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# First-order perturbations of the seismic response of fluid-filled stratified poro-elastic media

Louis de Barros<sup>1</sup> and Michel Dietrich<sup>2</sup>, *Laboratoire de Géophysique Interne et Tectonophysique, Université Joseph Fourier, Grenoble, France.*

## SUMMARY

We derive analytical formulas for the first-order effects produced by plane inhomogeneities on the seismic response of a stratified porous medium. The approach used for the derivation is similar to the one employed in the elastic case; it is based on a perturbation analysis of the poro-elastic wave propagation equations. The final forms of the sensitivity operators, which are often referred to as the Fréchet derivatives, are expressed in terms of the Green's functions of the solid and fluid displacements in the frequency-ray parameter domain. We compute here the Fréchet derivatives with respect to eight parameters, namely, the fluid and mineral density and bulk moduli, porosity, permeability, consolidation parameter and shear modulus.

The accuracy and stability of the derived expressions are checked by comparing differential seismograms computed from the analytical expressions of the Fréchet derivatives with solutions obtained by introducing discrete perturbations into the model properties. We find that the Fréchet derivative approach is generally accurate for perturbations of the medium properties of up to 10%, and for layer thicknesses less than one fifth of the dominant wavelength.

## INTRODUCTION

The evaluation of the sensitivity of a seismic wavefield to small perturbations of the medium properties is a classical problem in seismology (Aki and Richards, 1980; Tarantola, 1984). The so-called Fréchet derivatives play an important role in least-square inversion schemes, however, their implementation requires a fast and efficient numerical evaluation method. Tarantola (1984) in the general case and Dietrich and Kormendi (1990) in the one-dimensional case applied a perturbation analysis to the wave equations to analytically express the sensitivity operators for elastic media. The latter are classically calculated for  $P$ - and  $S$ -waves, and for density.

Since the pioneering work of Biot (1956), many authors (de la Cruz and Spanos, 1985; Johnson et al., 1994; Geerits and Kedler, 1997) have contributed to improve the poro-elastodynamic equations, either by averaging or by integrating techniques. The forward problem, i.e., the computation of synthetic seismograms in poro-elastic media has been developed and solved with several techniques (Dai et al., 1995; Carcione, 1996; Haartsen and Pride, 1997; Garambois and Dietrich, 2002). The model involves more parameters than the elastic case, but on the other hand, the wave velocities, attenuation and dispersion characteristics are computed from the medium's intrinsic properties without resorting to empirical relationships. However, the inverse problem has only been rarely addressed (Chotiros, 2002; Berryman et al., 2002) despite the fact that it can provide useful information on the material properties, especially the permeability and porosity, from the seismic waveforms and attenuation.

The aim of this study is to extend the methodology used in the elastic case (Dietrich and Kormendi, 1990) to derive the Fréchet derivatives for stratified poro-elastic media. We consider here a depth-dependent, fluid-saturated porous medium representing reservoir rocks. The 1-D forward modeling is carried out with the Generalized Reflection and Transmission Matrix Method (Kennett, 1983), already used by Garambois and Dietrich (2002) and Pride et al. (2002) for the computation of the theoretical seismograms in porous media.

We first present the governing equations for porous media before expressing the wave propagation equations for depth-dependent media.

Next, we develop the calculation of the Fréchet derivatives in the frequency-ray parameter domain, for the  $P-SV$  and  $SH$  wave cases. Finally, we check the accuracy of the operators obtained in an infinite medium and in a complex seismic model and conclude with the sensitivity of the seismic waveforms with respect to the different model parameters.

