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One way to design the control law of a mini-UAV

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This paper deals with a method used to design the control law of the µDrone MAV. This vehicle uses six propellers to fly and the dynamic model approximation for the motion is a MIMO linear time-invariant system. As we want to design a linear regulator, it is necessary to build a robust feedback control law. The LQ state feedback regulator design is applied to a standard model, tacking into account some perturbations. This is why the model is augmented with a perturbation vector and an observable subsystem is extracted in order to build a state estimator whose gain is the solution of a LQ problem. The subsystem is then decomposed into a controllable set and an uncontrollable one. The use of an asymptotic rejection strategy of the influence of uncontrollable modes gives the possibility to find a state feedback applied only to the controllable ones. Here again feedback matrix is chosen as the solution of a LQ problem. To compute the weighting matrices of quadratic criterions we use a "partial observability gramian". The great advantage of this method is due to the use of only three scalars to synthesize the control law.

Nomenclature

x, y, z	=	centre of gravity coordinates of the MAV
α	=	roll angle
β	=	pitch angle
λ	=	yaw angle
u_i	=	actuator control input for solid S_i
F_i	=	thrust produced by the *i*th actuator
M_i	=	moment produced by the *i*th actuator
τ_i	=	time constant of the *i*th actuator
k_i	=	DC gain of the *i*th actuator
$\underline{u}$	=	$[u_1 \quad u_2 \quad u_3 \quad u_4 \quad u_5 \quad u_6]^T$ input vector
$\underline{y}$	=	$[\dot{x} \quad \dot{y} \quad z \quad \alpha \quad \beta \quad \lambda]^T$ output vector
$\underline{y}_r$	=	reference vector
$\underline{x}$	=	state vector of initial linear MAV model $\in R^{18}$
A, B, C	=	matrices of initial state-space model

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$\underline{x}_p$ = perturbation vector

I. Introduction

THIS paper deals with a method used to design the control law of the µDrone. This MAV uses six propellers to fly and it is not obvious to control the six brushless motors simultaneously. In order to build a dynamical model of the MAV, we worked on the assumption that it can be described with seven solids in interaction. After a linearization of the equations in the vicinity of a horizontal trim, the model is a multi-input, multi-output (MIMO) linear time-invariant dynamic system. This approximation can be used only if it is possible to build a robust feedback control law.

The interest of the present paper is to provide a simple method to reach the needed robustness, using the LQ state-feedback regulator design applied to a "standard model", tacking into account some perturbations. This method is detailed in Ref. 1 and very recently in Ref. 2.

The paper is organized as follows. Section II gives some details on the MAV model and the control design is explained in section III. Section IV gives complements about the choice of adjustment parameters.

II. Model of the µDrone

The µDrone is a "gyro dyne" with six propellers. We worked on the assumption that it can be described with seven solids in interaction.

Solid S_0 is the body.

Solids S_1 and S_2 are made up of a fix pitch propeller, the motor shaft and the blades holder. The two propellers are counter-rotating and provide the lift of the MAV.

Solids S_3 and S_4 include a variable pitch propeller, the motor shaft and the variable pitch system. These propellers provide the trim stabilization.

Solids S_5 and S_6 are like the two precedents and provide MAV propulsion.

The detailed description of this mechanical system modeling can be found in Ref. 3.

Let F_i denotes the thrust produced by the *i*th actuator and M_i the associated moment. If u_i denotes the control input of the actuator, we assume that

$$\tau_i \frac{dF_i}{dt} + F_i(t) = k_i.u_i(t) \qquad (1)$$
$$M_i(t) = k_i.F_i(t)$$

With this assumption and after linearization of the mechanical model, the MIMO linear time-invariant system, denoted DRO, is an approximation of the MAV behavior[4,5]. DRO is a system with 6 inputs, 6 outputs, and state-vector $\underline{x}$ is of size 18.

$$\frac{d\underline{x}}{dt} = A.\underline{x} + B.\underline{u} \qquad (2)$$
$$\underline{y} = C.\underline{x}$$

The eigenvalues of the A matrix are so defined: 10 are zero, 6 are real negative and 2 are purely imaginary.

The rank of controllability matrix is 2 and that of observability matrix is 4.

