Jitter measurement of an ADC by statistical analysis

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This paper presents a method to measure the jitter of a ADC based on the locked histogram test. We study the conditions for its validation as a function of the sampling position. As all instruments of the test bench have their own independant jitter, we use a mathematical model to extract the jitter of the ADC. Further we give some experimental results.

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1. Introduction.

There are two basic methods to measure the jitter of a analogue-to-digital converters (ADC), spectral analysis (Wakimoto et al. 1988, Shinagawa et al. 1990) and statistical analysis (Hewlett-Packard Product Note 1982, Delmer 1991). The first method consists of deducting the jitter from the signal to noise ratio (SNR), which is calculated by spectral analysis. The second uses the statistical properties of a data acquisition at locked frequency ($f_{in} = f_{clk}$).

Both methods require a signal generator with a very low phase noise (Hewlett-Packard Product Note 1982). In this paper we present a method which allows to measure the jitter by statistical analysis, using signal generators which don't fulfil this condition. First, we describe the principle of the locked histogram test and give the conditions that are necessary for its validation. Secondly, we explain the methods that we used to distinguish and to separate the different noise sources.

2. Principle of the locked histogram test

The locked histogram test gives the distribution of the different codes obtained as a function of their probability of occurrence, $f_i = n_i/N$ (where n_i is the number of occurrences of code i and N is the total number of samples). We get a distribution of gaussian form with a mean value and a variance as following :

$$M_c = \frac{1}{N}\sum_i n_i.i \quad \text{and} \quad \sigma_c{}^2 = \frac{1}{N}\sum_i n_i(i - M_c)^2 \qquad (1)$$

The input test signal $V_{in}(t)$ is a sine wave ($V_{in}(t)$=Asin($2\pi f_{in}t$)) and the variation of this signal (ΔV), during a time interval ΔT which is smaller or equal to the jitter, is regarded to be linear (as shown in figure 1). Thus, the relation between the sampling time error and the amplitude noise error can be described as :

$$\Delta V = \Delta T \left\{ \frac{\partial V_{in}(t)}{\partial t} \right\}_{t=t_o} = \Delta T\, A\, 2\pi f_{in} \cos(2\pi f_{in} t_o) \qquad (2)$$

Expression (2) allows to deduct a relation between the jitter Δt and the standard deviation $\sigma_V = \sigma_C.q$ (where q is the quantisation step of the conversion and Δt is a rms value).

$$\Delta t = \sigma_t = \sigma_c q \frac{1}{A 2\pi f_{in} \cos(2\pi f_{in} t_o)} \qquad (3)$$

3. Condition to validate the locked histogram test

If the sampling position is not centred on the middle of the quantum which represents the most frequent code (the code that appears most frequently), a more or less important error deforms the gaussian form of the histogram (see figure 2). A study of the influence of the sampling position within the quantum is based on p(v), the probability density function of an analogue sample :

$$p(v) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(v-(n+y)q)^2}{2\sigma^2}\right) = \frac{1}{xq\sqrt{2\pi}} \exp\left(-\frac{(v-(n+y)q)^2}{2x^2q^2}\right) \qquad (4)$$

with $\sigma = x^*q$ where x fixes the number of code values of the distribution. $(n+y)q$ is the analogue mean value on which the distribution is centred and y represents the variation of the mean sampling position ($-0.5 \leq y \leq 0.5$). The occurrence frequency of code n is given by the integral of p(v). We introduce the variable i which corresponds to the code next to the code n (i.e. i = 0 corresponds to the code n , i = 1 to the code n+1).

$$f(n+i) = f(i) = \int_{(n+i-\frac{1}{2})q}^{(n+i+\frac{1}{2})q} p(v)dv = \frac{1}{2}\operatorname{erf}\left(\frac{(1-2i+2y)}{2x\sqrt{2}}\right) - \frac{1}{2}\operatorname{erf}\left(\frac{(-1-2i+2y)}{2x\sqrt{2}}\right) \qquad (5)$$