## WAVE PROPAGATION IN STRATIFIED POROUS MEDIA

### Governing equations

Assuming a  $e^{-i\omega t}$  dependence, Pride (1994, 2003) rewrote Biot's (Biot, 1956) equations of poro-elasticity in the form

$$\begin{cases} \nabla \cdot \bar{\tau} = -\omega^2 (\rho \bar{u} + \rho_f \bar{w}) \\ \bar{\tau} = [K_U \nabla \cdot \bar{u} + C \nabla \cdot \bar{w}] \bar{I} + G [\nabla \bar{u} + (\nabla \bar{u})^T - 2/3(\nabla \cdot \bar{u}) \bar{I}] \\ -P = C \nabla \cdot \bar{u} + M \nabla \cdot \bar{w} \\ -\nabla P = -\omega^2 \rho_f \bar{u} - \bar{p} \bar{w}, \end{cases} \quad (1)$$

where  $\bar{u}$  and  $\bar{w}$  respectively denote the average solid displacement and the relative fluid-to-solid displacement.  $P$  represents the interstitial pressure and  $\bar{\tau}$  is a  $3 \times 3$  stress tensor.  $\rho$  is the density of the porous medium. It is related to the fluid density  $\rho_f$ , solid density  $\rho_s$  and porosity  $\phi$  via

$$\rho = (1 - \phi)\rho_s + \phi\rho_f. \quad (2)$$

$K_U$  is the undrained bulk modulus and  $G$  is the shear modulus.  $M$  (fluid storage coefficient) and  $C$  (C-modulus) are mechanical parameters. In the quasi-static limit, at low frequencies, these parameters as well as the Lamé parameter  $\lambda_U$  defined below are real, frequency-independent and can be expressed in terms of the drained bulk modulus  $K_D$ , porosity  $\phi$ , mineral modulus of the grains  $K_s$  and fluid modulus  $K_f$ ,

$$\begin{aligned} K_U &= \frac{\phi K_D + \left[1 - (1 + \phi) \frac{K_D}{K_s}\right] K_f}{\phi(1 + \Delta)}, & \lambda_U &= K_U - \frac{2}{3}G \\ C &= \frac{\left[1 - \frac{K_D}{K_s}\right] K_f}{\phi(1 + \Delta)}, & M &= \frac{K_f}{\phi(1 + \Delta)} \end{aligned} \quad (3)$$

$$\text{with } \Delta = \frac{1 - \phi}{\phi} \frac{K_f}{K_s} \left[1 - \frac{K_D}{(1 - \phi)K_s}\right].$$

It is also possible to link the bulk properties  $K_D$  and  $G$  to the porosity and constitutive mineral properties (Pride, 2003):

$$K_D = K_s \frac{1 - \phi}{1 + c_s \phi} \quad \text{and} \quad G = G_s \frac{1 - \phi}{1 + 3c_s \phi / 2}. \quad (4)$$

The consolidation parameter  $c_s$  varies between 2 to 20 in a consolidated medium, but is much greater than 20 in an unconsolidated soil. Finally, the wave attenuation is explained by Darcy's law which uses a complex, frequency-dependent dynamic permeability (Johnson et al., 1994):

$$\bar{p} = i\omega \frac{\eta}{k(\omega)} \quad \text{with} \quad k(\omega) = k_0 / \left[ \sqrt{1 - i \frac{4}{n_f} \frac{\omega}{\omega_c}} - i \frac{\omega}{\omega_c} \right]. \quad (5)$$

The dynamic permeability  $k(\omega)$  tends toward the dc permeability  $k_0$  at low frequencies (where viscous effects are dominant) and includes

<sup>1</sup>louis.debarros@ujf-grenoble.fr

<sup>2</sup>michel.dietrich@ujf-grenoble.fr

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a correction to introduce the inertial effects at higher frequencies. The two domains are separated by the relaxation frequency  $\omega_c = \eta/\rho_f F k_0$ . The formation factor  $F = \phi^{-m}$  is expressed in terms of the porosity  $\phi$  and cementation factor  $m$ . Parameter  $n_j$  is considered constant and equal to 8 to simplify the equations. For more information on the parameters used in this study, we refer the reader to the work of Pride (2003).

