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Experimental observation of the Anderson transition with atomic matter waves

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Phase transitions are major phenomena in physics, where a small change in a parameter may induce a dramatic change in the physical properties of a system. Of particular interest in modern physics is the metal-insulator Anderson transition¹ in disordered systems, ultimately due to the interplay between quantum interference and disorder. Here we present the first experimental observation of the Anderson transition using atomic matter waves. The Anderson model describes the effect of random impurities on the quantum properties of a particle in a crystal lattice. The model predicts the existence of a quantum phase transition between an insulator phase, where the wavefunction is exponentially localized at long times, and a metallic phase, where the wavefunction is delocalized and displays diffusive transport. The metal-insulator transition is a fundamental quantum process which plays a central role in the study of quantum disordered systems, and has progressively been extended from its original solid-state physics scope¹⁻⁴ to many different fields, including dynamical systems^{5,6}, electromagnetic

* <http://www.phlam.univ-lille1.fr/atfr/cq>

radiation⁷⁻¹⁰, acoustics^{11,12}, biology^{13,14} and even cosmology¹⁵. Many studies have pointed out strong analogies between cold atoms trapped in optical lattices and condensed matter physics¹⁶. Using an atom-optics system formally equivalent to the Anderson model in three dimensions¹⁷⁻¹⁹ we unambiguously demonstrate the Anderson transition. Sensitive measurements of the atomic wavefunction dynamics and the use of finite-size scaling techniques make it possible to extract both the critical disorder strength and the critical exponent of the transition. In doing so, we make full use of the physical concepts underlying phase transition phenomena such as universality classes (i.e., the insensitivity of the critical phenomena to the microscopic properties of the system) and renormalization (which describes the scaling laws in the vicinity of the transition).

The Anderson model¹⁻³ starts from the “tight-binding” description of an electron in a lattice, which is essentially composed of two terms: a “diagonal”, on-site, term that describes the energy associated with the site, and a “hopping” term that describes the probability amplitude for the electron to jump to a neighbouring site. Anderson postulated in 1958 that the dominant effect of impurities in the lattice is to randomize the diagonal term, and showed that this generally leads to a localization of the wavefunction, in sharp contrast with the Bloch-wave solution valid for a perfect crystal. This model and its extensions can in fact describe a variety of systems where a wave propagates in a disordered medium (not only a wavefunction but also light^{7,8} or acoustic^{11,12} waves). The large popularity that the Anderson model has enjoyed for five decades is also due to its mathematical simplicity, allowing analytical approaches; yet, it predicts a wealth of interesting phenomena. One of its most interesting predictions^{1,2} is that the wavefunction is always localized in 1D; in 2D it is also localized but with a localization length that increases exponentially with the disorder strength; in 3D it predicts that there should be a phase transition at a well defined mobility edge between

localized and delocalized phases, with the density of impurities or the energy being the control parameter.

Despite the wide interest on the Anderson transition, few experimental results are available. In a crystal, it is very difficult to obtain the conditions for a clean observation of the Anderson localization. Firstly, one has no direct access to the electronic wavefunction and must rely on modifications of bulk properties like conductivity. Secondly, it is hard to reduce decoherence sources to a low enough level to allow quantum effects to be clearly observed. Finally, it is not easy to obtain a high directional confinement so as to produce the equivalent of a 1D or 2D system. It is thus interesting to find other systems that display the Anderson transition, but are more favourable for experimental studies. Anderson localization of photons propagating in a disordered medium has been observed in the microwave regime^{11,12} and recently in the visible range^{7,8}. Theoretical propositions have also been made recently to observe it with ultracold-atomic matter waves^{20,21}. In the present work, we use another system having these properties, the atomic *kicked rotor*^{5,6,17-19}.

