A polynomial time approximation scheme for the single machine total completion time scheduling problem with availability constraints

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Abstract

In this paper, we study the single machine total completion time scheduling problem with availability constraints. This problem is known to be NP-complete. Sadfi et al gave an approximation algorithm with relative error $\frac{3}{17}$. The main contribution of this paper is a polynomial time approximation scheme for this problem. Our scheme is a simple generalization of the algorithm proposed by Sadfi et al.

1 Introduction

We consider the problem of scheduling tasks on a machine having a period of maintenance. This period of maintenance is known in advance, and is such that no job can be done during it. In other words, preemption is not allowed, and the machine is not available for processing jobs during the maintenance. We wish to minimize the total completion time of the jobs. Scheduling problems with availability constrainsts are widely studied (see [6, 7] for surveys). This particular problem is usually denoted $1, h_1 // \sum C_i$. Adiri et al [1] and Lee and Liman [2] showed that this problem is NP-hard.

Lee and Liman also showed that the SPT algorithm leads to a heuristic of relative error $\frac{2}{7}$.

Sadfi et al [5] proposed an improved heuristic for this problem, having a relative error of $\frac{3}{17}$. Their heuristic is a post-optimisation of SPT using a 2-OPT procedure. More precisely, let us denote A and B the sets of jobs scheduled respectively after and before the maintenance by the SPT algorithm. The heuristic consists in exchanging one job of A with one job of B in order to improve the total completion time. They call their procedure MSPT, for Modified SPT.

In this paper we study a generalization of MSPT, that we call MSPT-k, which consists of exchanging at most k jobs of A with at most k jobs of B, with k a fixed positive constant. We prove that for all $k \geq 2$, MSPT-k has a relative error of α_k, with $\alpha_k \to 0$ when $k \to +\infty$. In other words, MSPT-k provide a polynomial time approximation scheme (PTAS) for the problem $1, h_1 // \sum C_i$. We also prove that the value α_k is tight, that is to say we give a construction of a family of instances such that, asymptotically, the error bound of MSPT-k is α_k.

2 Notations

Let $J = \{J_i \mid i = 1, ...n\}$ be the set of jobs. We use the following notations, which, for convenience, are the same as in [5]:

$J_{[i]}$	job scheduled at position i
p_i	processing time of job J_i
$p_{[i]}$	processing time of job scheduled at position i
C_i	completion time of job J_i
$C_{[i]}$	completion time of job scheduled at position i
R	starting time of maintenance
L	duration of maintenance
D	ending time of maintenance (hence $D = R + L$)
δ	idle time of the machine before the maintenance

The MSPT-k heuristic tries to improve the result given by the SPT algorithm by trying all possible exchanges of at most k jobs scheduled before the maintenance with at most k jobs scheduled after the maintenance. More formally, the algorithm is the following :

MSPT-k heuristic

1. Schedule the jobs according to the SPT rule.
2. Denote A the set of jobs scheduled after the maintenance, and B the set

of jobs scheduled before.
3. Try all possible exchanges of at most k jobs of A with at most k jobs of B (the jobs before and after the maintenance being scheduled in non-decreasing order of their processing times).
4. Output the best exchange found in step 3.

Note that MSPT-0 is just the SPT algorithm, and MSPT-1 is the MSPT heuristic of Sadfi et al [5].

3 A polynomial time approximation scheme

In this section we derive some properties of the solutions obtained using SPT and MSPT-k, for a fixed $k \geq 2$. The following lemmata enable us to analyze the MSPT-k heuristic and to compute an error bound tending to zero when k grows to infinity ; that is to say we prove that MSPT-k leads to a polynomial time approximation scheme (PTAS).

We will use the following notations, which are the same as in [5]: The schedule generated by the SPT algorithm will be denoted S, the optimal schedule will be denoted S^*, and the schedule generated by MSPT-k will be denoted S'. Clearly, any schedule can be seen as a partition of the jobs into two sets: Those which are scheduled before the maintenance, and those which are scheduled after the maintenance. Indeed, once the partition of jobs is fixed, it is dominant to schedule them in non increasing order of their processing time.

Let A be the set of jobs scheduled after the maintenance in S, and let B be the set of jobs scheduled before the maintenance in S. With straightforward notations, A' and B' represents the job partition in S'. Now, let X be the set of the $|B|$ first jobs scheduled in S^*, and let Y be the set of remaining jobs (note that $|Y| = |A|$).

