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Density Modulations in an Elongated BEC Released from a Disordered Potential

D. Clément, P. Bouyer, A. Aspect, and L. Sanchez-Palencia

Laboratoire Charles Fabry de l'Institut d'Optique, CNRS and Univ. Paris-Sud,
Campus Polytechnique, RD 128, F-91127 Palaiseau cedex, France

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We observe large density modulations in time-of-flight images of elongated Bose-Einstein condensates, initially confined in a harmonic trap and in the presence of weak disorder. The development of these modulations during the time-of-flight and their dependence with the disorder are investigated. We render an account of this effect using numerical and analytical calculations. Our analysis shows how the observed large modulations originate from the weak initial density modulations induced by the disorder, and do not require phase fluctuations (thermal or quantum) in the initial condensate.

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Recently, Bose-Einstein condensates (BEC) have been created in controlled disordered potentials [1, 2, 3, 4, 5, 6], opening new opportunities to study transport phenomena (*e.g.* Anderson Localization [7]) and quantum-state diagrams in disordered mesoscopic systems [8, 9]. There exist many tools to probe ultracold gases. For instance, *time-of-flight* (TOF) techniques, which amount to image the atoms after a free-expansion stage, give information on the quantum state (density profile and momentum distribution) of the initial gas at equilibrium in the trap. In the presence of weak disorder ($V_R \lesssim \mu$), the density profile of a trapped BEC shows modulations with a relative amplitude of the order of or smaller than V_R/μ where V_R is the amplitude of the disordered potential and μ is the BEC chemical potential [10]. However, surprisingly, TOF images of disordered, elongated BECs exhibit significantly larger modulations (see Fig.1 and [1, 11]).

A possible interpretation is that disorder may enhance phase fluctuations in the trapped BEC before the TOF [1, 12]. Indeed, the observed density modulations are similar to those reported in single-shot images of non-disordered but elongated quasi-BECs after a long-enough TOF [13]. In this case, the observed density fluctuations are a signature of *initial* random phase fluctuations in the trapped quasi-BEC [13, 14, 15, 16]. However, for disordered BECs, neither systematic study nor unambiguous interpretation of these modulations has been reported so far.

In this Letter, we investigate the dynamics of density modulations during the TOF of an elongated 3D BEC initially placed in a 1D disordered potential as well as their dependence with the amplitude of the disorder. We consider a weak-enough disorder such that the fragmentation of the BEC can be neglected. Our experimental results allow us to rule out the scenario based on disorder-induced initial phase fluctuations for the considered range of parameters. In contrast, they are consistent with numerical and analytical results for the dynamical TOF process of a disordered BEC *with* initial weak density modulations but *without* any phase fluctuations. We elaborate the following scenario for the enhancement of density modulations during the TOF. In a first stage dominated by a radial expansion, the initial weak density modulations induced by the weak disorder before the TOF imprint a phase

with longitudinal modulations and transversal invariance. In a second stage, the phase modulations are converted into large density modulations.

The experiment is detailed in Refs. [2, 5]. In brief, we form a cigar-shaped BEC of ^{87}Rb atoms in a Ioffe-Pritchard trap of frequencies $\omega_z/2\pi = 6.7\text{Hz}$ and $\omega_\perp/2\pi = 660\text{Hz}$. The BEC atom number is typically $N_0 \sim 3.10^5$, the longitudinal length $2 \times L_{\text{TF}} \simeq 300\mu\text{m}$ and the chemical potential $\mu/2\pi\hbar \simeq 4.5\text{kHz}$. We create a speckle (disordered) potential along the z axis and invariant radially, whose correlation function is $C(z) = V_R^2 \text{sinc}^2(z/\sigma_R)$ [2, 5]. In the experiment, both amplitude V_R and correlation length σ_R down to $0.33\mu\text{m}$ can be controlled [2, 5]. Here, we operate at $0 < V_R < \mu$ and $\sigma_R \simeq 1.7\mu\text{m}$. We wait 300ms for the BEC to reach equilibrium in the presence of disorder. We then switch off abruptly both magnetic trap and speckle potential and we take absorption images of the expanding cloud after a variable time of flight t_{TOF} . Typical TOF images are shown in Fig. 1.