We engineered a matter-wave system described by an Anderson-like model. It allows us to probe the physics of disordered systems in much better controlled conditions than what is possible in condensed matter physics, namely: (almost) no interaction between the particles exposed to the disordered potential, no absorption in the medium, no coupling with a thermal reservoir which could destroy localization, possibility of preparing an initially localized state and possibility of measuring the final state of the system after a chosen interaction time. Our system consists in a one-dimensional particle exposed to pulses (equally separated in time) of a spatially sinusoidal potential whose amplitude is quasi-periodically modulated at frequencies f_2 and f_3 . The corresponding Hamiltonian reads:

$$H = \frac{p^2}{2} + K \cos x [1 + \varepsilon \cos(2\pi f_2 t) \cos(2\pi f_3 t)] \sum_{n=0}^{N-1} \delta(t - n) \quad (1)$$

where x and p are the particle position and momentum, K is the pulse intensity, and $\delta(t)$ is the Dirac delta function. We have chosen normalized variables such that x is measured in units of the spatial period of the potential divided by 2π , the particle's mass is unity and time is measured in units of pulse period T_1 .

By taking $\varepsilon = 0$ we obtain the standard kicked rotor (strictly time-periodic) which has been widely used in theoretical and experimental studies of both classical and quantum chaos. This system is known to display a phenomenon called *dynamical localization*⁵, which manifests itself by an exponential localization of the wavefunction in *momentum space*. Dynamical localization has been shown to be a direct analogue of Anderson localization in one dimension, with the following correspondences: Localization takes place in real space for the Anderson model and in momentum space for the kicked rotor, the so-called “stochasticity parameter” K [see eq. (1)] is the equivalent of the diagonal disorder in the Anderson model. K will thus be the control parameter for the observation of the Anderson transition in our system, the mobility edge corresponding to a critical value K_c . It should be noted that dynamical localization exists only when the classical dynamics of the kicked rotor eq. (1) is a chaotic diffusion (inhibited in the quantum case by destructive interference). The experimental observation of dynamical localization in the kicked rotor¹⁷ actually constitutes the first observation of Anderson-like 1D localization with matter waves.

As the Anderson transition between localized and delocalized states exists only in three (or more) dimensions, one must generalize the kicked rotor to obtain a system analogous to the 3D Anderson model. This can be done by introducing further temporal dependencies^{18,19} making the Hamiltonian quasi-periodic in time, i.e. by taking $\varepsilon \neq 0$ in (1) and choosing f_2 and f_3 as incommensurate irrational numbers. One can then use a

generalization of the transformation introduced by Grempel *et al.*⁶ to show that the resulting system is substantially equivalent to the 3D Anderson model¹⁸, the new spatial coordinates in the Anderson model corresponding to the new time dependences in the quasi-periodic kicked rotor. Generally, to introduce additional incommensurable frequencies in the kicked rotor is equivalent to increase the dimensionality of the corresponding Anderson model.

Our atom-optics realization of the kicked rotor has been described in detail elsewhere²². Basically, we cool caesium atoms in a standard magneto-optical trap. After a Sisyphus-molasses phase, we obtain a cloud of 10^7 atoms at a temperature of $3.2 \mu\text{K}$. The laser-cooling setup thus prepares a sample of atoms in a thermal state whose momentum distribution is much narrower than the expected localization length. These atoms then interact with the potential generated by a horizontal standing wave. The laser beams forming the standing wave pass through an acousto-optical modulator driven by an arbitrary-form synthesizer, which allows us to modulate the optical potential at will. One generates in this way short pulses of duration $0.8 \mu\text{s}$ at $T_1 = 27.778 \mu\text{s}$, to which is superimposed a modulation of the form (1), with $\varepsilon = 0.3$, $f_2 = 2.236$ and $f_3 = 3.606$. The standing wave, of typical power 160 mW, is far off-resonant (17.3 GHz to red of the atomic transition, corresponding to 3.2×10^3 natural widths) in order to reduce spontaneous emission, which is a source of decoherence in the system. Our setup also allows sensitive measurements of the atomic momentum distribution by means of stimulated Raman transitions²³. The momentum exchange between the atom and the laser radiation is aligned along the axis of the standing wave, insuring that the spatial dynamics is essentially 1D. Atom-atom interactions are negligible on the time scale of the experiment, which may thus be considered as the superposition of 10^7 independent one-particle experiments.