Note that MSPT-k exchanges t jobs of A with t' jobs of B, with $t \leq k$, $t' \leq k$, and $t' \geq t$. Hence we have $|A'| \geq |A| = |Y|$ and $|B'| \leq |B| = |X|$.

Finally, let us denote C_i, C'_i, and C^*_i the completion times of job J_i in schedules S, S', and S^* respectively. Since MSPT-k clearly improves SPT, then we have $\sum_i C'_i \leq \sum_i C_i$. Let us also denote $J'_{[i]}$ and $J^*_{[i]}$ the jobs scheduled at position i in S' and S^*, respectively. The straightforward notations $p'_{[i]}$, $p^*_{[i]}$, $C'_{[i]}$ and $C^*_{[i]}$ will also be used in the sequel. Throughout this section we consider the case where $|A| \geq k+1$ and $|B| \geq k+1$, because if not, then MSPT-k is clearly optimal.

3.1 Preliminary lemmata

This lemma generalizes Lemma 1 in [5], that stated the result only for the case $\widehat{S} = S'$ and $\widehat{S} = S^*$. Its proof is essentially the one of [5].

Lemma 1 *Let $\widehat{S}$ be a schedule better than S, that is to say its total completion time is better that the one of S. Let us denote δ and $\widehat{\delta}$ the idle time of the machine before the maintenance in S and $\widehat{S}$, respectively. Then we have $\widehat{\delta} \le \delta$.*

Proof : Let $\widehat{S}$ be a schedule whose total completion time is better that the one of S. Let us denote $\widehat{J}_{[i]}$ the job scheduled at position i in $\widehat{S}$, $\widehat{p}_{[i]}$ its processing time, and $\widehat{C}_{[i]}$ its completion time. Let us denote $\widehat{\delta}$ the idle time of the machine before the maintenance in $\widehat{S}$. Let $J_{[t]}$ be a job scheduled at position t in S, with $t > |B|$. By definition, this job belongs to A, hence we have:

$$C_{[t]} = \sum_{i=1}^{t} p_{[i]} + \delta + L.$$

Since SPT maximizes the number of jobs scheduled before the maintenance, then we also have:

$$\widehat{C}_{[t]} = \sum_{i=1}^{t} \widehat{p}_{[i]} + \widehat{\delta} + L.$$

In addition, since SPT is the optimal sequencing without period of maintenance, then we have:

$$\sum_{i=1}^{t} \widehat{p}_{[i]} \ge \sum_{i=1}^{t} p_{[i]}.$$

This implies that $\widehat{C}_{[t]} \ge C_{[t]} + \widehat{\delta} - \delta$ for all $t = |B| + 1, \dots, n$.
Notice that for $t = 1, ..., |B|$, the SPT strategy ensures that $\widehat{C}_{[t]} \ge C_{[t]}$.
By way of contradiction, assume now that $\widehat{\delta} > \delta$. Summing the previous inequalities we obtain:

$$\sum_{i=1}^{n} \widehat{C}_{[i]} > \sum_{i=1}^{n} C_{[i]}.$$

This would implie that the SPT solution S is better than $\widehat{S}$, a contradiction. Therefore we have $\widehat{\delta} \le \delta$. □

Note that this implies in particular that $\delta^* \leq \delta$ and $\delta' \leq \delta$, where δ^* and δ' denote the idle times on the machine before the maintenance in schedules S^* and S', respectively.

The following lemma is Lemma 2 of [5], that we recall here without any proof.

Lemma 2 (Sadfi et al.) *Let $C_{[i]}$ and $C^*_{[i]}$ be the completion times of the job scheduled at position i in the SPT and in the optimal solution, respectively. Then we have:*

$$\sum_{J_{[i]} \in A} C_{[i]} \;\leq\; \sum_{J_{[i]} \in Y} C^*_{[i]} + |Y|(\delta - \delta^*).$$

The following lemma generalizes Lemma 3 of [5], that stated the result for the case $t = 1$.

