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Speeding up Simplification of Polygonal Curves using Nested Approximations

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Abstract: We develop a multiresolution approach to the problem of polygonal curve approximation. We show theoretically and experimentally that, if the simplification algorithm A used between any two successive levels of resolution satisfies some conditions, the multiresolution algorithm MR will have a complexity lower than the complexity of A . In particular, we show that if A has a $O(N^2/K)$ complexity, the complexity of MR is in $O(N)$. We experimentally compare the outcomes of MR with those of the optimal “full search” dynamic programming solution and of classical merge and split approaches. The experimental evaluations confirm the theoretical derivations and show that the proposed approach evaluated on 2D coastal maps either shows a lower complexity or provides polygonal approximations closer to the initial curves.

Keywords: *Polygonal approximation; Dynamic programming; Multiresolution;*

1. Introduction

Approximation of multi dimensional discrete curves has been widely studied essentially to speed up data processing required by resource demanding applications such as Computer Vision, Computer Graphics, Geographic Information Systems and Digital Cartography, Data Compression or Time Series Data Mining. For polygonal approximation of discrete curves, the problem can be informally stated as follows: given a digitized curve X of $N \geq 2$ ordered samples, find K dominant samples among them that define a sequence of connected segments which most closely approximates the original curve. This problem is known as the *min- ϵ* problem. Numerous algorithms have been proposed for more than thirty years to solve efficiently this optimisation problem. Most of them belong either to graph-theoretic approaches (Imai and Iri, 1986, 1988; Melkman and O’Rourke, 1988; Chan and Chin, 1996; Zhu, and Seneviratne, 1997; Chen and Daescu, 1998; Katsaggelos et al., 1998), dynamic programming (Perez and Vidal, 1994; Salotti, 2001; Horng, 2002, Kolesnikov et al., 2003,

2004) or to heuristic approaches (Douglas & Peucker, 1973; Pratt & Fink, 2002; Debled-Rennesson and Reveillès, 2003; Charbonnier al., 2004). This paper focuses on polygonal approximation of multidimensional curves using multiresolution. Our concern is the design of an algorithm that, starting from the finest resolution level, finds iteratively $min-\epsilon$ polygonal approximations for coarser resolution levels, exploiting at the any level the approximating segments obtained at the previous (finer) resolution level. In theory, such multiresolution approximation can exploit any polygonal simplification methods between two successive levels of resolution. Nevertheless, the effectiveness and efficiency of such approach is not guaranteed. The aim of this paper is to study the conditions required for efficiency and effectiveness of the multiresolution approach.

The first part of the paper addresses a complexity analysis that sketches conditions for the efficiency of the multiresolution approach. Some argumentation is also provided to outline the conditions for effectiveness. A proposal for a simple multiresolution algorithm that fulfils the previous conditions is then proposed.

The second part of the paper addresses the evaluation of the proposed algorithm through experimentations that compare the efficiency and effectiveness of the multiresolution approach with heuristic and dynamic programming solutions.

2. A Multi Resolution approach to Polygonal Curve Approximation (*MR algorithm*)

Basically, the idea behind the multiresolution approach to polygonal curve simplification is to successively re-approximate previous approximations obtained by using some given simplification algorithm, this process being initiated from an original discrete time series. Following (Kolesnikov & al. 2004), we take a sequence of polygonal curves $\{X_0, X_1, X_2, \dots, X_r\}$ as a multiresolution (multiscale) approximation of a N -vertex input curve X , if the set of curves $\{X_i\}$ satisfies the following conditions:

i) A polygonal curve X_i is an approximation of the curve X for a given number of segments K_i (*min- ϵ problem*) or error tolerance ϵ_i (*min-# problem*), where i is a resolution level ($i=0,1,2,\dots, r$).

ii) The set of vertices of curve X_i for resolution level i is a subset of vertices of curve X_{i-1} for the previous (higher) resolution level ($i-1$). The lowest resolution level r is represented by the coarsest approximation of X . The highest resolution level $i=0$ is represented by the most detailed approximation (namely the original curve $X_0=X$) with the largest number of segments $K_0 = N$. ($N=K_0 > K_1 > K_2 > \dots > K_r$) or smallest error tolerance $\epsilon_0=0$ for some distance measure (e.g. L_2) ($\epsilon_0 < \epsilon_1 < \epsilon_2 < \dots < \epsilon_r$).

Thus, an approximation curve X_i is either obtained by inserting new points into the approximation curve X_{i+1} (bottom-up approach), or, conversely, X_{i+1} is obtained by deleting points from the approximation curve X_i (top-down approach). These two variants have led to the development of two very popular heuristic approaches: respectively *Split* and *Merge* methods. In the *Split* approach, an iterative mechanism splits the input curve into smaller and smaller segments until the maximum deviation is smaller than a given error tolerance ϵ (*min-# problem*), or the number of linear segments equals to the given K_i (*min- ϵ problem*) for the current resolution level i . A famous split method is the Douglas-Peucker algorithm (Douglas and Peucker, 1973); this algorithm is known to have a $O(K.N)$ complexity; it has been used for multiresolution approximation in (Le Buhan Jordan & al., 1998, Battenfield, 2002).