These images show large density modulations along the axis z of the disorder. To measure their amplitude, we first extract a 1D longitudinal density $n_{1D}(z)$ by integrating the column density over the second transverse direction x . We

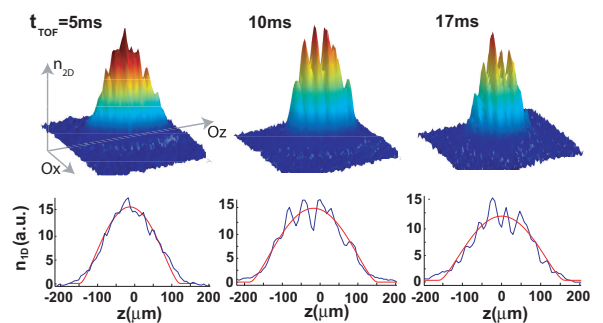


FIG. 1: Upper panel: TOF images of an expanding disordered BEC for three different times of flight. The vertical axis represents the column density along the y axis. Lower panel: longitudinal 1D density profiles $n_{1D}(z)$ (column density integrated along the x axis) (blue) and 1D TF parabolic profiles $n_{1D}^0(z)$ (red). The amplitude of the disorder is $\gamma = V_R/\mu = 0.41$.

then define $\eta(z)$ as the normalized deviations of the 1D density from the 1D parabolic Thomas-Fermi (TF) profile $n_{1D}^0(z)$ which fits best the experimental data (red line in Fig. 1), so that $n_{1D}(z) = n_{1D}^0(z)[1 + \eta(z)]$. Finally we calculate the standard deviation of $\eta(z)$ over a given length L : $\Delta\eta = \sqrt{\frac{1}{L} \int_L dz \eta^2(z)}$ (here $\int_L dz \eta(z) = 0$). The calculation of $\Delta\eta$ is restricted to 70% of the BEC total length ($L = 1.4L_{TF}$) to avoid the sides where thermal atoms are present and the TF profile does not fit the data. In the experiment, two imperfections reduce the amplitude $\Delta\eta$ of the measured density modulations compared to the real ones. First, our imaging system has a finite resolution $L_{res} = 8.5\mu\text{m}$, larger than the variation scale σ_R of the disorder. This effect is easily quantified by measuring the Modulation Transfer Function of the imaging system. Second, a slight mis-alignment of the probe beam which is not exactly perpendicular to axis z also reduces the contrast of the fringes [13]. This effect is more difficult to quantify as angles smaller than our uncertainty on the alignment of the probe (1°) can drastically reduce the contrast. Nevertheless, numerical calculations reproduce our experimental results if we admit a mis-alignment of 0.33° (see below).

We first study the amplitude of the normalized density modulations $\Delta\eta$ as a function of the amplitude of the disorder $\gamma = V_R/\mu$ at a given time-of-flight $t_{TOF} = 15.3\text{ms}$ ($\omega_\perp t_{TOF} = 62.2$) with experimental results plotted in Fig. 2a. In the absence of disorder, we observe non-vanishing density modulations ($\Delta\eta_0 \simeq 0.037$) larger than the noise in the background of the absorption images ($\Delta\eta_n \simeq 0.015$). They are interpreted as small but non-zero phase fluctuations initially present in our elongated BEC [16]. Their contribution to the density modulations in the TOF images after some expansion time ($\tau = \omega_\perp t_{TOF}$) has been calculated in Ref. [13]. Our experimental results agree with this prediction as shown in Fig. 2a (see also Fig. 3). In the presence of disorder, the experimental results show that for small values of γ (typically $\gamma < 0.2$), $\Delta\eta$ grows linearly with γ from the value obtained in the absence of disorder, $\Delta\eta = \Delta\eta_0 + 0.64(3)\gamma$. For larger values of γ , the disordered BEC is fragmented either in the trap or during the expansion, and $\Delta\eta$ saturates at $\Delta\eta \simeq 0.17$ for the experimental parameters.