Lemma 3 *Let $t \geq 1$ be an integer. If (at least) t jobs of X are scheduled after the period of maintenance in the optimal solution, then we have:*

$$\sum_{i=1}^{n} C'_i \;\leq\; \sum_{i=1}^{n} C^*_i + (|Y| - (t+1))\,(\delta - \delta^*).$$

Proof : Since at least t jobs of X are scheduled after the period of maintenance in the optimal solution, then we have:

$$C^*_{[i]} \;\geq\; C_{[i]} \;\text{ for all } i = 1, \ldots, |B| - t,$$

and

$$\begin{aligned} C^*_{[|B|-i]} \;&\geq\; C_{[|B-i|]} + \delta + L \\ &\geq\; C_{[|B-i|]} + (\delta - \delta^*) \;\text{ for all } i = 0, \ldots, t-1. \end{aligned}$$

As for the job scheduled at position $|B| - t$ in S^*, we have the following more accurate inequality :

$$C^*_{[|B|-t]} \;\geq\; C_{[|B|]} + (\delta - \delta^*) \;\geq\; C_{[|B|-t]} + (\delta - \delta^*).$$

These inequalities imply that:

$$\sum_{J^*_{[i]} \in X} C^*_{[i]} \;\geq\; \sum_{J_{[i]} \in B} C_{[i]} + (t+1)(\delta - \delta^*).$$

Using Lemma 2, we then get

$$\begin{aligned}\sum_{i=1}^{n} C_i &= \sum_{J_{[i]}\in B} C_i + \sum_{J_{[i]}\in A} C_i \\ &\leq \sum_{J^*_{[i]}\in X} C^*_i + \sum_{J^*_{[i]}\in Y} C^*_i + (|Y| - (t+1))(\delta - \delta^*).\end{aligned}$$

Since $\sum_{i=1}^{n} C'_i \leq \sum_{i=1}^{n} C_i$, we conclude that

$$\sum_{i=1}^{n} C'_i \;\leq\; \sum_{i=1}^{n} C^*_i + (|Y| - (t+1))\,(\delta - \delta^*).$$

□

The following lemma generalizes Lemma 4 of [5], that stated the result for the case $t = 2$.

Lemma 4 *Let $t \geq 1$ be an integer. If (at least) t jobs of B are scheduled after the period of maintenance in the optimal schedule S^*, then we have:*

$$\sum_{i=1}^{n} C^*_i \;\geq\; \left\{\frac{|Y|(|Y|+1)}{2} + t\right\}(\delta - \delta^*)$$

Proof : We first decompose the sum $\sum_{i=1}^{n} C^*_i = \sum_{J^*_{[i]}\in B} C^*_{[i]} + \sum_{J^*_{[i]}\in A} C^*_{[i]}$. The SPT heuristic implies that $p_i \geq \delta \geq \delta - \delta^*$ for all job $J_i \in A$. Hence we have:

$$\begin{aligned}\sum_{J^*_{[i]}\in A} C^*_{[i]} &\geq \left\{\frac{|A|(|A|+1)}{2}\right\}(\delta - \delta^*) \\ &= \left\{\frac{|Y|(|Y|+1)}{2}\right\}(\delta - \delta^*).\end{aligned}$$

If at least t jobs of B are processed after the period of maintenance in the optimal schedule, then we have:

$$\sum_{J^*_{[i]}\in B} C^*_{[i]} \;\geq\; t(R+L) \;\geq\; t(\delta - \delta^*).$$

Summing up these to inequalities we get:

$$\sum_{i=1}^{n} C^*_i \;\geq\; \left\{\frac{|Y|(|Y|+1)}{2} + t\right\}(\delta - \delta^*),$$

which is the desired result. □

Finally we will need later on the following lemma.

Lemma 5 *Let $p \geq 1$ and $q \geq 1$ be two integers such that $p \geq q$. Let us consider the solution S obtained with the SPT heuristic. We claim that if it is possible to exchange p jobs of B with q jobs of A, then it is possible to exchange $p - q + 1$ jobs of B with 1 job of A.*

Proof : Let us assume that it is possible to exchange p jobs of B with q jobs of A, with $p \geq q \geq 1$. If $p = q$ then we are done. Else, let $A_{\mathrm{e}} \subseteq A$ and $B_{\mathrm{e}} \subseteq B$ denote the sets of jobs of A and B that may be exchanged. Note that the exchange of these jobs is possible if and only if

$$\sum_{J_j \in A_{\mathrm{e}}} p_j \leq \sum_{J_i \in B_{\mathrm{e}}} p_i + \delta.$$

