Nurse scheduling using integer linear programming and constraint programming

Lorraine Trilling, Alain Guinet, Dominique Le Magny

NURSE SCHEDULING USING INTEGER LINEAR PROGRAMMING AND CONSTRAINT PROGRAMMING

Lorraine Trilling*, Alain Guinet* and Dominique Le Magny**

** PRISMa Laboratory, INSA of Lyon, France*
E-mail: lorraine.trilling@insa-lyon.fr

*** Hospital of Valence CHV, Valence, France*
E-mail: dlemagny@ch-valence.fr

Abstract: In order to reduce cost and to optimize the use of resources, hospitals are prompted to regroup facilities and human resources, especially in the surgical suite. This paper focuses on the anaesthesiology nurse scheduling problem (ANSP) of a French public hospital, where the anaesthesiology nurses constitute one of the most shared resources. They work in a cross way over surgical specialities and assume various activities. Two methods are proposed to solve the ANSP based on integer programming and constraint programming. The objective is to maximize the fairness of the schedule. Theses two techniques are tested in order to be compared.

Keywords: Personnel scheduling, Integer linear programming, Constraint satisfaction problem.

1. INTRODUCTION

Owing to the medical and non-medical demography on one hand, and to the limitation of national health spending on the other hand, the hospitals are prompted to optimize the use of human and material resources relative to surgery, and thus to regroup them in a larger surgical suite.

The assignment of working shifts to nurses over a period of several days is a difficult and time-consuming task, which is usually handled manually by the head nurses. The principle of sharing resources within larger surgical suite, in order to reduce the costs, makes more complex the nurses scheduling problem. This reinforces the need of computer-aided tools for this particularly complicated task.

More generally, in any organisation, the management of staff scheduling is a task that consists in assigning working shift to each employee on each day over a given period, while respecting a certain number of constraints. The requirements that must be considered in the generation of schedules include: legal regulation, personnel policies, workload coverage, individual preferences, etc., depending on the application field (Weil et al., 1995; Ernst et al., 2004). Employee scheduling and nurse scheduling in particular have been addressed by many scientists for more than 40 years (Burke et al., 2004), and many approaches have been proposed to tackle the difficult problem of constructing employee schedules, including operational research and artificial intelligence. The constraints taken into account and the objective in scheduling can vary from one hospital to another, and thus from one approach to another as well. The objective function (for optimization problems) or the target used to guide the solution generation (for other decision-type problems), can also vary and can be designed to minimize the cost, the number of soft constraints violations, or the unfairness of the schedules (Blöchliger, 2004). This diversity has resulted in a whole range of nurse scheduling models.

The surgical suite is a complex service, which includes two main parts: the operating rooms (ORs), and the post-anaesthesia care unit (PACU). Different classes of nurses working in this service have to be scheduled: the operating room nurses, assisting the surgeons during the surgery; the anaesthesiology nurses, taking care of the inpatient state during the surgery or supervising the PACU; the registered nurses, taking care of the inpatient during the recovery; and finally, the auxiliary nurses, performing the logistical tasks as well as the cleaning tasks.

The scheduling problem differs from one class to another, due to the tasks they have to perform and to the work organisation (staff divided in teams, types of shifts, particular scheduling rules, etc.). This paper focuses on the anaesthesiology nurses scheduling problem (ANSP) because this class of nurses is more and more organized in pool of equally-skilled nurses assuming various activities during the inpatient care process. They are also able to perform those activities in a cross way over the surgical specialities. Those equally-skilled nurses can be assigned to different activities from one day to another. One of the objectives is to find a schedule where the distribution of the tasks is balanced over the nursing staff.

The purpose of this article is to give a comparison of two suitable methods (integer linear programming and constraint programming) to solve an instance of the nurse scheduling problem met in a real modern hospital, while seeking for a schedule that guarantees a high level of fairness between the nurses. Several authors have applied those techniques, but we think that the performance of the approaches is much more related to the problem treated, which changes from one paper to the other.