We also perform numerical integrations of the 3D Gross-Pitaevskii equation (GPE) for the expanding disordered BEC and extract $\Delta\eta$ from integrated 1D density profiles as in the experiments. The numerics do not include any phase fluctuations and we find a linear dependence of $\Delta\eta$ versus γ , $\Delta\eta_{th} \simeq 3.5\gamma$ for the bare numerical results, and $\Delta\eta \simeq 1.23\gamma$ if we take into account the finite resolution of the imaging system but not the mis-alignment of the probe (see Fig. 2b). In fact, we find that the numerics are in agreement with the experimental data if, in addition to an offset $\Delta\eta_0$ to mimic the small initial phase fluctuations, we include the systematic correction associated with a probe angle of 0.33° .

We now examine the TOF dynamics of the disordered BEC and plot $\Delta\eta$ versus t_{TOF} in Fig. 3. In the presence of disorder,

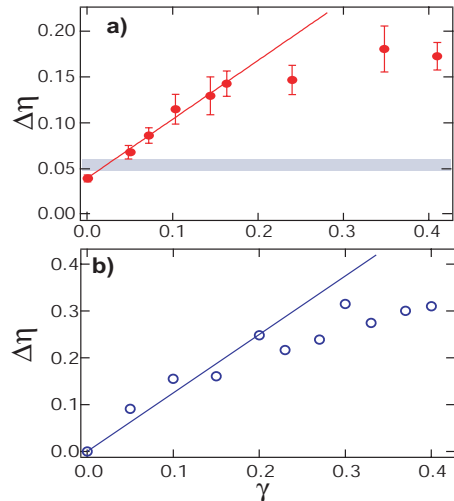


FIG. 2: **a)** Standard deviation $\Delta\eta$ of the normalized 1D density modulations at $\omega_\perp t_{TOF} = 62.2$ versus the amplitude of the disorder $\gamma = V_R/\mu$. The shaded area corresponds to phase fluctuations in our initial elongated BEC, as calculated in Ref. [13] (the error bar reflects the uncertainty on the temperature). **b)** Same as a) but obtained from numerical integrations of the 3D GPE, taking into account the finite resolution of our optics but not the misalignment of the probe beam.

der, the observed density modulations (red points) are clearly enhanced compared to those in the absence of disorder (blue points). We also observe that the density modulations first develop, and then saturate. The dynamics of the development of the density modulations (time scale and amplitude) is again reproduced well by our numerical calculations (solid red line), if we take into account all imperfections of our imaging system.

After correcting for the finite resolution of our optics and the probe angle, the density modulations in the TOF images turn out to be larger ($\Delta\eta \simeq 3.5\gamma$) than the ones expected in the trapped BEC before TOF ($\Delta\eta = 2\gamma$) [17]. One may wonder whether these large density modulations in the TOF images may reveal phase fluctuations induced in the initial BEC by the disorder [1, 12]. Actually, several arguments lead us to conclude that it is not so. First, our numerical calculations, which reproduce well the experimental data, do *not* include initial phase fluctuations. Second, numerical diagonalization of the Bogolyubov equations indicate that the disorder hardly affect the excitation spectrum of a 1D BEC with similar parameters as in the experiment [18]. Last but not least, we have observed identical density modulations in successive experiments performed with the *same* realization of the disordered potential [11, 19]. It thus excludes initial random fluctuations, quantum or thermal.

We now develop an analytical model for the evolution of the BEC density profile during the TOF, which agrees with numerical results, and shows explicitly how a weak disorder leads to large density modulations after a long enough TOF, without initial phase fluctuations. Although the probability of

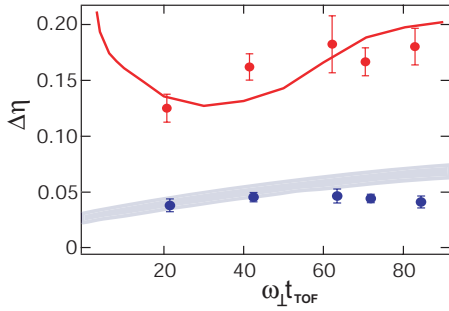


FIG. 3: Time evolution of the measured density modulations $\Delta\eta$ during a TOF for $\gamma = 0$ (no disorder; blue points) and $\gamma = 0.41$ (with disorder, red points). The solid red line is the result of numerical calculations for $\gamma = 0.4$, taking into account the finite resolution of our optics, an offset to mimic initial phase fluctuations, and the probe mis-alignment of 0.33° .

fragmentation is small for weak enough disorder, it may happen that the BEC is fragmented into a small number of fragments. However, for the considered $t_{\text{TOF}} \lesssim 1/\omega_z$, the fragments will only weakly overlap as the longitudinal expansion is small. Therefore, we neglect fragmentation in our model.