As we can see, the frequency of occurrence depends on the values of x and y. The standard deviation $\sigma_C(x,y)$, deducted from relation (1) is given by :

$$\sigma_c(x,y) = \sqrt{\sum_i f(i).i^2 - \left\{\sum_i f(i).i\right\}^2} \qquad (6)$$

A code i is considered to be existent when its probability of occurrence f(i) is superior to 10^{-6}. Figure 3 shows the variation of $\sigma_c(x,y)$ in relation to $\sigma_c(x,0)$ for different values of x as a function of y (the numbers in brackets on figure 3 give the number of different codes obtained for a given x value and y = 0).

If the number of different codes values in the distribution is larger than 5 (see figure 3.b), the error can be ignored. Whereas, if this number is equal or smaller than 5, a more or less important error depending on the value of x appears (see figure 3.a). In figure 2, two data acquisitions of a 10-bit ADC are shown. For this measurement, we used the Hewlett Packard HP 3326 generator (two channel synthesiser). Unlike in figure 2.a, in figure 2.b the acquisition is not centred on the middle of the quantum, but nevertheless there is no important difference comparing the two measurements of Δt. This can be explained by the existence of more than 5 different codes values, 7 in this particular case.

4. Separation of the different jitters noises

We consider that the three major elements of the test bench (ADC, input signal and clock generator) have their own independent jitter (Δt_{adc}, Δt_{in}, Δt_{clk}) which influence the measurement. In general a jitter noise is normally distributed (Shinagawa et al. 1990). With this assumption, the total jitter Δt is given by the following relation :

$$\Delta t^2 = \Delta t_{adc}^2 + \Delta t_{in}^2 + \Delta t_{clk}^2 \qquad (7)$$

To extract Δt_{adc} from Δt, the frequency stability properties of the generator can be used. In fact, BARNS J. A. et al. (BARNS J. A. et al. 1971) determined the clock error (jitter) of a signal generator in function of the phase noise density and the carrier frequency. Considering the phase noise density of the generator to be independent of the frequency of the output signal, we can write :

$$\Delta t_{sg}{}^2 = \frac{C}{f_{sg}{}^2} \qquad (8)$$

C is a constant and f_{sg} is the carrier frequency of the generator which provides the input signal and the clock signal.

We use the normalised frequency f_{sgo} and transform expression (8) to :

$$\Delta t_{sg}{}^2 = \left(\frac{f_{sgo}}{f_{sg}}\right)^2 \Delta t_{sgo}{}^2 \qquad (9)$$

Combining the expressions (7) and (9) with $f_{in} = f_{clk} = f_{sg}$ and $f_{ino} = f_{clko} = f_{sgo}$ results in :

$$\Delta t^2 = \Delta t_{adc}{}^2 + \left\{ \left(\frac{f_{sgo}}{f_{sg}}\right)^2 \Delta t_{sgo}{}^2 \right\} \text{ with } \Delta t_{sgo}{}^2 = \Delta t_{ino}{}^2 + \Delta t_{clko}{}^2 \qquad (10)$$

Let us $f_{sg} = k.f_{sgo}$, we take M measurements of Δt as a function of k (Δt_{ki}, k_i , $i = 1...M$). Now it's possible to extract Δt_{adc} by using the least square method with the following model :

$$\Delta t_k = \sqrt{\Delta t_{adc}{}^2 + \left\{ \left(\frac{1}{k}\right)^2 \Delta t_{sgo}{}^2 \right\}} \qquad (11)$$

To simplify, let us $y_i = (\Delta t_{ki})^2$, $\alpha = (\Delta t_{adc})^2$, $\beta = (\Delta t_{sgo})^2$. Thus, the error function in the least square notation can be written as :

$$E = \sum_{i=1}^{M} \left\{ \frac{y_i - g(k_i;\alpha,\beta)}{y_i} \right\}^2 \quad \text{with} \quad g(k_i;\alpha,\beta) = \alpha + \frac{1}{k_i{}^2}\beta \,;\, i \in [1..M] \qquad (12)$$