The paper is organized as follows. The next section presents a brief literature review of the existing approaches to the nurse scheduling problem (NSP). In Section 3, a precise definition of the studied problem is given. Section 4 is dedicated to show how this problem can be modelled as an Integer Linear Programming (ILP) model as well as a constraint satisfaction problem (CSP). Section 5 gives the results of each experimented techniques, compares their performances, highlights their differences, and outlines their main drawbacks and advantages. Finally, section 6 draws conclusion and gives some future research tracks.

2. LITERATURE SURVEY

Nurse scheduling problems (NSP) or nurse rostering problems (NRP), which involves the creation of individual schedules, have been widely studied over the last decades. Several very recent and complementary bibliographic surveys have been published and give a good overview of the problem modelling and the various approaches (Cheang et al., 2003; Burke et al., 2004; Ernst et al., 2004).

In the literature, two types of scheduling can be found: cyclical and non-cyclical scheduling. A popular approach is to construct cyclical schedules, where the same schedule is repeated as long as the requirements do not change. These kinds of schedules are easy to build but may be very rigid, and may adapt difficultly to changes. In a non-cyclical scheduling process, a new schedule is generated for each scheduling period. This process is more time-consuming but is much more flexible to changes such as the variability of demand (Valouxis and Housos, 2000).

Several studies have employed optimization methods to solve the NSP, like linear, integer or mixed integer programming (Jaumard et al., 1998), goal programming (Berrada et al., 1996) or constraint programming (Weil et al., 1995; Adbennadher and Schlenker, 1999). Many of more recent papers tackle the NSP with metaheuristic methods such as genetic algorithms (Aickelin and Dowsland, 2004), tabu search (Berrada et al., 1996; Valouxis and Housos, 2000; Aickelin and Dowsland, 2004) or simulated annealing (Brusco and Jacobs, 1995).

We believe that resolution techniques involving the use of solvers are more easily transferable to hospitals services. Other approaches, like heuristics or meta-heuristics are less accessible, and could be time-consuming (Guinet, 1995). Hence our contribution, related to existing approaches, is focused on the comparison of using mixed linear programming on one hand and constraint programming on the other hand. Jaumard et al. (1998) present a generalized 0-1 column generation model, that seeks to satisfy the demand coverage while minimizing the salary costs and maximizing the nurses' preferences as well as team balance. In their approach the authors include the shift selection and sizing stage, which is considered previously done in our problem. However the search objective is quite similar except regarding the cost factor. The Constraint Programming (CP) approaches adopt a different way to model the problem (decision variables, parameters and domains) than the 0-1 models. Weil et al (1995) reduce the complexity of a constraint satisfaction problem by eliminating interchangeable values, and thus reducing the variable domains (from 10 to 4 values). Other authors (Adbennadher and Schlenker, 1999) propose an approach in a three phases distribution: free shifts, night shifts and then morning and evening shifts. The CP approach described later is based on the previously cited studies, but the model and the resolution search have been adapted to the real problem, which is defined in next section.

3. ANSP DESCRIPTION

Anaesthesiology nurses work together with the anaesthesiologists during the surgery in the ORs but also during the recovery time in the PACU. They can work in a cross way over different surgical specialities and several types of surgery (scheduled cases, ambulatory cases or emergency cases). In hospitals that include a surgical emergency service, some nurses work around the clock (on call duty or on stand-by duty), even in the operating suite, where surgical teams have to be rapidly ready to welcome a new patient. Nurses can be assigned to either day or night shifts, and each day to one activity. This daily activity can evolve from one day to another.

The problem presented in this paper has been studied in a French hospital, with a surgical suite containing 9 ORs. The part-time and full-time nurses are all equally-skilled, and can be assigned to any of the following shifts during on day (or night):

Table 1: Types of shifts - start and end time

Day shift (DS)	8:00 – 16:00
Emergency day shift (EDS)	8:00 – 20:00
Emergency night shift (ENS)	20:00 – 8:00
PACU supervision (PS)	9:00 – 17:00 / 11:00 – 19:00

The emergency shifts are on stand-by duty and have to be performed on each day of the week (including the week-ends) whereas the other shifts concern the days from Monday to Friday. The shifts involving the supervision of the PACU are considered as equivalent.