In the absence of disorder, the TOF expansion of a BEC initially trapped in a harmonic potential in the Thomas-Fermi regime is self-similar [20], so that

$$\psi(\vec{r}, t) = \left[\prod_j b_j(t) \right]^{-1/2} \phi(\{x_j/b_j(t)\}, t) e^{i\theta_0(\vec{r}, t)} \quad (1)$$

where $j = 1, 2, 3$ are the spatial directions, $\theta_0(\vec{r}, t) = (m/2\hbar) \sum_j (\dot{b}_j/b_j) x_j^2$ is the dynamical phase and ϕ is the (time-independent) wavefunction of the BEC in the trap. The expansion of the (non-disordered) BEC is thus completely determined by the scaling factors $b_j(t)$ which are governed by the equations $\dot{b}_j = \omega_j^2/(b_j \prod_k b_k)$ with the initial conditions $b_j(0) = 1$ and $\dot{b}_j(0) = 0$. In the presence of disorder, we use the scaling (1) and we write the (now time-dependent) reduced wavefunction as $\phi(\rho, z, t) = \sqrt{\tilde{n}(\rho, z, t)} e^{i\tilde{\theta}(\rho, z, t)}$ where $\rho = \sqrt{x^2 + y^2}$ is the radial coordinate. In the absence of phase fluctuations, $\phi(\rho, z, t=0)$ is real (up to a homogeneous phase) as it is the ground state of the trapped, disordered BEC. The TOF dynamics is then governed by the equations

$$\begin{aligned} \partial_t \tilde{n} &= -\frac{\hbar}{mb_\perp^2} \nabla_\rho \left(\tilde{n} \nabla_\rho \tilde{\theta} \right) - \frac{\hbar}{mb_z^2} \partial_z \left(\tilde{n} \partial_z \tilde{\theta} \right) \quad (2) \\ -\hbar \partial_t \tilde{\theta} &= \frac{V_{\text{ho}} + g\tilde{n} - \mu}{b_\perp^2 b_z} + \frac{\hbar^2}{2mb_z^2} \left(|\partial_z \tilde{\theta}|^2 - \frac{\partial_z^2 \sqrt{\tilde{n}}}{\sqrt{\tilde{n}}} \right) \\ &\quad + \frac{\hbar^2}{2mb_\perp^2} \left(|\nabla_\rho \tilde{\theta}|^2 - \frac{\Delta_\rho \sqrt{\tilde{n}}}{\sqrt{\tilde{n}}} \right) \quad (3) \end{aligned}$$

with the initial conditions $\tilde{\theta} = 0$ and, for $\xi \ll \sigma_R$, \tilde{n} equal to $[\mu - V_{\text{ho}}(\rho, z) - V(z)]/g$ if positive and zero otherwise. Here, $V_{\text{ho}}(\rho, z) = m\omega_\perp^2 \rho^2/2 + m\omega_z^2 z^2/2$ is the harmonic trap, and

$V(z)$ is the disordered potential. Note that during the TOF, both harmonic and disordered potentials are switched off. The term V_{ho} in Eq. (3) is actually reminiscent from the scaling (1) [20].