Since $p_j \geq p_i$ for all jobs $J_j \in A$ and $J_i \in B$, then we still have

$$\sum_{J_j \in A_{\mathrm{e}} \setminus A'_{\mathrm{e}}} p_j \leq \sum_{J_i \in B_{\mathrm{e}} \setminus B'_{\mathrm{e}}} p_i + \delta$$

where $A'_{\mathrm{e}} \subseteq A_{\mathrm{e}}$ and $B'_{\mathrm{e}} \subseteq B_{\mathrm{e}}$ are any arbitrary sets having the same cardinality. This leads to the desired result if we consider A'_{e} and B'_{e} of cardinality $q - 1$. □

Now we are ready to analyze the MSPT-k heuristic. Two cases follow, depending on the possibility of exchanging jobs from A with jobs from B.

3.2 Analysis of the MSPT-k heuristic when no exchange of jobs is possible

Let us assume that no exchange of jobs is possible, that is to say we can not exchange at most k jobs of B with at most k jobs of A in the SPT schedule S. This implies that $p_{[|B|]} + \delta > p_k$ for all $J_k \in A$, and that the MSPT-k solution is identical to the SPT solution.

If $B = X$, then the SPT schedule is clearly optimal, and so is the MSPT-k schedule. If $B \neq X$, then SPT is not optimal, and we have to exchange at least $k + 1$ jobs from B with jobs from A to get the optimal solution.

Hence there exists $p \geq k + 1$ jobs in set B that can be exchanged with $q \leq p$ jobs in set A. According to Lemma 5, this implies that there exists $p - q + 1$ jobs in set B that can be exchanged with 1 job in set A. Since there is no exchange possible of at most k jobs of set B with at most k jobs of set A, then we have $p-q+1>k$. This implies that $p-q\geq k$, that is to say there are at most $|B|-k$ jobs scheduled before the period of maintenance in the optimal solution. Thus, at least k jobs of X are scheduled after the period of maintenance in the optimal solution.

To summarize, we have at least $k+1$ jobs of B, and k jobs of X, which are scheduled after the period of maintenance in the optimal solution. According to Lemma 3 and Lemma 4, we have the two following inequalities:

$$\sum_{i=1}^{n} C_i' \;\leq\; \sum_{i=1}^{n} C_i^* + (|Y|-(k+1))(\delta-\delta^*) \tag{1}$$

and

$$\sum_{i=1}^{n} C_i^* \;\geq\; \left\{\frac{|Y|(|Y|+1)}{2} + (k+1)\right\}(\delta-\delta^*). \tag{2}$$

Let us denote ε_k the error bound of the MSPT-k heuristic, that is to say:

$$\varepsilon_k \;=\; \frac{\sum_{i=1}^{n} C_i' - \sum_{i=1}^{n} C_i^*}{\sum_{i=1}^{n} C_i^*}.$$

Combining inequalities (1) and (2), we obtain:

$$\varepsilon_k \;\leq\; \frac{2(|Y|-(k+1))}{|Y|(|Y|+1)+2(k+1)}. \tag{3}$$

3.3 Analysis of the MSPT-k heuristic when the exchange of jobs is possible

Now, let us assume that we can exchange at most k jobs in set B with at most k jobs in set A in the SPT schedule S. If MSPT-k is not optimal, then at least $k+1$ jobs from set B are scheduled after the period of maintenance in the optimal solution. Using Lemma 4, again, we have the following inequality:

$$\sum_{i=1}^{n} C_i^* \;\geq\; \left\{\frac{|Y|(|Y|+1)}{2} + (k+1)\right\}(\delta-\delta^*). \tag{4}$$

Now we study two cases, according to the number of jobs from set X scheduled after the period of maintenence in the optimal solution.

Case 1: At least k jobs of X are scheduled after the period of maintenance in the optimal solution. In this case we can again use the result of Lemma 3 to get:

$$\sum_{i=1}^{n} C'_i \;\leq\; \sum_{i=1}^{n} C^*_i + (|Y| - (k+1))(\delta - \delta^*). \tag{5}$$

Case 2: At most $k-1$ jobs of X are scheduled after the period of maintenance in the optimal solution. Let $z \leq k-1$ be the number of jobs of X scheduled after the period of maintenance in the optimal solution. Clearly, the optimal solution is obtained by exchanging q jobs from set A with $q+z$ jobs from set B, with $q \geq 1$. Notice that $q+z > k$ since we assumed that MSPT-k is not optimal.