The anaesthesiology nurses schedule is constructed by the anaesthesiology head nurse in a non-cyclical process. First the head nurse gathers nurses' preferences concerning the day they would like to be off and from this, tries to elaborate schedules that satisfies all the constraints listed below. Given that nurses work under work annualization, if during one week a nurse has an overload compared to the standard number of working hours per week, then he/she can recover the over-time worked during the following week or later.

The constraints that have to be satisfied are the following:

C1: Coverage constraints require a number of nurses for each shift (DS, EDS, ENS, PS) and each day.
C2: Working hours must not exceed 12 hours per day
C3: Working hours must be close to 38 hours per week, and must not exceed 48 hours per week
C4: A nurse cannot work more than three night shifts during a week.
C5: If a nurse works an EDS (respectively ENS) on Saturday, then he/she also works an EDS (respectively ENS) on Sunday, and the next Monday and Tuesday are free.
C6: Succession of activities constraints, that allow minimal rest time between two shifts (equal to 11 hours), have to be satisfied such as:

- If a nurse works a NDS, the following day is free;
- If a nurse works an EDS, the following day could be either an ENS or could be free. Given that twice more nurses are required for the EDS than for the ENS during the week, half of the nurses performing an EDS would work an ENS the following day, and the rest would be free.

The constraints C1 to C4 are considered as compulsory constraints that have to be respected (hard constraints). Usually, constraints C5 and C6, as well as the preferences constraints expressed by the nurses, could be optional and an acceptable planning could violate some of them (soft constraints). Since the objective is to find a good planning that maximizes the nurses' satisfaction, soft constraints become hard constraints and the required solution would have to satisfy all the constraints from C1 to C6.

In order to generate the fairest schedule, the popular and unpopular shifts have to be distributed among the nurses in a balanced way, taking into account the difficulty of each shift, according to the head nurse knowledge. To traduce the popularity of the shifts, a penalty associated to each type of shifts and to each day of the week is defined. The penalties are rational values and are included in the interval $[1,...2]$.

Table 2: Penalty associated to the shifts

Type of shift	*Mon*	*Tue*	*Wen*	*Thur*	*Fri*	*Sat*	*Sun*
DS	1	1	1	1	1	0	0
EDS	1.2	1.2	1.2	1.2	1.2	1.4	1.4
ENS	1.4	1.4	1.4	1.4	1.4	1.6	1.6
PS	1.6	1.6	1.6	1.6	1.6	0	0

The best schedule would be the one that minimizes total penalty associated to each employee. In order to find a schedule with the lower penalty for each nurse and the lower difference in comparison to the other nurses, the aim will be to optimize the difference of total penalty between the most loaded nurse and the less loaded nurse. The following section presents how this problem can be modelled and solved, using first ILP and then considering the problem as a CSP.

4. PROBLEM RESOLUTION

4.1. Integer linear programming formulation

Variables. In the ILP formulation, decision variables are 0-1 integer variables defined for each nurse, for each day on each shift as X_{ijk}, where $1 \le i \le N$ indexes the nurses, $1 \le j \le H$ indexes the day within the scheduling horizon and $1 \le k \le S$ indexes the S possible shifts. Those binary variables can take two values as following:

$$X_{ijk} = \begin{cases} 1 & \text{nurse } i \text{ works on shift } k \text{ on day } j, \\ 0 & \text{otherwise.} \end{cases}$$