In order to observe the Anderson transition we apply a sequence of kicks to the atomic cloud and measure its dynamics. In the localized regime, the evolution of its momentum distribution is “frozen” after some characteristic time (~ 12 kicks) into an exponential curve $\exp(-|p|/p_{loc})$ (where p_{loc} is the *localization length*). In the diffusive regime, the initial Gaussian shape is preserved and the distribution gets broader as kicks are applied, corresponding to a linear increase of the average kinetic energy. Observing the Anderson transition would thus mean to obtain narrow, exponentially localized distributions for $K < K_c$ and increasingly broad, Gaussian-shaped distributions for $K > K_c$. In practice, however, it is much easier to measure the population $\Pi_0(t)$ of the zero velocity class. By virtue of the constancy of the total number of atoms, $\Pi_0^{-2}(t)$ is proportional to p_{loc}^2 in the localized regime and to the average kinetic energy in the diffusive regime. If the dynamics is diffusive then $\langle p^2 \rangle \propto t$, which means, by a simple area-conservation argument, that $\Pi_0(t) \propto t^{-1/2}$. In the localized regime, $\Pi_0(t)$ tends asymptotically to a constant value p_{loc}^{-1} . At the critical point (see below), we expect $\Pi_0(t) \propto t^{-1/3}$. So, if we multiply the $\Pi_0(t)$ curves by $t^{1/3}$ we obtain curves of positive slope in the localized case and of negative slope in the diffusive (delocalized) regime. This provides us a simple way to clearly distinguish the two regimes, the zero slope curve being the critical curve corresponding to the phase transition.

We performed 44 experimental runs corresponding to different values of K , below and above the transition. In each run we recorded the value of $\Pi_0(t)$ as the kicks were applied. We also recorded the background signal obtained by not applying the Raman detection sequence, and the total number of atoms in the cold-atom cloud. These signals were used to correct the experimental data from background signals and from long-term drifts of the cloud population. Fig. 1 displays the experimental data $\Pi_0(t) \times t^{1/3}$ as a function of time, in the various regimes. It clearly shows the changing of behaviour from localized (positive slope) to diffusion (negative slope) around $K \approx 3.8$. However, this is not enough to prove that we observe a true phase transition, characterized by

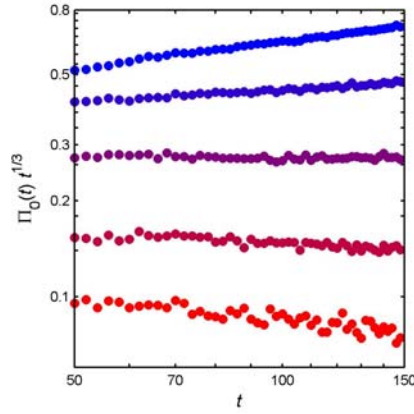


Fig. 1: Direct experimental observation of the Anderson transition. We plot the experimentally measured quantity $\Pi_0(t) t^{1/3}$ vs. time t in a log-log scale. The values of the stochasticity parameter K are (from top to bottom) 0.9, 2.1, 3.8, 6.3 and 9.0, and $\varepsilon = 0.3$. The inversion of the slope from positive (localization, blue) to negative (diffusion, red) is clearly visible around $K \approx 3.8$.

singular behaviours. In order to do so, we will show that the data follow the scaling laws predicted by renormalization theory applied to the 3D Anderson model. This will permit us to extract the critical singularities, thus to measure the critical disorder and the critical exponents.