The parameters α and β can be obtained by searching for the minimum value of E which leads to the following system of linear equations :

$$
\begin{aligned}
\frac{\partial E}{\partial \alpha} = 0 \quad &\Rightarrow \quad \sum_{i=1}^{M} \frac{1}{y_i} = \alpha \sum_{i=1}^{M} \frac{1}{y_i^2} + \beta \sum_{i=1}^{M} \frac{1}{y_i^2 k_i^2} \\
\frac{\partial E}{\partial \beta} = 0 \quad &\Rightarrow \quad \sum_{i=1}^{M} \frac{1}{y_i k_i^2} = \alpha \sum_{i=1}^{M} \frac{1}{y_i^2 k_i^2} + \beta \sum_{i=1}^{M} \frac{1}{y_i^2 k_i^4}
\end{aligned}
\tag{13}
$$

Its solutions are given by :

$$
\alpha = \frac{\sum_{i=1}^{N} \frac{1}{y_i} \sum_{i=1}^{N} \frac{1}{y_i^2 k_i^4} - \sum_{i=1}^{N} \frac{1}{y_i k_i^2} \sum_{i=1}^{N} \frac{1}{y_i^2 k_i^2}}{\sum_{i=1}^{N} \frac{1}{y_i^2} \sum_{i=1}^{N} \frac{1}{y_i^2 k_i^4} - \left(\sum_{i=1}^{N} \frac{1}{y_i^2 k_i^2} \right)^2}
$$

$$
\beta = \frac{\sum_{i=1}^{N} \frac{1}{y_i^2} \sum_{i=1}^{N} \frac{1}{y_i k_i^2} - \sum_{i=1}^{N} \frac{1}{y_i} \sum_{i=1}^{N} \frac{1}{y_i^2 k_i^2}}{\sum_{i=1}^{N} \frac{1}{y_i^2} \sum_{i=1}^{N} \frac{1}{y_i^2 k_i^4} - \left(\sum_{i=1}^{N} \frac{1}{y_i^2 k_i^2} \right)^2}
$$

5. Experimental results

We used this method to measure the jitter of a 10-bit ADC. A generator Hewlett Packard HP3326 furnished the input and the clock signal (two channel synthesiser). The amplitude of the input signal is equal to the full-scale range of conversion of the ADC. For the locked histogram test, we made a data acquisition of $N = 512*2^{10}$ samples centred on code 300. These samples were processed by a Macintosh based software program, designed for ADC testing. In figure 4, the data samples and the corresponding fitted curve (least square method) are shown.

We consider the difference between the given samples and the fitted curve as error function. For high k values ($k \geq 8$), the jitter of the converter Δt_{adc} becomes the dominant term in expression (11). The maximum deviation between a data sample

(Δt_{kl}, k_l ; l = 8...13) and the fitted curve $g(k_l, \Delta t_{adc}, \Delta t_{sgo})$ is thus a good approximation for the error of the calculated jitter Δt_{adc}. We find $\Delta t_{adc} = 9.65 \pm 0.63$ ps.

6. **Conclusion**

Determination of the jitter of a ADC by the locked histogram test gives precise results if :

- the number of different code values that occur in the distribution is larger than 5, so that the error which depends on the exact acquisition position can be ignored.

- or if the number of the different codes is equal or smaller than 5, and the acquisition position is well centred on the middle of the most frequent code. In this case, centring can be done by varying the phase position of the input signal.

If one of these two conditions is fulfilled, the method is interesting because it allows to extract the jitter of a ADC despite of using a generator which has a quite important phase noise. This makes, it generally impossible to use other existing measurement methods.