Let W_A (resp. W_B) be the set of jobs of A (resp. B) scheduled before (resp. after) the maintenance in S^* and let p_A (resp. p_B) be the sum of their processing times. Notice that $p_A \geq p_B$ by Lemma 1. See Figure 1 for these notations.

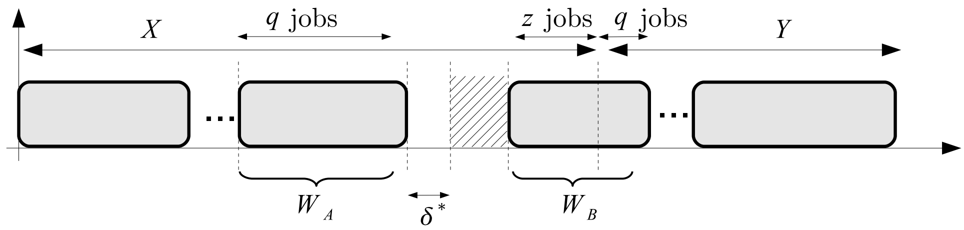

Figure 1: The optimal schedule S^*.

Now let us consider the schedule $\widetilde{S}$, obtained from the optimal schedule S^* by exchanging the jobs of W_A with the jobs of W_B. Let Δ be the quantity $\Delta = p_A - p_B \geq 0$. It is easy to see that

$$\sum_{i=1}^{n} \widetilde{C}_i \;\leq\; \sum_{i=1}^{n} C^*_i + (|Y| - q)\Delta - z(p_A + \delta^* + L), \tag{6}$$

where $\widetilde{C}_i$ denotes the completion time of job J_i in the schedule $\widetilde{S}$. Indeed, we have $\widetilde{C}_i = C^*_i + \Delta$ for the $|Y| - q$ jobs $J_i \in Y$ scheduled after W_B in the optimal solution. Similarly, we have $\widetilde{C}_i = C^*_i - (p_A + \delta^* + L)$ for the $q+z$ jobs $J_i \in W_B$, and $\widetilde{C}_i = C^*_i + (p_A + \delta^* + L)$ for the q jobs $J_i \in W_A$.

Obviously, the idle time of the machine before the maintenance in schedule $\widetilde{S}$ is equal to δ. Hence $\Delta = p_A - p_B = \delta - \delta^*$. It is also immediate to observe that $p_A + \delta^* + L \geq \delta - \delta^*$, since $p_A + \delta^* = p_B + \delta$. Moreover, it is clear that $\sum_{i=1}^n C_i' \leq \sum_{i=1}^n C_i \leq \sum_{i=1}^n \widetilde{C}_i$, because SPT is optimal once the set of jobs before and after the maintenance is fixed. Thus we obtain the following inequality

$$\sum_{i=1}^{n} C_i' \;\leq\; \sum_{i=1}^{n} C_i^* + (|Y| - q - z)(\delta - \delta^*).$$

Now, using the fact that $q + z > k$, we have

$$\sum_{i=1}^{n} C_i' \;\leq\; \sum_{i=1}^{n} C_i^* + (|Y| - (k+1))(\delta - \delta^*). \tag{7}$$

Conclusion To summarize this section, we have shown that if at least k jobs of X are scheduled after the period of maintenance in the optimal solution, then we have

$$\sum_{i=1}^{n} C_i' \;\leq\; \sum_{i=1}^{n} C_i^* + \{|Y| - (k+1)\}\,(\delta - \delta^*). \tag{8}$$

(this is equation (5)). We also showed that the previous equation also holds if less than k jobs of X are scheduled after the period of maintenance in the optimal solution (this is equation (7)).