Objective. The objective of the scheduling is to reach a load distribution well balanced between the nurses. As explained in the previous section, this fairness objective is traduced by the use of a penalty P_{jk}, for each day j and each shift k. The values of P_{jk} are given in table 2. The objective function could be formulated as:

$$\textbf{Minimize} \quad Z = P_{\max} - P_{\min} \qquad (1)$$

where $P_{\max}$ is the upper bound of the total penalty of each nurse, and $P_{\min}$ is the lower bound. Theses bounds are defined as following:

$$P_{\min} - \left(\sum_{j=1}^{H} \sum_{k=1}^{S} \frac{1}{R_i} .X_{ijk} .P_{jk} \right) \leq 0 \quad \forall i = 1,...N \qquad (2)$$

$$P_{\max} - \left(\sum_{j=1}^{H} \sum_{k=1}^{S} \frac{1}{R_i} .X_{ijk} .P_{jk} \right) \geq 0 \quad \forall i = 1,...N \qquad (3)$$

with R_i designating the working rate of each nurse ($=1$ for a full-time, <1 for a part-time).

Constraints. With ILP it is not easy to express constraints dealing with the EDS followed either by the ENS or by a day off. Hence a new type of shift EDS' is introduced, raising to 5 the number of possible shifts: DS ($k=1$), EDS ($k=2$), EDS' ($k=3$), ENS ($k=4$) and PS ($k=5$).

The coverage constraints **C1** are expressed as in equation (4), with R_{jk} representing the number of required nurses for shift k on day j :

$$\sum_{i=1}^{N} X_{ijk} = R_{jk} \quad \forall i = 1,...,N \quad \forall j = 1,...,H \qquad (4)$$

Constraints **C2** limiting the working hours per day are formulated like in equation (5):

$$\sum_{k=1}^{S} X_{ijk} \leq 1 \quad \forall i = 1,...,N \quad \forall j = 1,...,H \qquad (5)$$

Constraints **C3,** limiting the working hours per week, are formulated taking into account the working rate R_i , and N_k the number of hours worked during shift k:

$$\sum_{j=1}^{H} \sum_{k=1}^{S} X_{ijk} .N_k \leq \tfrac{H}{7} .48.R_i \quad \forall i = 1,..., N \qquad (6)$$

The constraints **C4,** limiting the number of the night shifts, are expressed thanks to a new index h where $1 \leq h \leq H/_7$ corresponds to the week.

$$\sum_{j=7h-6}^{7h} X_{ij4} \leq 3 \quad \forall i = 1,..., N \quad \forall h = 1,..., \tfrac{H}{7} \qquad (7)$$

The constraints **C5,** dealing with the work during the week-end, implies several equations as:

- The same shift is assigned during both days of the week-end:

$$X_{i,(7h-1),k} - X_{i,7h,k} = 0 \quad \forall i = 1,..., N \quad \forall h = 1,..., \tfrac{H}{7} \quad \forall k = 3,4 \qquad (8)$$

- After a week-end on, the two following days are off:

$$X_{i,7h,k} + \sum_{k'=1}^{4} X_{i,7h+d,k'} \leq 1 \quad \forall d = 1,2 \quad \forall i = 1,..., N \quad \forall h = 1,..., \tfrac{H}{7} - 1 \quad \forall k = 3,4 \qquad (9)$$

The sequence constraints **C6** concern only the days of the week, and not the week-ends.

- Each EDS' and ENS must be followed by a rest:

$$X_{i,7h-7+j,k} + \sum_{k'=1}^{4} X_{i,7h-6+j,k'} \leq 1 \quad \forall k = 2,4 \quad \forall i = 1,..., N \quad \forall j = 1,...,5 \quad \forall h = 1,..., \tfrac{H}{7} \qquad (10)$$

- Each EDS must be followed by an ENS:

$$X_{i,7(h-1)+j,3} - X_{i,7(h-1)+j+1,4} = 0 \quad \forall i = 1,..., N \; \forall j = 1,...,4 \; \forall h = 1,..., \tfrac{H}{7} \qquad (11)$$

Finally the integration of nurses' preferences is translated by adding constraints like fixing all the decision variables related to the nurse i on the day j he wants to be off:

$$X_{ijk} = 0 \quad \forall k = 1,...,5 \qquad (12)$$

Resolution. This model was solved using the LINGO mixed-integer optimization software, and by executing its own branch-and-bound algorithm.