### Coupled second-order equations for plane P-SV waves

In a depth-dependent poro-elastic medium and  $P-SV$  case, equations (1) reduce to the following system of second-order differential equations:

$$\left\{ \begin{array}{l} F_{1z} = \frac{\partial}{\partial z} \left[ (\lambda_U + 2G) \frac{\partial U}{\partial z} + C \frac{\partial W}{\partial z} - \omega p (\lambda_U V + CX) \right] \\ \quad - \omega p G \frac{\partial V}{\partial z} + \omega^2 [\rho U - p^2 GU + \rho_f W] \\ F_{1r} = \frac{\partial}{\partial z} \left[ G \frac{\partial V}{\partial z} + \omega p GU \right] + \omega p \left[ \lambda_U \frac{\partial U}{\partial z} + C \frac{\partial W}{\partial z} \right] \\ \quad + \omega^2 [\rho V - p^2 (\lambda_U + 2G)V - p^2 CX] \\ F_{2z} = \frac{\partial}{\partial z} \left[ C \frac{\partial U}{\partial z} - C \omega p V + M \frac{\partial W}{\partial z} - M \omega p X \right] \\ \quad + \omega^2 [\rho_f U + \tilde{\rho} W] \\ F_{2r} = \omega p \left[ C \frac{\partial U}{\partial z} + M \frac{\partial W}{\partial z} \right] + \omega^2 [\rho_f V + \tilde{\rho} X \\ \quad - p^2 (CV + MX)] \end{array} \right. \quad (6)$$

These expressions were obtained by performing a series of changes of variables which are described in detail in Kennett (1983) for the elastic case. In the above equations,  $U = U(z_R, \omega; z_S)$ ,  $V = V(z_R, \omega; z_S)$ ,  $W = W(z_R, \omega; z_S)$  and  $X = X(z_R, \omega; z_S)$  respectively denote the vertical and horizontal displacements for the solid and for the fluid. Variables  $z_R$  and  $z_S$  are the receiver and seismic source depths. Equations (6) are valid in the presence of vertical and horizontal body forces: force  $F_1$  is applied on an average volume of porous medium and force  $F_2$  is related to the pressure gradient in the fluid. We can rewrite equations (6) in the form of a matrix differential equation as

$$\bar{L}\bar{Y} = \bar{F} \quad \text{where} \quad \bar{Y} = \begin{bmatrix} U \\ V \\ W \\ X \end{bmatrix} \quad \text{and} \quad \bar{F} = \begin{bmatrix} F_{1z} \\ F_{1r} \\ F_{2z} \\ F_{2r} \end{bmatrix}. \quad (7)$$

Equations (7) admit a solution for the displacement fields in terms of the Green's functions, i.e.,

$$U(z_R, \omega) = \int_{\mathcal{M}} [G_{11zz}(z_R, \omega; z') F_{1z}(z', \omega) + G_{12zz}(z_R, \omega; z') F_{2z}(z', \omega) \\ + G_{11zr}(z_R, \omega; z') F_{1r}(z', \omega) + G_{12zr}(z_R, \omega; z') F_{2r}(z', \omega)] dz' \quad (8)$$

where  $G_{ijkl}(z_R, \omega; z_S)$  is the Green's function corresponding to the displacement at depth  $z_R$  of phase  $i$  (solid or fluid) in direction  $k$  generated by a harmonic point force  $F_{jl}(z_S, \omega)$  ( $j = 1, 2$ ) at depth  $z_S$  in direction  $l$ . A total of 32 different Green's functions are needed to express the 4 displacements  $U$ ,  $V$ ,  $W$  and  $X$ . In order to simplify this problem, we use the reciprocity theorem (eq. 9) (Pride and Haartsen, 1996; Sahay, 2001), and assume that the average force  $F_1$  acting on the porous medium and fluid force  $F_2$  are similar (Garambois and Dietrich, 2002). This simplification allows us to drop the  $j$  index of the Green's functions (eq. 10):

$$G_{ijkl}(z_R, \omega; z_S) = G_{jikl}(z_S, \omega; z_R) \quad (9)$$

$$G_{ijkl}(z_R, \omega; z_S) = G_{ikl}(z_R, \omega; z_S). \quad (10)$$

The problem then requires only 8 Green's functions. The integral of equation (8) is taken over the depths  $z'$  of a region  $\mathcal{M}$  including forces