III. Control design of the MAV

In order to build a "standard model" we introduce a perturbation vector $\underline{x}_p$ and we work on the assumption that each perturbation is constant. It is then possible to give a simple model for these perturbations

(3) $$\frac{d\,\underline{x}_p}{dt} = A_p.\underline{x}_p$$

where A_p is a square matrix of zeros. In our application $\underline{x}_p \in R^2$ and the perturbations are moments acting on roll and pitch rates. The initial system DRO is augmented and becomes

(4) $$\frac{d\,\underline{x}_e}{dt} = A_e.\underline{x}_e + B_e.\underline{u} \qquad \text{with} \qquad A_e = \begin{bmatrix} A & A_{12} \\ O & A_p \end{bmatrix},\ B_e = \begin{bmatrix} B \\ O \end{bmatrix},\ C_e = \begin{bmatrix} C & O \end{bmatrix}$$
$$\underline{y} = C_e.\underline{x}_e$$

As we want to elaborate the control law with a state-feedback, it is necessary to build a state estimator. The first step is to compute the observability staircase form of (A_e, B_e, C_e) and extract an observable subsystem

(5) $$\frac{d\,\underline{x}_o}{dt} = A_{eo}.\underline{x}_o + B_{eo}.\underline{u}$$
$$\underline{y} = C_{eo}.\underline{x}_o$$

It is then possible to build a state estimator

(6) $$\frac{d\,\hat{\underline{x}}_o}{dt} = A_{eo}.\hat{\underline{x}}_o + B_{eo}.\underline{u} + K_o.(\underline{y} - C_{eo}.\hat{\underline{x}}_o)$$

The gain matrix K_o is chosen as the solution of the LQ problem

find $\underline{v} = -KK_o.\underline{x}_o$ that minimize $J_o = \int_0^\infty \left(\underline{x}_o^T.Q_o.\underline{x}_o + \underline{v}^T.R_o.\underline{v}\right)dt$ for $\frac{d\,\underline{x}_o}{dt} = A_{eo}^T.\underline{x}_o + C_{eo}^T.\underline{v}$

and define a positive scalar k_o such as

(7) $$K_o = k_o.KK_o^T$$

At this point, the problem is to make a good choice for the weighting matrices Q_o and R_o of the quadratic criterion J_o. In order to adjust easily Q_o, we use a "partial observability gramian" defined as

(8) $$G_o(T_{oo}) = \int_0^{T_{oo}} \exp(A_{eo}^T.t).C_{eo}^T.C_{eo}.\exp(A_{eo}.t).dt$$

where $T_{oo} = k_o.T_o$ with T_o positive, called the filtering horizon and k_o the shape factor.

As the pair (A_{eo}, C_{eo}) is observable, $G_o(T_{oo})$ is asymmetric positive matrix, and it is possible to choose

(9) $$Q_o = \left[T_{oo}.G_o(T_{oo})\right]^{-1} \qquad \text{with } I \text{ the identity matrix.}$$
$$R_o = I$$

The state estimator is then

(10) $$\frac{d\,\hat{\underline{x}}_o}{dt} = (A_{eo} - K_o.C_{eo}).\hat{\underline{x}}_o + \begin{bmatrix} B_{eo} & K_o \end{bmatrix}.\begin{bmatrix} \underline{u} \\ \underline{y} \end{bmatrix}$$

Now let us compute the controllability staircase form of (A_{eo}, B_{eo}, C_{eo}) with the similarity transformation matrix T_{oc}

$$(11)\qquad \frac{d}{dt}\begin{bmatrix} \underline{x}_{nc} \\ \underline{x}_c \end{bmatrix} = \begin{bmatrix} A_{eonc} & O \\ A_{eo\,21} & A_{eoc} \end{bmatrix}.\begin{bmatrix} \underline{x}_{nc} \\ \underline{x}_c \end{bmatrix} + \begin{bmatrix} O \\ B_{eoc} \end{bmatrix}.\underline{u} \qquad \text{with} \quad \begin{bmatrix} \underline{x}_{nc} \\ \underline{x}_c \end{bmatrix} = T_{oc}.\underline{x}_o$$

$$\underline{y} = \begin{bmatrix} C_{eonc} & C_{eoc} \end{bmatrix}.\begin{bmatrix} \underline{x}_{nc} \\ \underline{x}_c \end{bmatrix}$$