4.2. Constraints satisfaction problem formulation

The nurse scheduling problem is typically a constraint satisfaction problem (CSP), since it consists in assigning a value from a finite domain to each variable of a finite set. Thus we suggest to formulate the ANSP as a CSP, to solve it using the constraint programming library ILOG (ILOG, 1998) and to compare the results with those given by the MIP approach.

Problem representation. The CSP is defined as a three-tuple (X , D , C) where:

- X is a set of n variables X_{ij} , corresponding to the shift assigned to nurse i on the day j .
- D is a set of n domains D_{ij} , such that each variable X_{ij} takes its value k in $D_{ij} = \{0,1,2,3,4\}$. The value k corresponds to the possible shifts: DS ($k=1$), EDS ($k=2$), ENS ($k=3$) and PS ($k=4$).
- C is a finite set of q constraints, each of which connects a subset of variables in X reducing the possible values those variables can take.

In the CP approach, the constraints are globally the same as the ILP approach. But some differences can be noticed, owing to the method used. For instance, the constraints **C2** limiting the working hours per day (or the assignment of only one shift to each nurse on each day) are integrated in the CSP since a variable can take only one value in the domain.

Coverage constraints **C1** are formulated using $Card\{\omega\}$, that is the cardinal of a set ω .

$$Card\left\{ i \in E \middle| X_{ij} = k \right\} = R_{jk} \quad \forall j \in T, \forall k \in D_{ij} \qquad (13)$$

where E is the set of nurses, T is the set of the days in the scheduling period, and R_{jk} is the number of required nurses for shift k on day j .

Constraints **C4,** limiting the number of night shifts, uses the index h as well as the set of the weeks W :

$$Card\left\{j \in \{7h-6,...,7h\} \middle| X_{ij} = 3\right\} \leq 3 \quad \forall j \in T, \forall h \in W \tag{14}$$

Without adding a virtual shift EDS' like in the ILP model, the constraints **C5** on the shift EDS followed either by an ENS or a rest can be simply expressed:

$$\text{if } X_{i,7(h-1)+j} = 2 \text{ then } X_{i,7(h-)+j+1} = 0 \text{ or } X_{i,7(h-1)+j+1} = 3 \quad \forall i \in E, \forall j \in \{1,...,4\}, \forall h \in W \tag{15}$$

$$\text{if } X_{i,7(h-1)+j} = 3 \text{ then } X_{i,7(h-)+j+1} = 0 \quad \forall i \in E, \forall j \in \{1,...,5\}, \forall h \in W \tag{16}$$

The constraints **C6** are easily written in a CP mode, using constraints "if…then…".

- The same shift is assigned during both days of the week-end:

$$\text{if } X_{i,7h} = k \text{ then } X_{i,7h-1} = k \quad \text{if } X_{i,7h-1} = k \text{ then } X_{i,7h} = k \quad \forall i \in E, \forall h \in W, \forall k \in \{0,2,3\} \tag{17}$$

- After a week-end on, the two following days are off:

$$\text{if } X_{i,7h} \neq 0 \text{ then } X_{i,7h+1} = X_{i,7h+2} = 0 \quad \forall i \in E, \forall h \in \left\{1,..., \tfrac{H}{7} - 1\right\}, \forall k \in \{0,2,3\} \tag{18}$$

Since the length of the shifts can vary from one shift to another, new variables L_{ij} representing the length of the working day for nurse i on day j have to be introduced to express the constraints **C3** that limits the working hours per week. Those variables take a value in the domain $D_{ij}^{L} = \{0,8,12\}$ and are defined using "if…then…" constraints:

$$\text{if } X_{ij} = 4 \text{ then } L_{ij} = 8 \quad \forall i \in E, \forall j \in T \tag{19}$$

Hence, constraints C3 become:

$$\sum_{j=7h-7}^{7h} \frac{L_{ij}}{R_i} \leq 48 \quad \forall i \in E, \forall h \in W \tag{20}$$