Putting (4) and (8) together, we get the following bound on the error bound of MSPT-k:

$$\varepsilon_k \;=\; \frac{\sum_{i=1}^n C_i' - \sum_{i=1}^n C_i^*}{\sum_{i=1}^n C_i^*} \;\leq\; \frac{2(|Y| - (k+1))}{|Y|(|Y|+1) + 2(k+1)}. \tag{9}$$

3.4 Computing the error bound

Putting (3) and (9) together, we can claim that the error bound ε_k of MSPT-k is bounded as follows:

$$\varepsilon_k \;=\; \frac{\sum_{i=1}^n C_i' - \sum_{i=1}^n C_i^*}{\sum_{i=1}^n C_i^*} \;\leq\; \frac{2(|Y| - (k+1))}{|Y|(|Y|+1) + 2(k+1)}.$$

For all $k > 0$, the function $f_k : x \mapsto f_k(x) = \frac{2(x-(k+1))}{x(x+1)+2(k+1)}$, $x \in \mathbb{N}^+$ reaches its maximum for $x_k = 2k+3$ (straightforward computation). Then we have

$$\max_{|Y| \in \mathbb{N}^+} \varepsilon_k \leq f_k(x_k) = \frac{k+2}{2k^2+8k+7}.$$

Since

$$\alpha_k = \frac{k+2}{2k^2+8k+7} \underset{k\to\infty}{\longrightarrow} 0,$$

then MSPT-k provides a polynomial time approximation scheme for the problem $1, h_1//\sum C_i$. This is the main result of the present paper. In the next section we exhibit a family of instances of $1, h_1//\sum C_i$ such that, asymptotically, MSPT-k has an error bound of α_k.

4 Tightness of the error bound of MSPT-k

In the previous section we proved that, for all $k \geq 1$, the MSPT-k algorithm had an error bound bounded by $\alpha_k = \frac{k+2}{2k^2+8k+7}$. For $k = 1$, we get $\alpha_1 = \frac{3}{17}$, hence this bound is tight in this case [5]. Note that for $k = 0$ too the previous bound is tight, since $\alpha_0 = \frac{2}{7}$ [2].

Actually we can prove that, for all k, α_k is the tight error bound of MSPT-k. Indeed, let us consider the following example, which is a generalization of instances proposed by Lee and Liman [2] and Sadfi et al [5].

Let $M \in \mathbb{N}^+$ be a number greater than or equal to $k+1$. Let us consider $3k+4$ jobs such that with $p_i = 1$ for $i \in \{1, 2, .., k+1\}$ and $p_i = M$ for $i \in \{k+2, .., 3k+4\}$. Let $R = M$ and $L = 1$. It is easy to see that the MSPT-k schedule is the one where the set of jobs scheduled before the maintenance is $\{J_i \mid i = 1, \ldots, k+1\}$, and that the optimal schedule is the one where the set of jobs scheduled before the maintenance is $\{J_{k+2}\}$ (see Figures 2 and 3).

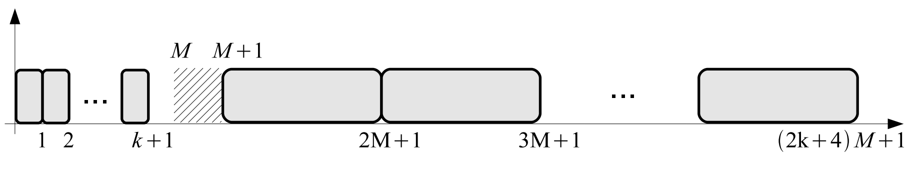

Figure 2: The schedule S' obtained using the MSPT-k heuristic.

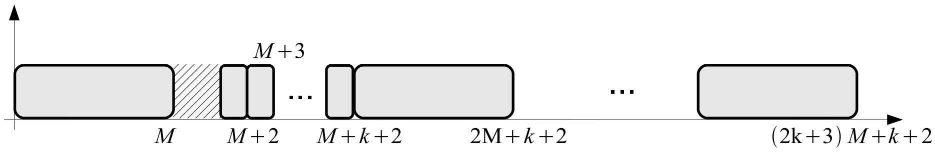

Figure 3: The optimal schedule S^*.

Let us assume that M is significantly larger than k, for instance $k^2 = o(M)$. Straightforward computations show that:

$$\sum_{i=1}^{n} C'_i = M(2k^2 + 9k + 9) + o(M),$$

and

$$\sum_{i=1}^{n} C^*_i = M(2k^2 + 8k + 7) + o(M).$$

Hence, we have

$$\frac{\sum_{i=1}^{n} C'_i - \sum_{i=1}^{n} C^*_i}{\sum_{i=1}^{n} C^*_i} = \frac{M(k+2) + o(M)}{M(2k^2 + 8k + 7) + o(M)}.$$

When M tends to infinity, this quantity tends to $\frac{k+2}{2k^2+8k+7}$. Hence the error bound of MSPT-k is exactly $\frac{k+2}{2k^2+8k+7}$.