As $A_{eo\,21}$ is not a matrix of zeros, uncontrollable modes $\underline{x}_{nc}$ perturb controllable modes $\underline{x}_c$. In order to reject the influence of uncontrollable modes it is possible to use a linear transformation

$$(12)\qquad \begin{aligned} \underline{u}(t) &= \underline{v}(t) - G_a.\underline{x}_{nc}(t) \\ \underline{\tilde{x}}_c(t) &= \underline{x}_c(t) + T_a.\underline{x}_{nc}(t) \end{aligned}$$

where G_a and T_a are constant matrices, leading to

$$(13)\qquad \begin{aligned} \frac{d\,\underline{\tilde{x}}_c}{dt} &= A_{eoc}.\underline{\tilde{x}}_c + \left[A_{eo\,21} - A_{eoc}.T_a + T_a.A_{eonc} - B_{eoc}.G_a\right].\underline{x}_{nc} + B_{eoc}.\underline{v} \\ \underline{y} &= C_{eoc}.\underline{\tilde{x}}_c + \left[C_{eonc} - C_{eoc}.T_a\right].\underline{x}_{nc} \end{aligned}$$

Now, it is often possible to find G_a and T_a such as

$$(14)\qquad \begin{aligned} \left[A_{eo\,21} - A_{eoc}.T_a + T_a.A_{eonc} - B_{eoc}.G_a\right] &= [O] \\ \left[C_{eonc} - C_{eoc}.T_a\right] &= [O] \end{aligned}$$

With these conditions equations (11) become

$$(15)\qquad \begin{aligned} \frac{d\,\underline{\tilde{x}}_c}{dt} &= A_{eoc}.\underline{\tilde{x}}_c + B_{eoc}.\underline{v} \\ \frac{d\,\underline{x}_{nc}}{dt} &= A_{eonc}.\underline{x}_{nc} \\ \underline{y} &= C_{eoc}.\underline{\tilde{x}}_c \end{aligned}$$

where the pair (A_{eoc}, B_{eoc}) is controllable

Once more, it is possible to compute the solution of the LQ problem

find $\underline{v} = -K_c.\underline{\tilde{x}}_c$ which minimize $J_c = \int_0^\infty \left(\underline{\tilde{x}}_c^T.Q_c.\underline{\tilde{x}}_c + \underline{v}^T.R_c.\underline{v}\right).dt$ for $\frac{d\,\underline{\tilde{x}}_c}{dt} = A_{eoc}.\underline{\tilde{x}}_c + B_{eoc}.\underline{v}$

In order to adjust weighting matrices Q_c and R_c of the quadratic criterion J_c, define a positive scalar k_c, a positive control horizon T_c such as

$$(16)\qquad T_c = T_o / k_c$$

and the "partial observability gramian"

$$(17)\qquad G_{oc}(T_c) = \int_0^{T_c} \exp(A_{eoc}^T.t).C_{eoc}^T.C_{eoc}.\exp(A_{eoc}.t).dt$$

It is then possible to choose

$$(18)\qquad \begin{aligned} R_c &= T_c.B_{eoc}^T.G_{oc}(T_c).B_{eoc} \\ Q_c &= C_{eoc}^T.C_{eoc} \end{aligned}$$

Now, it is possible to elaborate the feedback control

(20) $\underline{v} = -K_c.\underline{\tilde{x}}_c + K_r.\underline{y}_r$

where K_r is a constant matrix and $\underline{y}_r$ the reference vector.

In order to determine K_r matrix we use equations (15) in which vector $\underline{v}$ is replaced by expression (20)

(21)
$$\frac{d\,\underline{\tilde{x}}_c}{dt} = \left(A_{eoc} - B_{eoc}.K_c\right).\underline{\tilde{x}}_c + B_{eoc}.K_r.\underline{y}_r$$
$$\underline{y} = C_{eoc}.\underline{\tilde{x}}_c$$

In permanent rate we want $\underline{y} = \underline{y}_r$. As $\left(A_{eoc} - B_{eoc}.K_c\right)$ is a stability matrix, it is invertible and

(22) $\underline{y} = -C_{eoc}.\left(A_{eoc} - B_{eoc}.K_c\right)^{-1}.B_{eoc}.K_r.\underline{y}_r$

Then, the choice

(23) $K_r = -\left[C_{eoc}.(A_{eoc} - B_{eoc}.K_c)^{-1}.B_{eoc}\right]^{-1}$

leads to the solution. In our application, this last matrix is not invertible; it is the reason why we use the expression of the pseudo inverse and it appears that it is not possible to choose reference α_r except for the zero value.