Equations (2),(3) are equivalent to the complete time-dependent GPE. Let us introduce now a couple of approximations. First, in elongated 3D BECs, the expansion for $\omega_z t \lesssim 1$ is mainly radial so that $b_z(t) \simeq 1$ [20]. Second, we assume not too large perturbation of the density in the absence of disorder \tilde{n}_0 , so that $\tilde{n} = \tilde{n}_0 + \delta\tilde{n}$ with $\delta\tilde{n} \ll \tilde{n}_0$. Using a local density approximation, we can neglect all the spatial derivatives of \tilde{n}_0 . We also neglect the radial derivatives of $\delta\tilde{n}$ since the disorder introduces short-range spatial inhomogeneities mainly along the z -axis. We are thus left with the equations

$$\begin{aligned} \partial_t \delta\tilde{n} &= -(\hbar/m) \tilde{n}_0 \partial_z^2 \tilde{\theta} \quad (4) \\ -\hbar \partial_t \tilde{\theta} &= g\delta\tilde{n}/b_\perp^2 + (\hbar^2/2m) [|\partial_z \tilde{\theta}|^2 - \partial_z^2 \delta\tilde{n}/2\tilde{n}_0]. \quad (5) \end{aligned}$$

In a first stage, the initial small inhomogeneities of the density induced by the disorder before TOF, $\delta\tilde{n} \simeq -V(z)/g$ [10], hardly evolve since $\partial_t \delta\tilde{n}(t=0) = 0$. They are however crucial as they act as an inhomogeneous potential which induces the development of a phase modulation $\tilde{\theta}(z, t)$ at the beginning of the TOF [21] and which will be at the origin of the subsequent development of large density modulations. From Eqs. (4),(5), we find

$$\begin{aligned} \tilde{\theta}(z, t) &\simeq \arctan(\omega_\perp t) [V(z)/\hbar\omega_\perp] \quad (6) \\ \text{and } \delta\tilde{n} &\simeq -V(z)/g - (\tilde{n}_0 \partial_z^2 V(z)/m\omega_\perp^2) F(\omega_\perp t) \quad (7) \end{aligned}$$

where $F(\tau) = \int_0^\tau d\tau' \arctan(\tau') = \tau \arctan(\tau) - \ln \sqrt{1 + \tau^2}$. From Eq. (7), we then find

$$\Delta\eta(t) \simeq 2 \left(\frac{V_R}{\mu} \right) \left[1 - \frac{2}{3} \left(\frac{\mu}{\hbar\omega_\perp} \right)^2 \left(\frac{\xi}{\sigma_R} \right)^2 F(\omega_\perp t) \right]. \quad (8)$$

Hence, $\Delta\eta$ first slightly decreases. It is easily understood as a BEC initially at rest will tend to fill its holes when released from the disordered potential. The solution (6),(7) is valid as long as the contribution of the last two terms in Eq. (5) remain small [*i.e.* for $\omega_\perp t \ll (\sigma_R/\xi)^2$ and $\omega_\perp t \ll (\sigma_R/\xi)(\hbar\omega_\perp/\sqrt{V_R\mu})$]. In addition, it requires that the density modulations do not vary much (*i.e.* the second rhs term in Eq. (7) is small compared to the first one). For the experimental parameters, the last condition is the most restrictive. It defines a typical time $t_0 = (1/\omega_\perp) F^{-1} [(\sigma_R/\xi)^2 (\hbar\omega_\perp/\mu)^2/2]$ during which the radial expansion imprints a phase modulation due to the initial inhomogeneities of the BEC density created by the disorder before the TOF. In particular, if $t_0 \gg 1/\omega_\perp$, the phase modulations freeze at $\tilde{\theta}(z) \simeq (\pi/2)[V(z)/\hbar\omega_\perp]$.

In a second stage, the phase modulations convert into density modulations similarly as thermal phase fluctuations do during the TOF of an elongated quasi-BEC [13]. For $t \gtrsim t_1$ where t_1 is a typical time much longer than $1/\omega_\perp$ (see below), the scaling parameter $b_\perp(t)$ becomes

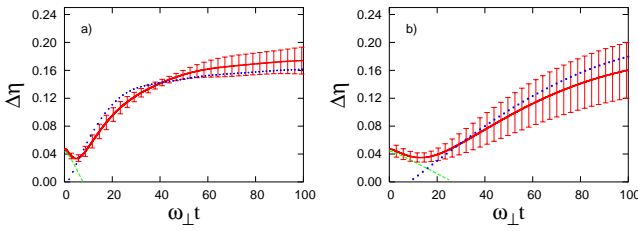


FIG. 4: Dynamics of density modulations as obtained numerically (solid red line with error bars) and comparisons with Eq. (8) (green dashed line) and Eq. (9) (blue dotted line). **a)** The parameters are the same as in the experiment (in particular, $\sigma_R = 1.7\mu\text{m}$) with $V_R = 0.02\mu$. Here $\omega_\perp t_0 \simeq 3$ and we have used $t_1 = 0.5t_0$. **b)** Same as a) but with $\sigma_R = 3.4\mu\text{m}$. Here, $\omega_\perp t_0 \simeq 8.5 \gg 1$, so that we have used $t_1 = t_0$ (see text).