$F_1$  and  $F_2$ . In case of a vertical point force at depth  $z_S$ , the expressions of the forces become:

$$\left\{ \begin{array}{l} F_{1z}(z_S, \omega) = \delta(z - z_S) S_1(\omega) \\ F_{1r}(z_S, \omega) = 0 \\ F_{2z}(z_S, \omega) = \delta(z - z_S) S_2(\omega) \\ F_{2r}(z_S, \omega) = 0 \end{array} \right. \quad (11)$$

where  $S_1(\omega)$  and  $S_2(\omega)$  are the Fourier transforms of the source time functions associated with forces  $F_1$  and  $F_2$ . With the hypothesis that  $F_1 \simeq F_2$ , we take  $S(\omega) = S_1(\omega) = S_2(\omega)$ . The displacements fields can then be written in simple forms with the Green's functions,

$$\left\{ \begin{array}{l} U(z_R, \omega; z_S) = G_{1zz}(z_R, \omega; z_S) S(\omega) \\ V(z_R, \omega; z_S) = G_{1zr}(z_R, \omega; z_S) S(\omega) \\ W(z_R, \omega; z_S) = G_{2zz}(z_R, \omega; z_S) S(\omega) \\ X(z_R, \omega; z_S) = G_{2zr}(z_R, \omega; z_S) S(\omega). \end{array} \right. \quad (12)$$

The displacement fields corresponding to a horizontal point force are defined in the same way.

## FRÉCHET DERIVATIVES OF THE PLANE WAVE REFLECTIVITY

### Statement of the problem

The Fréchet derivatives are usually introduced by considering the forward problem of the wave propagation, in which a set of synthetic seismograms  $d$  is computed for an earth model  $m$  using the nonlinear relationship  $d = f(m)$ . Tarantola (1984) used a Taylor expansion to express the relation between the variations of  $m$  and  $d$ ,

$$f(m + \delta m) = f(m) + D \delta m + o(\|\delta m\|^2) \quad (13)$$

where  $D = \partial f / \partial m$  is the matrix of the Fréchet derivatives.

Our aim is to calculate the various Fréchet derivatives corresponding to slight modifications of the model parameters at a given depth  $z$ . In the  $P-SV$  case, this problem reduces to finding analytical expressions for the quantities

$$\left\{ \begin{array}{l} A_1(z_R, \omega; z_S | z) = \frac{\partial U(z_R, \omega; z_S)}{\partial \rho(z)} \\ A_2(z_R, \omega; z_S | z) = \frac{\partial V(z_R, \omega; z_S)}{\partial \rho(z)} \\ A_3(z_R, \omega; z_S | z) = \frac{\partial W(z_R, \omega; z_S)}{\partial \rho(z)} \\ A_4(z_R, \omega; z_S | z) = \frac{\partial X(z_R, \omega; z_S)}{\partial \rho(z)}. \end{array} \right. \quad (14)$$

We can similarly define the Fréchet derivatives  $B_i$ ,  $C_i$ ,  $D_i$ ,  $E_i$ ,  $F_i$  and  $G_i$  for model parameters  $\rho_f$ ,  $\tilde{\rho}$ ,  $C$ ,  $M$ ,  $\lambda_U$  and  $G$ ;  $i = 1..4$  describing the four displacements considered. This is the natural choice of parameters to carry out a perturbation analysis because of the linear dependence of these parameters with the wave equations. We also introduce the set of Fréchet derivatives  $A'_i$ ,  $B'_i$ ,  $C'_i$ ,  $D'_i$ ,  $E'_i$ ,  $F'_i$ ,  $G'_i$  and  $H'_i$  corresponding to model parameters  $\rho_s$ ,  $\rho_f$ ,  $k_0$ ,  $\phi$ ,  $K_s$ ,  $K_f$ ,  $G_s$  and  $c_s$  that we will use in a second stage. The sensitivity operators are derived by following the procedure presented in Dietrich and Kormendi (1990).

