity and mass density are known.

ACKNOWLEDGEMENTS

This work was sponsored in part by the French Agency for Environment and Energy Management (ADEME) and the "Région Aquitaine".

REFERENCES

1. R. D. Blevins, *Formulas for natural frequency and mode shape*, Van Nostrand Reinhold Company, 1978, p 108.
2. D. Lange, C. Hagleitner, A. Hierlemann, O. Brand, H. Baltes, *Analytical Chemistry*, 74 (2002), pp 3084-3095.
3. J. E. Sader, *Journal of applied physics*, 84 (1998), pp 64-76.
4. Z. Hao, A. Erbil, F. Ayazi, *Sensors and Actuators A*, 109 (2003), pp 156-164.
5. W. K. Blake, *Journal of Sound and Vibration*, 33 (1974), pp 427-450.
6. R. Lifshitz, M. L. Roukes, *Physical Review B*, 61 (2000), pp 5600-5609.