In the *Merge* approach (Pikaz and Dinstein, 1995, Visvalingam and Whyatt 1993), the polygonal approximation is performed by using a cost function that determines sequential elimination of the vertices with the smallest cost value, while the two adjacent segments of the eliminated vertex are merged into one segment. The approximation curve X_i is obtained

by discarding vertices from the curve X_{i-1} until the desired number of vertices K_i (*min-ε problem*) is reached. This *Merge* approach is known to have a $O(N \cdot \log(N))$ complexity.

2.1. Complexity of multiresolution approach

We address in this section the efficiency issue of the multiresolution approach comparatively to the complexity of an algorithm that does the job in a single step. Let \mathbf{C} be the complexity of an algorithm \mathbf{A} that simplifies a polygonal curve X having N segments into a polygonal curve having K segments ($K < N$). We suppose first that \mathbf{C} is a polynomial function of N and K , namely $\mathbf{C} = \alpha \cdot N^p \cdot K^q$ where p and q are integers and α is a positive constant ($\alpha > 0$). For all N , $K < N$ and ρ in $]0;1[$ there exists a natural number r such that $N \cdot \rho^{r+1} < K \leq N \cdot \rho^r$. Given N , $K < N$ and ρ , we construct the multiresolution $\{X_0, X_1, X_2, \dots, X_r\}$ of curve X such that $K_j = \rho^j \cdot N$ is the number of segments that compose X_j , the j^{th} nested polygonal approximation of X .

In this context, using algorithm \mathbf{A} during the first step, we obtain from an original curve (X) of size N a polygonal curve approximation (X_1) having $K_1 = \rho \cdot N$ segments with complexity: $\mathbf{C}_1 = \alpha N^p K_1^q = \alpha N^p (\rho N)^q = \alpha \rho^q N^{p+q}$

Considering as a second step the simplification of the X_1 polygonal curve seen as a polygonal curve having $N_1 = K_1$ samples, still using algorithm \mathbf{A} , we get a polygonal curve approximation X_2 having $K_2 = \rho^2 \cdot N$ segments from the X_1 curve with time complexity:

$$\mathbf{C}_2 = \alpha (K_1)^p (K_2)^q = \alpha (\rho N)^p (\rho^2 N)^q = \alpha \rho^{p+2q} N^{p+q}$$

Iterating the process we get successively:

$$\mathbf{C}_3 = \alpha (K_2)^p (K_3)^q = \alpha (\rho^2 N)^p (\rho^3 N)^q = \alpha \rho^{2p+3q} N^{p+q}$$

By induction, it is easy to show that for all j in $\{1, \dots, r\}$ we have :

$$\mathbf{C}_j = \alpha (K_{j-1})^p (K_j)^q = \alpha (\rho^{j-1} N)^p (\rho^j N)^q = \alpha \rho^{(j-1)p+jq} N^{p+q}$$

The final iteration - required to ensure that the last approximation has exactly K segments

– is performed using algorithm **A** with the following complexity:

$$\begin{aligned} \mathfrak{C}_{r+1} &= \alpha K_r^p K^q = (\rho^r N)^p (K)^q \\ \text{as } \rho^{r+1} N \leq K < \rho^r N, \quad \mathfrak{C}_{r+1} &\leq \alpha (\rho^r N)^p (\rho^r N)^q = \alpha \rho^{r(p+q)} N^{p+q} \quad \text{if } q > 0, \\ \mathfrak{C}_{r+1} &\leq \alpha (\rho^r N)^p (\rho^{r+1} N)^q = \alpha \rho^q N^{p+q} \rho^{r(p+q)} \quad \text{otherwise.} \end{aligned}$$

Finally, from the original time series of size N , we get after $r+1$ iterations of the previous process a polygonal approximation having K segments with time complexity:

$$\mathfrak{C}_{MR} = \mathfrak{C}_{r+1} + \sum_{j=1}^r \alpha \rho^{(j-1)p+jq} N^{p+q} = \mathfrak{C}_{r+1} + \alpha \rho^q N^{p+q} \sum_{j=1}^r \rho^{(j-1)(p+q)} \quad (1)$$

In general, the multiresolution approach does not offer computation complexity reduction comparatively to the complexity of the algorithm **A** applied in a single step. This is due to the presence of term N^{p+q} in the polynomial function \mathfrak{C}_{MR} . But for some specific cases, \mathfrak{C}_{MR} simplifies nicely as shown below.