### Perturbation analysis

We consider small changes in the model parameters at depth  $z$  that result in perturbations  $\delta U$ ,  $\delta V$ ,  $\delta W$  and  $\delta X$  of the seismic waves. We write the perturbed displacements fields  $U'$ ,  $V'$ ,  $W'$  and  $X'$  in matrix form as  $\bar{Y}' = \bar{Y} + \Delta \bar{Y}$  where, for instance,

$$U'(z_R, \omega; z_S) = U(z_R, \omega; z_S) + \delta U(z_R, \omega; z_S) \quad (15)$$

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with

$$\begin{aligned} \delta U(z_R, \omega; z_S) = & \int_{\mathcal{M}} [A_1(z_R, \omega; z_S|z) \delta \rho(z) + B_1(z_R, \omega; z_S|z) \delta \rho_f(z) \\ & + C_1(z_R, \omega; z_S|z) \delta \bar{\rho}(z) + D_1(z_R, \omega; z_S|z) \delta C(z) \\ & + E_1(z_R, \omega; z_S|z) \delta M(z) + F_1(z_R, \omega; z_S|z) \delta \lambda_U(z) \\ & + G_1(z_R, \omega; z_S|z) \delta G(z)] dz. \end{aligned} \quad (16)$$

With the model parameterization used, the wave operator  $\bar{L}$  in the perturbed medium can be written as  $\bar{L} = \bar{L} + \Delta \bar{L}$ , so that equations (7) become (Hudson and Heritage, 1981):

$$(\bar{L} + \Delta \bar{L})(\bar{Y} + \Delta \bar{Y}) = \bar{F}. \quad (17)$$

We use the single scattering (Born) approximation to solve the above equation under the assumption that the scattered wavefields  $\Delta \bar{Y}$  are weak compared to the incident wave fields  $\bar{Y}$ . We then obtain

$$\bar{L} \Delta \bar{Y} \simeq -\Delta \bar{L} \bar{Y} \equiv \Delta \bar{F}. \quad (18)$$

This matrix equation has a solution similar to equations (8) by substituting  $\delta F_{ij}$  for  $F_{ij}$  and  $\delta U$  for  $U$ . The secondary Born sources  $\Delta \bar{F}$  are obtained from equations (6). As the expressions of  $\Delta \bar{F}$  depend on the perturbations of the model parameters, we rearrange the modified equations (8) in order to separate their contributions. This is done via integrations by parts of the integrals. Finally, we identify the Fréchet derivatives by comparing term by term these new expressions with equations (16). For the vertical displacement  $U$  of the solid, we find

$$\begin{aligned} A_1 &= -\omega^2 (U G_{1zz} + V G_{1zr}) \\ B_1 &= -\omega^2 (U G_{1zz} + W G_{1zz} + V G_{1zr}) \\ C_1 &= -\omega^2 (W G_{1zz} + X G_{1zr}) \\ D_1 &= \left[ \frac{\partial W}{\partial z} - \omega p X + \frac{\partial U}{\partial z} - \omega p V \right] \left[ \frac{\partial G_{1zz}}{\partial z} - \omega p G_{1zr} \right] \\ E_1 &= \left[ \frac{\partial W}{\partial z} - \omega p X \right] \left[ \frac{\partial G_{1zz}}{\partial z} - \omega p G_{1zr} \right] \\ F_1 &= \left[ \frac{\partial U}{\partial z} - \omega p V \right] \left[ \frac{\partial G_{1zz}}{\partial z} - \omega p G_{1zr} \right] \\ G_1 &= \left[ \frac{\partial V}{\partial z} + \omega p U \right] \left[ \frac{\partial G_{1zr}}{\partial z} + \omega p G_{1zz} \right] \\ &+ 2 \frac{\partial U}{\partial z} \frac{\partial G_{1zz}}{\partial z} + 2 \omega^2 p^2 V G_{1zr} \end{aligned} \quad (19)$$

where  $U = U(z_R, \omega; z)$ ,  $V = V(z_R, \omega; z)$ ,  $W = W(z_R, \omega; z)$  and  $X = X(z_R, \omega; z)$  are the wavefields incident on the model perturbations, and  $G_{ijk} = G_{ijk}(z_R, \omega; z)$  are the Green's functions propagating the scattered wavefields back from the inhomogeneities to the receivers.