When one approaches the critical point, the localization length diverges on the insulator side while the diffusion constant vanishes on the metallic side. However, a strict divergence can be observed only in macroscopically large samples; in small samples the divergence is smoothed. This fact plagued the numerical studies of the solid-state Anderson transition, as only a finite (small) lattice can be dealt with in a computer, and no divergence is observed. In our system, this singular behaviour would show up only for prohibitively large numbers of kicks. Indeed, during our experiment, the atoms are falling under gravity action, and eventually go out of the detection region and are lost. In practice, this limits the maximum number of kicks to around 150. To overcome this limitation, a technique named “finite-size scaling”²⁴⁻²⁹ was introduced.

The basic idea is to infer the scaling law allowing proper extrapolation of the measured localization length to an infinite sample.

We have adapted the standard finite-size scaling approach used in numerical studies of the Anderson transition³ assuming that, for finite interaction time, the quantity $\Pi_0^{-2}(t)t^{-2/3} \equiv \Lambda(t)$ is an arbitrary function $f(\zeta t^{-1/3})$ (see methods section), with ζ the scaling parameter which depends *only* on K . By gathering the results obtained for various values of t and K , one can reconstruct both the function f and the scaling parameter $\zeta(K)$. It should be emphasized that no assumption about the form of f has been made. The result is shown in Fig. 2 for numerical simulations of the system and in Fig. 3 for the experimental results, which contain the most important results of this work. In both cases, the scaling hypothesis is justified by the fact that all points in Fig. 2a and 3a lie (within reasonably small errors) on a single curve. The remarkable feature is the existence of two branches: the upper one (asymptotic slope -1 in the log-log plot) corresponds to diffusive motion while the lower one (asymptotic slope +2 in the log-log plot) corresponds to the localized regime, while the critical point is at the right tip joining the diffusive and localized branches. The scaling parameter $\zeta(K)$ is plotted in Figs. 2b and 3b: it represents the localization length in the localized regime and scales as the inverse of the diffusion constant in the diffusive regime. Clearly, it increases rapidly in the vicinity of the critical value K_c , on both sides of the transition. Although no hypothesis has been made about its behaviour near the critical point, it is found to behave as $\zeta \sim |K - K_c|^{-\nu}$ when $K \rightarrow K_c$ for the numerical results in Fig. 2. The value of the critical exponent $\nu = 1.6 \pm 0.2$ can be directly extracted from Fig. 2b; the numerical uncertainties in the fitting procedure are the main source of error. Once the existence of the scaling law is established from Figs. 2 and 3, it is more convenient to use a *global* analysis of the numerical data at various values of K (see ref.^{25,26}). We then obtain the more precise value for the critical exponent $\nu = 1.58 \pm 0.10$.

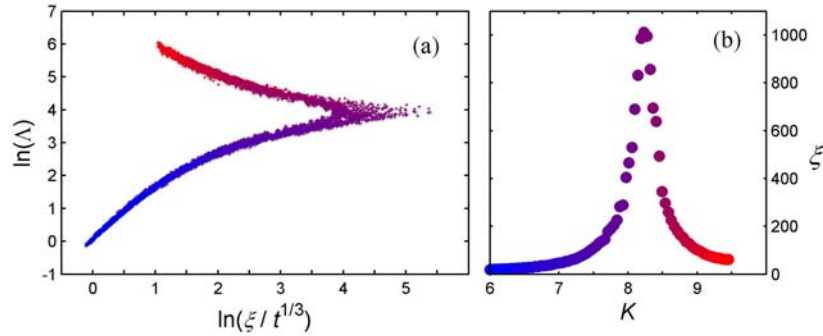


Fig. 2: Finite size scaling applied to the results of numerical simulations of the quasi-periodic kicked rotor ($\varepsilon = 0.3$). We compute the evolution of the zero-velocity class population $\Pi_0(t)$ as a function of time (from 10^2 to 10^4 kicks), for various values of K . The finite-size scaling makes it possible to determine both the scaling function f , shown in (a), with its upper branch (red), associated with the diffusive regime, and its lower branch (blue), associated with the localized regime. The dependence of the scaling parameter ξ on K is shown in (b). It displays a divergent behaviour around the critical point $K_c = 8.23$, which is a signature of the Anderson phase transition. The critical exponent is $\nu = 1.58 \pm 0.10$, in good agreement with numerical experiments on the Anderson model.