large so that the first rhs term in Eq. (5) can now be neglected. Assuming small phase gradients, we are left with the equation $\partial_t^2 \delta \tilde{n}_k + \hbar^2 k^4 \delta \tilde{n}_k / 4m^2 = 0$ where $\delta \tilde{n}_k(\rho, t)$ is the 1D Fourier transform of $\delta \tilde{n}$ along z , and whose solution reads $\delta \tilde{n}_k(t) = \delta \tilde{n}_k(t_1) \cos[(\hbar k^2 / 2m)(t - t_1)] + (2m \dot{\delta \tilde{n}}_k(t_1) / \hbar k^2) \sin[(\hbar k^2 / 2m)(t - t_1)]$. If $t_0 \gg 1/\omega_\perp$, we can take $1/\omega_\perp \ll t_1 \leq t_0$ and the exact value of t_1 does not matter much (we use $t_1 = t_0$). If $t_0 \lesssim 1/\omega_\perp$, the determination of t_1 is not straightforward but can be found through fitting procedures, for instance. Then, according to Eq. (7), $\delta \tilde{n}_k(t_1) \simeq -V(z)/g$ is mainly determined by the initial density modulations of the trapped BEC while $\dot{\delta \tilde{n}}_k(t_1) \simeq -(\tilde{n}_0 \partial_z^2 V(z) / m\omega_\perp) \arctan(\omega_\perp t_1)$ results from the phase modulation created in the first stage of the TOF. For $\hbar\omega_\perp \ll \mu$ as in the experiment, the cosine term can be neglected and we find

$$\Delta\eta(t) \simeq \sqrt{8}(V_R/\hbar\omega_\perp) \arctan(\omega_\perp t_1) I[\sigma_R, t - t_1] \quad (9)$$

where $I(\sigma_R, t) = \sqrt{\int_0^1 d\kappa (1 - \kappa) \sin^2[(2\hbar t/m\sigma_R^2)\kappa^2]}$ for a speckle potential.

Numerical integrations of the complete 3D GPE confirm the expected behavior of $\Delta\eta$ during the TOF at short [Eq. (8)] and long [Eq. (9)] times as shown in Fig. 4. This validates our scenario in a quantitative manner.

Three remarks are in order. First, we find that, due to the development of phase modulations in the first stage of the TOF, the density modulations in the expanded BEC ($\Delta\eta \propto V_R/\hbar\omega_\perp$) can be larger than those in the trapped BEC ($\Delta\eta \propto V_R/\mu$) if $\mu > \hbar\omega_\perp$. Second, the density pattern is completely determined by the realization of the disorder. Third, Eq. (9) shows that the density modulations saturate at $\Delta\eta \simeq \sqrt{2}(V_R/\hbar\omega_\perp) \arctan(\omega_\perp t_1)$ for very long times t [22]. These properties are in qualitative agreement with the experimental observations.

In conclusion, we have shown that large density modulations develop during the TOF of a disordered BEC without initial phase fluctuations. This effect results from the TOF process itself which first develops a phase modulation deter-

mined by the initial disorder which is later converted into density modulations. Our scenario agrees with numerical calculations and experimental observations. A natural extension of this work would be to explore the regime of fragmented BECs where quantum fluctuations of the phases in the various fragments may reduce the contrast of the fringes in TOF images. Another interesting direction would be to explore regimes where disorder might significantly change the coherence properties of trapped (quasi-)BECs. In this case, initial phase fluctuations would superimpose to the phase modulation which develops during the first expansion stage. Revealing this possible effect in TOF images demands to take into account the effect described in this work.

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