The Fréchet derivatives for the horizontal displacement  $V$  are easily deduced from the expressions (19) by changing  $G_{1zz}$  to  $G_{1zr}$ ,  $G_{2zz}$  to  $G_{2zr}$ ,  $G_{1rz}$  to  $G_{1rr}$  and  $G_{2zr}$  to  $G_{2rr}$ . In the same way, the Fréchet derivatives of the vertical and horizontal fluid displacements are obtained by changing  $G_{1ij}$  into  $G_{2ij}$ . It can be verified that the expressions corresponding to perturbations of parameters  $\lambda_U$ ,  $\rho$  and  $G$  are in agreement with the Fréchet derivatives calculated in the elastic case (Dietrich and Kormendi, 1990).

### Fréchet derivatives for relevant model parameters

To develop an inversion procedure for porous media, we should ideally concentrate on model parameters that are easily measurable and independent from each other. We consider here the second set of 8 parameters  $\rho_s, \rho_f, k_0, \phi, K_f, K_s, G_s$  and  $c_s$  which is more naturally related to the solid and fluid phases. To obtain the corresponding Fréchet derivatives, we use a  $8 \times 7$  Jacobian matrix  $J$  calculated from equations (2) to (5) and defined by

$$J_{ij} = \frac{\partial p_i}{\partial p'_j} \quad (20)$$

where  $p_i$  is one of the parameters  $\rho, \rho_f, \bar{\rho}, C, M, \lambda_U$  and  $G$ , and  $p'_j$  is one of the parameters  $\rho_s, \rho_f, k_0, \phi, K_s, K_f, G_s$  and  $c_s$ . The Fréchet derivatives can be further simplified by considering that the source and the receivers are located at the same depth  $z_0 = z_R = z_S$ . It should be noted that the Fréchet derivatives with respect to the permeability and fluid density are complex due to their role in the wave attenuation.

### SH case

We follow the same procedure to calculate the Fréchet derivatives for the *SH*-wave displacements and the transverse fluid displacements. Derivatives with respect to  $K_s$  and  $K_f$  are equal to zero and the other expressions are simpler than in the *P-SV* case.

## NUMERICAL SIMULATIONS AND SENSITIVITY OF THE PARAMETERS

### Computation of Green's functions

As mentioned in the introduction, the Green's function for layered porous media are computed with the Generalized Reflection and Transmission Matrix Method of Kennett (1983) which yields the plane-wave response in the frequency–ray parameter (or horizontal wavenumber) domain. The seismograms in the time–distance domain are finally computed with the discrete wavenumber integration method (Bouchon, 1981). In order to test our analytical formulation and assess its limitations, we compare the differential seismograms computed with the Fréchet derivative approach with those obtained with a discrete perturbation of the medium properties. The similarity between the seismograms is evaluated by computing correlation coefficients.

### Uniform medium

We first consider the simple case of a uniform infinite medium and small perturbations  $\delta \rho_s, \delta \rho_f, \delta k_0, \delta \phi, \delta K_s, \delta K_f, \delta G_s$  and  $\delta c_s$  at depth  $z = 50$  m. Source and receivers are located at the same depth  $z_0 = 0$  m, and therefore, we observe only reflected waves.