Now the use of expression (12) leads to the control vector

$$\underline{u} = \underline{v} - G_a.\underline{x}_{nc} = -K_c.\underline{\tilde{x}}_c + K_r.\underline{y}_r - G_a.\underline{x}_{nc}$$

which gives

(24) $\underline{u} = -\left[K_c.T_a + G_a \quad K_c\right].\begin{bmatrix} \underline{x}_{nc} \\ \underline{x}_c \end{bmatrix} + K_r.\underline{y}_r$

but with the transformation matrix T_{oc} in (11) it is possible to express $\underline{u}$ as a feedback control

(25) $\underline{u} = -\left[K_c.T_a + G_a \quad K_c\right].T_{oc}.\underline{x}_o + K_r.\underline{y}_r$

According to separation principle and using state estimator expression (10), it follows easily the regulator equations

(26)
$$\frac{d\,\underline{x}_o}{dt} = A_{reg}.\underline{x}_o + B_{reg}.\begin{bmatrix} \underline{y}_r \\ \underline{y} \end{bmatrix}$$
$$\underline{u} = C_{reg}.\underline{x}_o + D_{reg}.\begin{bmatrix} \underline{y}_r \\ \underline{y} \end{bmatrix}$$

with

$$A_{reg} = A_{eo} - K_o.C_{eo} - B_{eo}.\left[K_c.T_a + G_a \quad K_c\right].T_{oc}$$
$$B_{reg} = \left[B_{eo}.K_r \quad K_o\right]$$
$$C_{reg} = -\left[K_c.T_a + G_a \quad K_c\right].T_{oc}$$
$$D_{reg} = \left[K_r \quad O\right]$$

IV. Choice of the adjustment parameters

Among the main objectives, the stability of state matrix estimator A_{reg} in (26) is essential. It is clear that parameters T_o, k_o and k_c will influence stability, acting on K_o and K_c matrices. The first step is to assign a fixed value to k_o and k_c parameters; often 5 is a good value to start. Now the filtering horizon T_o is adjusted

so that A_{reg} is a stability matrix; it is often effective to start with the slowest time constant value of the initial model.

In order to quantify the robustness stability of the feedback, the regulator, without reference, can be converted to a transfer function matrix. Starting with

$$(27)\qquad \frac{d\,\underline{x}_o}{dt} = A_{reg}.\underline{x}_o + K_o.\underline{y}$$
$$\underline{u} = C_{reg}.\underline{x}_o$$

we obtain

$$(28)\qquad \underline{u}(s) = -K(s).\underline{y}(s)$$

where

$$(29)\qquad \begin{aligned} K(s) &= KK_c.\left(sI - A_{eo} + B_{eo}.KK_c + K_o.C_{eo}\right)^{-1}.K_o \\ KK_c &= \left[K_c.T_a + G_a \quad K_c\right].T_{oc} \end{aligned}$$

(It is interesting to notice the symmetry of expression (29) in relation to matrix gains KK_c and K_o).

Similarly, the initial model of the MAV is

$$(30)\qquad \underline{y}(s) = DRO(s).\underline{u}(s)$$

Now define

$$(31)\qquad L_u(s) = K(s).DRO(s) \quad \text{the input loop transfer}$$

$$(32)\qquad L_y(s) = DRO(s).K(s) \quad \text{the output loop transfer}$$

$$(33)\qquad M_{st} = \min\left\{\frac{1}{\max\left|L_u(j\omega)\right|}, \frac{1}{\max\left|L_y(j\omega)\right|}\right\} \quad \text{the static margin}$$

$$(34)\qquad M_{dyn} = \min\left\{\frac{1}{\max\left|\omega.L_u(j\omega)\right|}, \frac{1}{\max\left|\omega.L_y(j\omega)\right|}\right\} \quad \text{the dynamic margin}$$

Then, when A_{reg} is a stability matrix, the stability of the global feedback is robust in relation to any relative error such as

$$(35)\qquad \left|\frac{\Delta\, DRO(j\omega)}{DRO(j\omega)}\right| < M_{st} \quad \text{it is why } M_{st} \text{ is expressed in \%}$$

Similarly, the stability of the feedback is robust in relation to any relative error such as

$$(36)\qquad \left|\frac{\Delta\, DRO(j\omega)}{DRO(j\omega)}\right| < \omega.M_{dyn}$$

Notice that: $M_{dyn} \le M_{Td}$ where M_{Td} is the time delay margin.

Now it is not difficult to compute static and dynamic margin for different values of T_o, k_o and k_c. It then may be possible to find a triplet for which the stability margins and the dynamic performances are fulfilled.

The reason why this strategy is often efficient is due to the natural robustness of a LQ problem solution and to the notion of loop transfer recovery (LTR).

In our application the following results are obtained

$M_{st} = 42.2$ %, $M_{dyn} = 32.7$ ms.

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µDrone project : <URL : http://www.ens2m.fr/drone>