REFERENCES

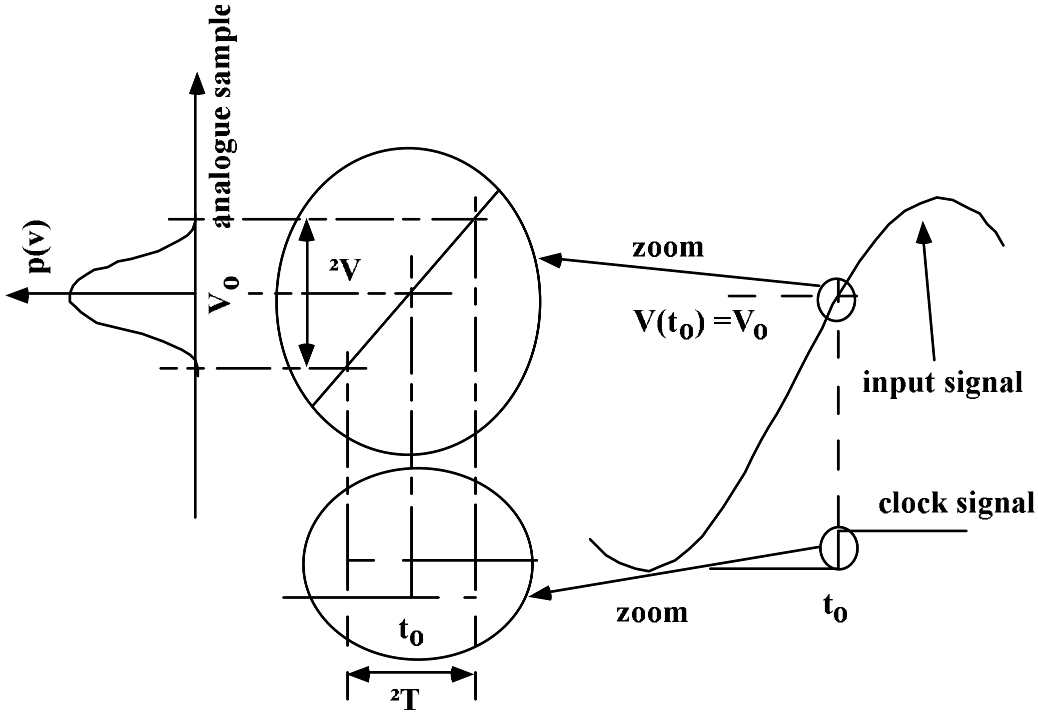
Barns, J. A., and al., 1971, Characterization of frequency stability. *I.E.E.E. Transactions on Instrumentation and Measurement*, **20**, pp. 105-120.

Delmer, M., 1991, *High-speed analogue-to-digital conversion* (Academic Press, U.K.), Ch. 6.

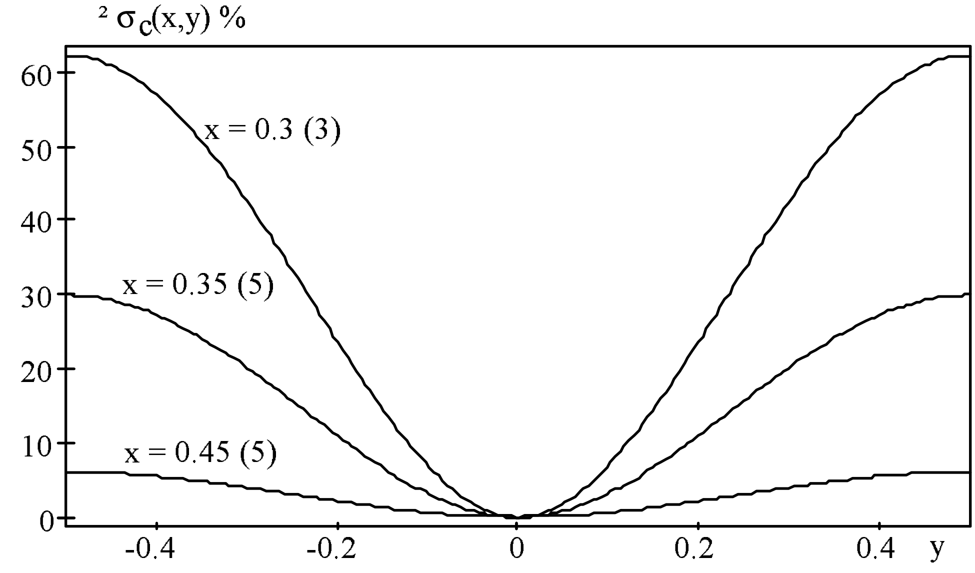
Hewlett-Packard Product Note 5180A-2, 1982, *Dynamic performance testing of A to D converters.*