In order to perform a similar analysis of the experimental data, we must consider decoherence processes, which make the strict localization impossible to maintain for arbitrarily long times. In our system, spontaneous emission is such a phase-breaking mechanism. The process where an atom absorbs a photon from the standing wave and performs a spontaneous emission is not included in the Hamiltonian eq. (1). Although this process is rare, it destroys the phase coherence of the atomic matter wave, and consequently breaks Anderson localization. Other sources of decoherence are most likely present in our experiment, such as acoustical vibrations and laser-intensity or frequency fluctuations, which would also break the quantum localization effect and contribute to make the transition less sharp than expected. This is a rather common situation also in solid state physics, where phase-breaking processes are due e.g. to

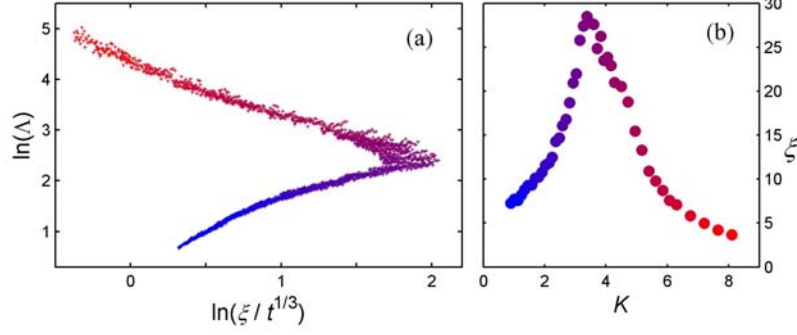


Fig. 3: Finite size scaling applied to the experimental results (with $\varepsilon = 0.3$ and 50 to 150 kicks). The scaling procedure is identical to the one used in Fig. 2. The fact that all experimental points lie on a single curve in (a), with a diffusive (red) and a localized (blue) branch, is a proof of the relevance of the one-parameter scaling hypothesis. The maximum displayed by the scaling parameter ξ in (b) in the vicinity of $K_c = 3.83$ is a clear-cut proof of the Anderson transition. Phase-breaking mechanisms (cf. text) smooth the divergence at the critical point. When these effects are properly taken into account, one obtains a critical exponent $\nu = 1.55 \pm 0.20$, compatible with the numerical result.

phonons at finite temperature or electron-electron interactions at zero temperature. We performed numerical experiments taking decoherence into account. The resulting scaling parameter $\xi(K)$ is plotted in Fig. 4 for various values of the decoherence rate η . Not surprisingly, the sharp divergence at zero rate is smoothed when η increases and the critical value K_c is shifted. We checked that there is a good agreement (in the perturbative regime $\eta \rightarrow 0$) between the numerical data and the experimental measurements. More precisely, the experimental data are well fitted by the formula:

$$\frac{1}{\xi(K)} = \alpha |K - K_c(\eta)|^\nu + \beta, \quad (2)$$

where α and β are the fitting parameters. This makes it possible to extract from the experimental data a value of the critical exponent $\nu = 1.55 \pm 0.20$, in good agreement with the critical exponent obtained above from numerical simulations in the absence of phase-breaking mechanisms.

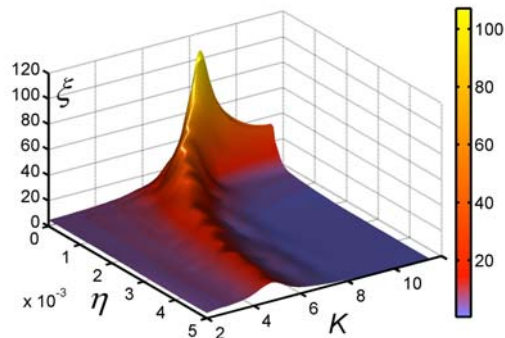


Fig. 4: The scaling parameter ξ – extracted from finite-size scaling analysis of numerical experiments in the presence of decoherence – as a function of the kick strength K and the decoherence rate η . One sees that decoherence shifts and smoothes the critical behaviour, but the transition is still clearly visible.