Our simulations include  $P_{fast}$ ,  $P_{slow}$  and *S*-waves whose velocities

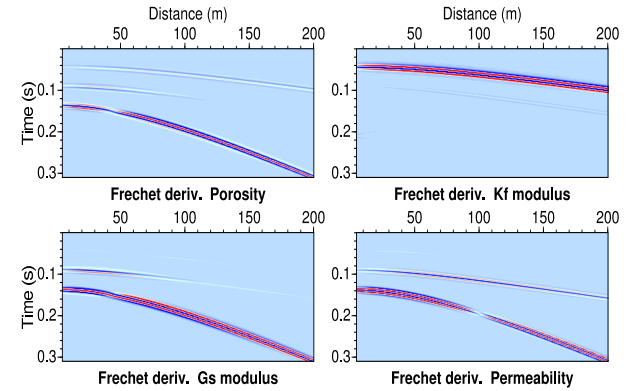


Figure 1: Fréchet derivatives with respect to porosity  $\phi$ , fluid modulus  $K_f$ , shear modulus  $G_s$  and permeability  $k_0$ .

are respectively equal to 2250, 130 and 750 m/s at a frequency of 43 Hz. Three reflected waves (compressional *PP*, converted *PS* and *SP*, and shear *SS*) are easily identified. Small contrasts in  $K_s$  and  $K_f$  have no influence on shear waves, whereas changes in the other parameters mainly generate *SS* reflections, as seen in figure 1.

With this simple model, all correlation coefficients are greater than 99%, except for the permeability in the *P-SV* case. However, this derivative is more stable in the *SH* case. The analytical expressions of the Fréchet derivatives are thus validated.

We further check the accuracy and stability of the first-order sensitivity operators by modifying the amplitude of the perturbations, with the

## Fréchet derivatives for poro-elastic media

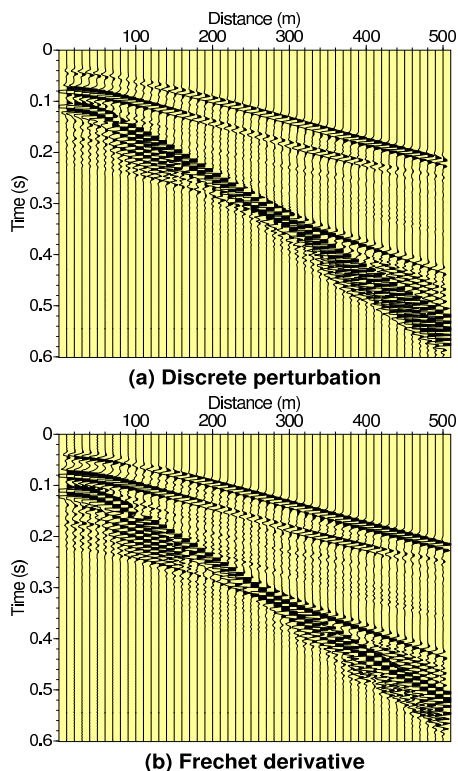


Figure 2: Comparison of the seismic sections obtained with the Fréchet derivatives and with the discrete perturbation approaches for a perturbation in solid density  $\rho_s$  at  $z = 50$  m in a 16-layer model.

following results: 1) perturbations of consolidation parameter  $c_s$  and porosity  $\phi$  show very similar wave patterns; 2) the same is true for bulk moduli  $K_s$  and  $K_f$ ; 3) Fréchet derivatives of the parameters that only influence  $P$ -waves are more stable than those that influence both  $P$ - and  $S$ -waves; 4) seismic waveforms are not exactly similar for positive and negative perturbations; 5) the first-order approximations are usually stable for parameter perturbations of up to 15%.

In this model, the wavelengths of  $P$ - and  $S$ -waves are respectively equal to 26 m and 9 m at the dominant frequency. The Fréchet derivatives are stable until the thickness of the perturbed layer reaches 20% of the dominant wavelength, which corresponds here to thicknesses of 5 m and 2 m for  $P$ - and  $S$ -waves. Derivatives with respect to  $K_s$  and  $K_f$  remain accurate for a larger range of layer thicknesses than the derivatives with respect to other parameters which depend more strongly on  $S$ -waves.