Shinagawa, M., Akazawa, Y., and Wakimoto, T., 1990, Jitter analysis of high-Speed Sampling Systems. *I.E.E.E. Journal of Solid-State Circuits*, **25**, pp. 220-224.

Wakimoto, T., Akazawa, Y., and Konaka S., 1988, Si Bipolar 2-GHz 6-bit Flash A/D Conversion LSI. *I.E.E.E. Journal of Solid-State Circuits*, **23**, pp. 1345-1350.

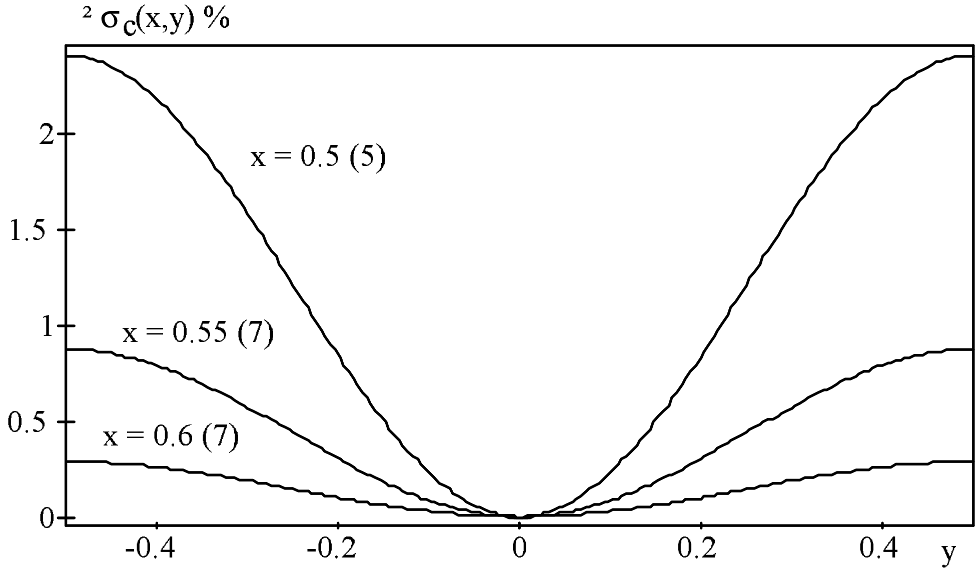


analogue sample
p(v)
V_o
²V
zoom
$V(t_o) = V_o$
input signal
clock signal
zoom
t_o
t_o
²T



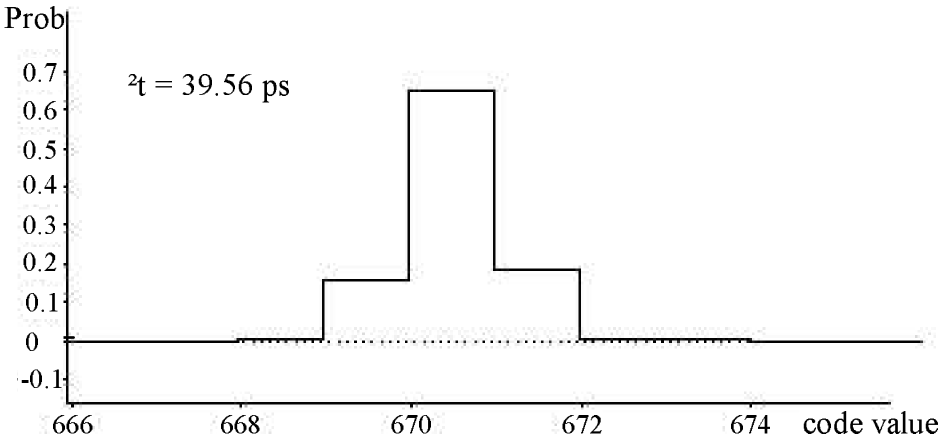
² σc(x,y) %
60
50
40
30
20
10
0
x = 0.3 (3)
x = 0.35 (5)
x = 0.45 (5)
-0.4
-0.2
0
0.2
0.4
y