The same principle of creating new variables P_{ij} for the penalty associated to nurse i on day j, taking their value in the domain $D_{ij}^{P} = \{0,1,1.2,1.4,1.6\}$ is applied to calculate the total penalty of the schedules for each nurse. This total penalty is involved in the objective criterion $Z = P_{\max} - P_{\min}$, where $P_{\max}$ and $P_{\min}$ are respectively the maximum and the minimum of this total penalty over the nurses:

$$P_{\min} \leq \left(\sum_{j=1}^{H} \frac{P_{ij}}{R_i} \right), P_{\max} \geq \left(\sum_{j=1}^{H} \frac{P_{ij}}{R_i} \right) \quad \forall i = 1,...N \tag{21}$$

Solving this CSP consists of selecting a value in the domain of each variable, so that all the constraints described above are satisfied. Nevertheless, as the aim is to look for an optimal solution minimizing the unfairness of the schedules, it is possible to include an objective function in the search for solution, as explained below.

Solution search. Like other constraints solvers, ILOG Solver uses the posted constraints during the search for the solution: it reduces the domains of the variables by removing the values that are inconsistent with the constraints.

In a CSP, the order in which variables are chosen and values are assigned to variables has a great impact on resolution efficiency. To solve the ASNP, a heuristic strategy, which customizes pre-defined search strategy of the solver, has been elaborated after a large number of tests (figure 2). The purpose of this strategy is to guide the search in order to find from the first iteration a good feasible solution.

As long as there exist unknown variables,

- For day j going form 1 to H,
- choose the first unbounded variable X_{ij}
- evaluate how often each possible value of the domain of X_{ij} has been already assigned to the nurse in the previous days of the schedule
- among the less frequently assigned values, choose the highest value of the domain
- propagate the effects of that assignment.

Figure 2: Customization of the standard algorithm

If the guided choice of value fails (i.e. this value assigned to this variable is inconsistent), the solver backtracks by undoing the assignment and trying the next value according to the heuristic developed. Once a first feasible solution has been found, it is possible to associate to the search an objective function to minimize, by adding a constraint on the maximal value of the objective criterion after each iteration.

5. RESULTS AND DISCUSSION

We have tested the ILP and CP techniques and evaluated their efficiency by solving some instances of ANSP of different sizes (table 3). In the samples considered, all the nurses are regarded as full-time nurses.

Table 3: Parameters of the experimented tests

	Nurses	*Days*	*Requirements*	*DS*	*EDS*	*ENS*	*PS*
Problem 1	10	14	Mon to Fri	2	2	1	1
			Sat to Sun	0	1	1	0
Problem 2	20	7	Mon to Fri	7	2	1	2
			Sat to Sun	0	1	1	0
Problem 3	16	7	Mon to Fri	7	2	1	2
			Sat to Sun	0	1	1	0
Problem 4	16	14	Mon to Fri	7	2	1	2
			Sat to Sun	0	1	1	0

For both approaches, the objective function values given in table 4 are not exactly the optimal values, but the last values obtained when cancelling solvers before completion, i.e. after one hour of search. The search have been rather stopped after a reasonable time for different reasons: (1) solvers, especially the ILP solver, have found a very good solution almost reaching the asymptotic value after a short time; (2)

even after 12 hours of search, the best value found was the one given after one hour; (3) a good schedule could be acceptable even if it has not exactly the minimal difference between $P_{\min}$ and $P_{\max}$.

Table 4: Experimental results

	ILP Model				CP Model			
	P_{max}	P_{min}	*Obj*	*CPU*	P_{max}	P_{min}	*Obj*	*CPU*
Problem 1	8.8	8.4	0.4	14	9	8.2	0.8	2168
Problem 2	4.2	3.6	0.6	62	5.6	2.0	3.6	25
Problem 3	5.0	4.6	0.4	183	5.4	4.0	1.4	28
Problem 4	9.6	9.4	0.2	241	10.6	8.8	1.6	1067

CPU is given in seconds

According to table 4, it appears that the ILP model gives far better results than the CP model, in the sense of the objective criteria $P_{\min} - P_{\max}$. However, for the smaller problem (problem 1), the results given by the CP are still acceptable. For different numbers of nurses but identical requirements (problems 2 and 3), the ILP model gives similar results, whereas objective value obtained by the CP model is worse when the number of nurses is oversized compared to the requirements. In almost all of the cases, the ILP is faster to find the "best" solution.