5 Computational experiments

In this section we present results of computational experiments we made to test our algorithm. For $k = 2, 3$, we compared results of MSPT-k with those obtained using MSPT-$(k-1)$ and with the optimal schedule. The optimal schedule was computed using a dynamic programming approach described in [3, Section 4.5] and in [4].

The algorithms have been implemented using the Java langage. Job processing times are uniformly generated as integers in the interval $[1, 100]$. The number n of jobs ranges from 10 to 100 jobs, and 50 instances are generated for each value of n. For the generation of the date of maintenance, we choose $R = 25\%$ of the sum of processing times of the jobs (in this case,

$B_{\max} \simeq 50\%$ of n, where $B_{\max}$ denotes the number of jobs scheduled before the maintenance in the SPT schedule).

Figure 4 shows the evolution with the number of jobs of the number of cases where MSPT-2 improves MSPT, while Figure 5 deals with the number of cases where MSPT-3 improves the MSPT-2 heurisitic.

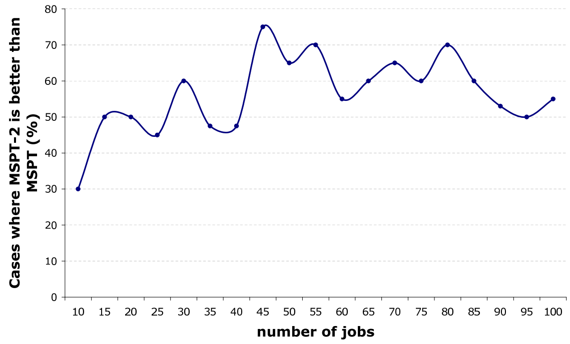

Figure 4: Proportion of cases where MSPT-2 improves MSPT.

Figure 5: Proportion of cases where MSPT-3 improves MSPT-2.

As shown in these two figures, the percentage of cases where MSPT-k improves MSPT-$(k-1)$ $(k = 2, 3)$ reaches at least 50%.

Figure 6 shows the comparison of MSPT-3, MSPT-2 and MSPT with the optimal schedule.

We can observe that MSPT-2 and MSPT-3 have effective relative errors of less than 1.5% for the (large) randomly generated instances we tested. These values can be compared to the theoretical relative errors $\alpha_2 \simeq 12.9\%$ and $\alpha_3 \simeq 10.2\%$. Hence the MSPT-k heuristics has an *effective* relative error far below the theoretical ones for the instances we generated.

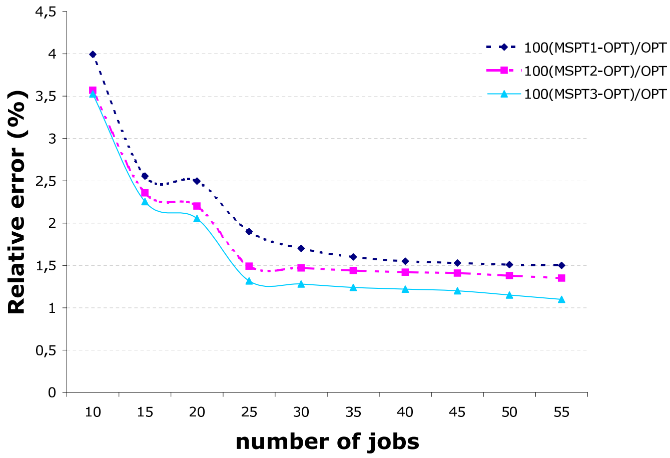

Figure 6: Relative errors of MSPT, MSPT-2 and MSPT-3.

6 Conclusion

In this paper we described a polynomial time approximation scheme (PTAS) for the single machine total completion time scheduling problem with availability constraints, denoted $1, h_1 // \sum C_i$. Our heuristic is inspired and generalizes the work of Lee and Liman [2] and Sadfi et al [5].

In order to prove the PTAS, we computed an error bound for our heuristic, and we proved our bound to be tight in Section 4. One can notice that the main results of [2] and [5] can be obtained as immediate consequences of ours.

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