(a)

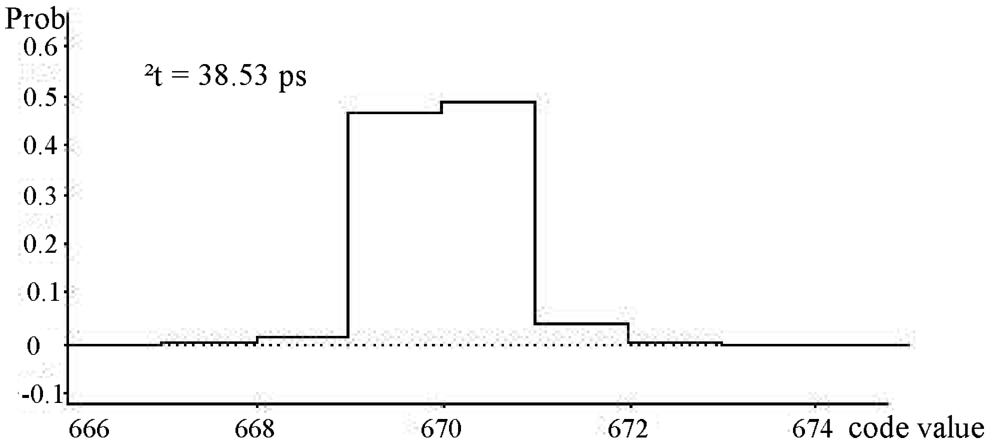


² σc(x,y) %
2
1.5
1
0.5
0
x = 0.5 (5)
x = 0.55 (7)
x = 0.6 (7)
-0.4
-0.2
0
0.2
0.4
y

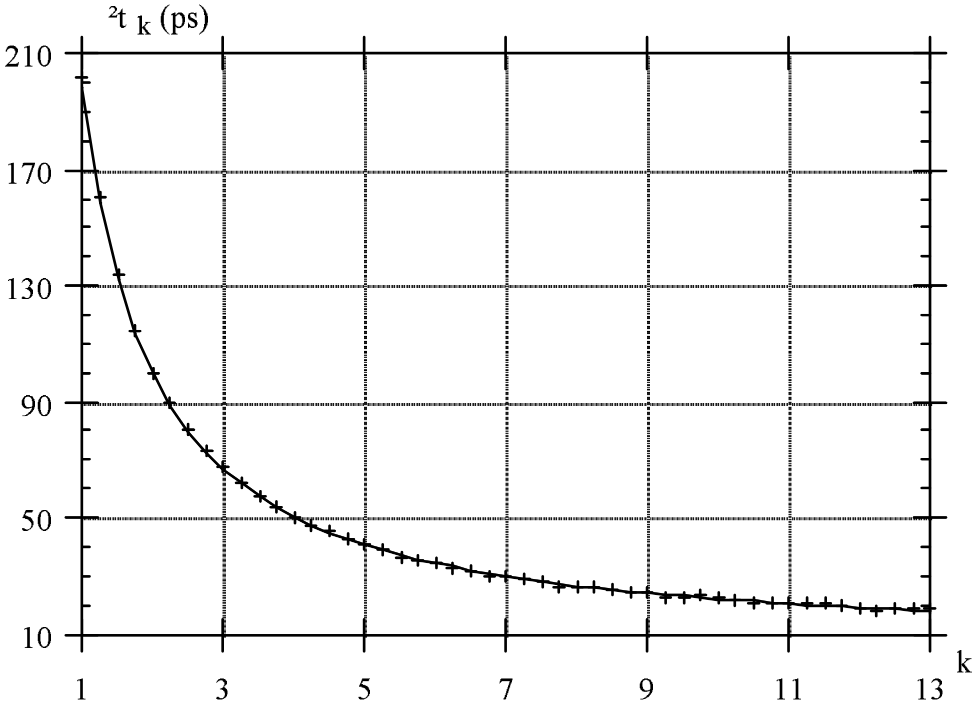
(b)