We observe in all cases that the accuracy of the Fréchet derivatives decreases when the source-receiver offset increases (i.e., when the angle of incidence increases).

### Complex model

Finally, we numerically check the stability and accuracy of the Fréchet derivative formulas in a complex model. Figure 2 is calculated for a 16-layer model and a perturbation in solid density. It is seen that the waveforms obtained with the two computation methods are very similar.

## CONCLUSION

We derived analytical expressions of the Fréchet derivatives in a depth-dependent porous medium in the frequency  $\omega$  and ray parameter  $p$  domain. In the  $P-SV$  case, we obtained 64 expressions for 2 different forces, 4 displacements and 8 parameters, while in the  $SH$  case, we obtained 12 sensitivity operators. We checked the accuracy of these expressions with a purely numerical computation method and found good results as long as the Born approximation criteria are satisfied, that is, for weak and localized perturbations of the model parameters. These operators will be especially useful in full waveform inversion algorithms implemented with gradients techniques.

## REFERENCES

- Aki, K. and P. Richards, 1980, Quantitative seismology: W. H. Freeman and Co., New York.
- Berryman, J. G., P. Berge, and B. Bonner, 2002, Estimating rock porosity and fluid saturation using only seismic velocities: Geophysics.
- Biot, M. A., 1956, Theory of propagation of elastic waves in a fluid-saturated porous solid. i. low-frequency range, ii. higher frequency range: J. of the Acoustical Society of America.
- Bouchon, M., 1981, A simple method to calculate green's functions for elastic layered media: Bull. Seismo. Soc. Am.
- Carcione, J. M., 1996, Wave propagation in anisotropic, saturated porous media: Plane-wave theory and numerical simulation.: J. of the Acoustical Society of America.
- Chotiros, N., 2002, An inversion for biot parameters in a water-saturated sand: J. of the Acoustical Society of America.
- Dai, N., A. Vafidis, and E. Kanasevich, 1995, Wave propagation in heterogeneous porous media: A velocity-stress, finite-difference method: Geophysics.
- de la Cruz, V. and T. Spanos, 1985, Seismic wave propagation in a porous medium: Geophysics.
- Dietrich, M. and F. Kormendi, 1990, Perturbation of the plane-wave reflectivity of a depth-dependent elastic medium by weak inhomogeneities: Geophy. J. Int.
- Garambois, S. and M. Dietrich, 2002, Full waveform numerical simulations of seismoelectromagnetic wave conversions in fluid-saturated stratified porous media: J. Geophys. Res.
- Geerits, T. and O. Kedler, 1997, Acoustic wave propagation through porous media: theory and experiments: J. of the Acoustical Society of America.
- Haartsen, M. and S. Pride, 1997, Electrostatic waves from point sources in layered media: J. Geophys. Res.
- Hudson, J. and J. Heritage, 1981, The use of the born approximation in seismic scattering problems: Geophys. J. R. Astr. Soc.
- Johnson, D. L., T. Plona, and H. Kojima, 1994, Probing porous media with first and second sound. i. dynamic permeability: Journal of Applied Physics.
- Kennett, B., 1983, Seismic wave propagation in stratified media: Cambridge University Press.
- Pride, S. R., 1994, Governing equations for the coupled electromagnetics and acoustics of porous media: Physical Review B.
- 2003, Hydrogeophysics, chapter 8, Relationships between seismic and hydrological properties. Kluwer.
- Pride, S. R. and M. Haartsen, 1996, Electrostatic wave properties: J. of the Acoustical Society of America.
- Pride, S. R., E. Tromeur, and J. G. Berryman, 2002, Biot slow-wave effects in stratified rock.: Geophysics.
- Sahay, P., 2001, Dynamic green's function for homogeneous and isotropic porous media: Geophy. J. Int.
- Tarantola, A., 1984, Inversion of seismic reflection data in the acoustic approximation: Geophysics.