In conclusion, we have presented the first unambiguous experimental evidence of the Anderson transition in 3D with atomic matter waves, characterized by a well defined critical point, a divergence of the localization length below the critical point (in the localized regime) and a vanishing of the diffusion constant above the critical point (in the diffusive regime). We have determined the scaling laws and the critical exponent $\nu \approx 1.6$ of the Anderson transition. This exponent is significantly larger than unity and very close to the one observed in recent numerical experiments^{25,26} on the “pure” Anderson model, enforcing the assumption^{18,19} that the two systems are substantially equivalent. Whether this exponent is universal (i.e. independent of the microscopic details) or not remains to be studied. A very interesting point is that our Anderson-equivalent system can be easily generalized to higher dimension, which opens perspectives for fascinating studies of the dependence of the critical exponent on the dimension. Finally, we have shown how to take into account quantitatively spurious phase-breaking mechanisms such as spontaneous emission which smooth the sharp Anderson transition.

METHODS

Finite-size scaling. A quantum phase transition obeys scaling laws in the critical regime. The single parameter scaling theory, successfully used for the standard 3D Anderson model, can be applied to analyze our experimental data, and especially to determine the critical properties of the Anderson transition, i.e. the critical exponents. How can we combine the localized and diffusive dynamics on both sides of the transition in a consistent frame, including possible anomalous diffusion at the critical point? Scaling theory can be applied to $\Pi_0(t)$ depending on the two variables $1/t$ and $(K - K_c)$. We thus assume the scaling law:

$$\frac{1}{\Pi_0^2(t)} = t^{k_1} F\left[(K - K_c)t^{k_2}\right]$$

with $F(x)$ an arbitrary function and k_1 and k_2 two exponents which can be determined as follows. In the diffusive regime, for sufficiently long times, the transport depends on the diffusion constant D which goes to zero as K approaches K_c from above:

$D \sim (K - K_c)^s$. In the localized regime, for sufficiently long times, the behaviour depends on the localization length which diverges as K goes to K_c : $p_{loc} \sim (K_c - K)^{-\nu}$,

with ν the localization ‘‘length’’ exponent. It is easy to see that these two asymptotic behaviours of F imply that $k_1 + sk_2 = 1$ and $k_1 - 2\nu k_2 = 0$. Using Wegner's scaling law^{27,28} which relates the exponents ν and s by the relation $s = (d - 2)\nu$ ($d = 3$ being the dimensionality of the system), we have: $k_1 = 2/3$ and $k_2 = 1/3\nu$. We therefore expect anomalous diffusion, with $\Pi_0^{-2}(t) \sim t^{2/3}$ to govern the dynamics right at the critical point^{29,30}. We can thus exhibit the scaling property by plotting the quantity:

$$\Lambda(t) = \frac{1}{\Pi_0^2(t)t^{2/3}} = f\left(\frac{\xi}{t^{1/3}}\right)$$

where the two branches f_{\pm} of the function f are defined as $f_{\pm}(x) = F(\pm|x|^{-1/\nu})$ and the scaling parameter ξ behaves as $\xi \propto |K - K_c|^{-\nu}$.

How can we evaluate the critical exponent ν and the critical stochasticity parameter K_c ? The behaviour close to the transition can be found by linearising the scaling function F around K_c to obtain:

$$\Lambda(t) = A + B(K - K_c)t^{(1/3\nu)} \quad (3)$$

Then K_c and the critical exponent ν can be obtained by fitting the data with eq. (3).

When taking into account systematic corrections to scaling in the data due to the practical limitations on the number of kicks which can be applied, this procedure leads to a more accurate estimation of the critical parameters. For further details we refer the reader to ref.^{25,26}.

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