Prob
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
²t = 39.56 ps
666
668
670
672
674
code value

(a)


Prob
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
²t = 38.53 ps
666
668
670
672
674
code value

(b)


²t k (ps)
210
170
130
90
50
10
1
3
5
7
9
11
13
k
(a)

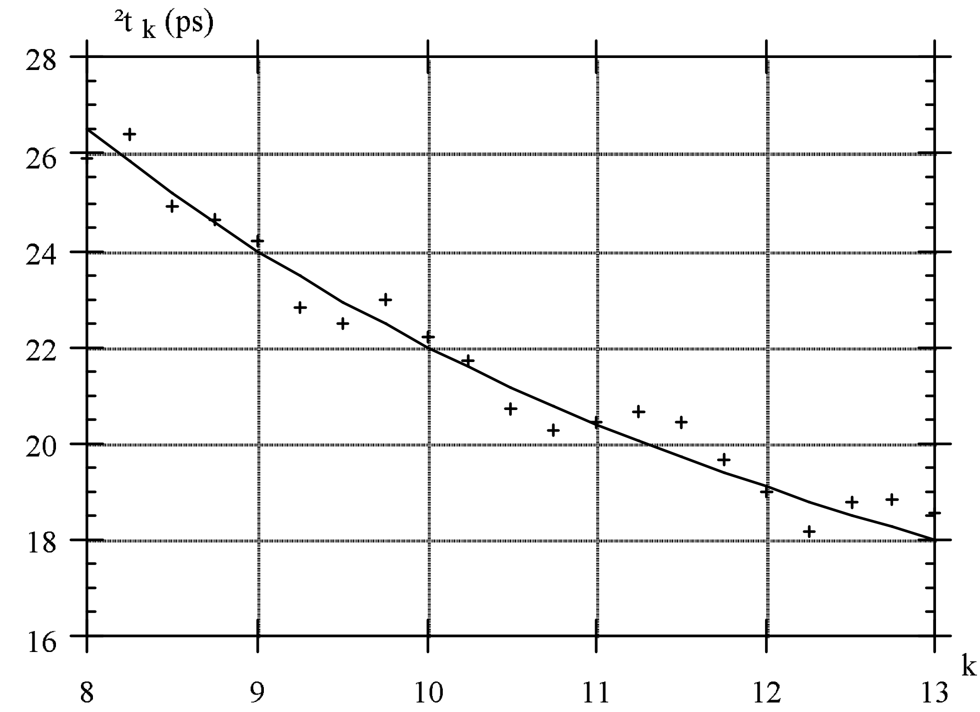

²t k (ps)
28
26
24
22
20
18
16
8
9
10
11
12
13
k
(b)

Figure 1. Noise amplitude ΔV versus times clock variation ΔT.

Figure 2. Acquisition of locked histogram : (a) the histogram is centred; (b) the histogram is not centred.

Figure 3. $\Delta\sigma_c(x,y)$ versus y for $0.3 \leq x \leq 0.6$.

Figure 4. Plot of Δt for different k values with fitted curve (a) and zoom of this plot for $8 \leq k \leq 13$ (b).

ADC jitter measurement