When analysing the schedules obtained with each method, it seams that they are not of comparable nature, and for this reason give so different results. With the CP model, shifts are well distributed among the first nurses, given that, following the search procedure, it harmonizes the tasks distribution from the beginning of the search. The ILP model does not include these "additional distribution constraints", and the branch-and-bound algorithm results in finding schedules corresponding to a low total penalty difference, but where a larger number of nurses could work several times the same shift during one week. Thus the solutions given by the two models in their current formulation are not really comparable with regard to the objective criteria.

6. CONCLUSION

In this paper, the use of integer linear programming and the use of constraint programming to solve the anaesthesiology nurse scheduling problem have been compared. The aim of this problem is to maximize the fairness of the schedule, while respecting all the constraints. In regards with the results obtained after some tests on various data sets, the ILP approach shows its superiority in finding a very good solution to the ANSP problem with the lower computation time. However, we found that the schedules given by each method are not completely comparable in their current state and theirs respective natures have been highlighted. Some perspectives of this work can be drawn. Firstly, better values of the penalties associated to the shifts could be defined in order to represent the reality more accurately, especially by taking into account the length of the shifts. It is also in our plan to write a better search procedure improving the efficiency of the CP model and to enhance the ILP model with additional constraints, in order to make both models comparable. The improved models constructed for the anaesthesiology nurses could be adapted to other nurse classes' problems, especially by adding the competency aspect to the current model.

ACKNOLEDGEMENTS

This study has been performed within the scope of the HRP² French regional project.

REFERENCES

Adbennadher, S. and H. Schlenker (1999). Nurse scheduling using constraint logic programming. *Proceedings of AAAI/IAAI.*

Aickelin, U. and K. A. Dowsland (2004). An indirect Genetic Algorithm for a nurse-scheduling problem. *Computers and Operations Research*, **31**, 761–778.

Berrada, I., J. A. Ferland and P. Michelon (1996). A multi-objective approach to nurse scheduling with both hard and soft constraints. *Socio-Economic Planning Sciences*, **30**, 183-193.

Blöchliger, I. (2004). Modeling staff scheduling problem. A tutorial. *European Journal of Operational Research*, 533-542.

Brusco, M. J. and L. W. Jacobs (1995). Cost analysis of alternative formulations for personnel scheduling in continuously operating organisations. *European Journal of Operational Research*, **86**, 249–261.

Burke, E. K., P. D. Causmaecker, G. V. Berghe and H. V. Landeghem (2004). The state of the art of nurse rostering. *Journal of scheduling*, **7**, 441-499.

Cheang, B., H. Li, A. Lim and B. Rodrigues (2003). Nurse Rostering Problems - A Bibliographic Survey. *European Journal of Operational Research*, **151**, 447-460.

Ernst, A. T., H. Jiang, M. Krishnamoorthy and D. Sier (2004). Staff scheduling and rostering : A review of applications, methods and models. *European Journal of Operational Research*, **153**, 3-27.

Guinet, A. (1995). Scheduling independent jobs on uniform parallel machines to minimize tardiness criteria. *Journal of Intelligent Manufacturing*, **6**, 95-103.

ILOG (1998). *ILOG Solver 4.4 Reference Manual*, Gentilly, France.

Jaumard, B., F. Semet and T. Vovor (1998). A generalised linear programming model for nurse scheduling. *European Journal of Operational Research*, **107**, 1-18.

Valouxis, C. and E. Housos (2000). Hybrid optimization techniques for the workshift and rest assignment of nursing personnel. *Artificial Intelligence in Medicine*, **20**, 155-175.

Weil, G., K. Heus, P. François and M. Poujade (1995). Constraint Programming for Nurse Scheduling. *IEEE Engeneering in Medicine and Biology*, **14**, 417-422.