From formula (1), we see that if $q \leq 0$,

$$\mathfrak{C}_{MR} = \alpha \rho^q N^{p+q} \rho^{r(p+q)} + \alpha \rho^q N^{p+q} \sum_{j=1}^r \rho^{(j-1)(p+q)} = \alpha \rho^q N^{p+q} \sum_{j=1}^{r+1} \rho^{(j-1)(p+q)}$$

$$\begin{aligned} \text{In that case:} \quad \mathfrak{C}_{MR} &= \alpha \rho^q N^{p+q} \frac{1 - \rho^{(r+1)(p+q)}}{1 - \rho^{(p+q)}} \leq \alpha \rho^q N^{p+q} \frac{1}{1 - \rho^{(p+q)}} \quad \text{if } p+q \neq 0 \\ &= \alpha \rho^q (r+1) \quad \text{otherwise.} \end{aligned} \quad (2)$$

hence, if $p+q \neq 0$ and $q < 0$, we note that \mathfrak{C}_{MR} is upper bounded by a polynomial function of N that does not depend upon r , the number of iterations required by the multiresolution algorithm. Furthermore, if $p+q > 0$ and $q < 0$, the multiresolution algorithm brings a complexity reduction of order N^{-q} .

$$\text{As an example, for } p=2 \text{ and } q=-1, \text{ we get the upper bound: } \mathfrak{C}_{MR} \leq \frac{\alpha \cdot N}{\rho \cdot (1 - \rho)} \quad (3)$$

Since this upper bound is independent from K and r , we show that the time complexity of the multiresolution approach is $O(N)$. This lower bound is minimized for $\rho = 1/2$.

It is not straightforward to generalize this result for non polynomial complexities.

Nevertheless if the complexity of \mathbf{A} is in $O(f(N))$, where f is independent from K , (e.g. $f(N)=N\text{Log}(N)$) one cannot expect to reduce this complexity using multiresolution.

2.2. Effectiveness considerations

There are two sources of error increasing in multiresolution approximation in comparison with polygonal approximation obtained in a single step:

1. In multiresolution approximation, vertices for the next level of resolution should be selected among the vertices available at the current level of resolution. This constraint does not exist for single step approximation.
2. The non-optimality of algorithm used to solve $\text{min-}\epsilon$ polygonal approximation produces error propagation inside multiresolution approximation.

We cannot reduce inaccuracies related to the first source of errors, but if we use near-to-optimal algorithm to solve $\text{min-}\epsilon$ problems between successive levels of resolution, one can expect to approach also near-to-optimal solutions for multiresolution approximation.

2.3 Conditions for optimizing the (*Efficiency.Effectiveness*) product

The condition to maximize the efficiency gain is to select an algorithm \mathbf{A} that solves $\text{min-}\epsilon$ problems in one step with a complexity $\mathcal{C} = \alpha.N^p K^q$ where $p+q > 0$ and $q < 0$: In that case, the complexity gain is N^{-q} .

The condition to maximize the effectiveness or accuracy of the multiresolution algorithm is to select the optimal algorithm \mathbf{A} that solves optimally the $\text{min-}\epsilon$ problem.

As we know, the optimal algorithm is the one proposed by (Perez and Vidal, 1994): it solves the $\text{min-}\epsilon$ problem with a complexity in $O(KN^2)$. Unfortunately, the condition $q < 0$ is not satisfied since $q=1$. In the other hand, the near-to-optimal solution developed by (Kolesnikov and Fränti, 2003), the so-called “Reduced Search Dynamic Programming”

(*RSDP*) has a complexity in $O(N^2 / K)$ that satisfies the conditions of a complexity gain for the multiresolution solution. If *RSDP* is selected as the algorithm **A** then the complexity of the multiresolution is in $O(N)$ as stated by equation (3), whatever the decimation factor ρ is. We experimentally verify this claim in the next section.

3. Experimentations and discussion

To evaluate the quality of suboptimal algorithms, Rosin (1997) introduced a measure known as fidelity (F). It measures how good (or how bad) a given suboptimal approximation is in respect to the optimal approximation in terms of the approximation error:

$$F = 100 \cdot \frac{E_{\min}}{E},$$

where E_{\min} is the approximation error of the optimal solution, and E is the

approximation error of the given suboptimal solution. In practice, we will identify E_{\min} to the error (the Euclidian distance between the original time series and the polygonal approximation) obtained using the ‘Full Search’ dynamic programming (*FSDP*) solution, namely the algorithm of Perez and Vidal (Perez and Vidal, 1994).

The *MR* algorithm we evaluate uses a simplified version of the *RSDP* algorithm for which the reduced search space is determined by a fixed size corridor β (Sakoe and Chiba, 1978, Kolesnikov and Fränti, 2003, Marteau and M  nier, 2006). This version of the *RSDP* is known to have a complexity in $O(N^2 / K)$ (Horng, 2002; Kolesnikov and Fränti, 2003). This ensures that the *MR* algorithm has a complexity of $O(N)$ as shown in the previous section.

The evaluation consists essentially in measuring the fidelity and runtime elapsed time for various parameter settings (i.e. N , K , β , ρ) of the *MR* algorithm compared to other solutions (*Split*, *Merge*, *FSDP*). We have tested the *MR* algorithm on 2D coastal maps extracted from the National Geophysical Data Center (NGDC, 2006) dataset.

FIG.1 shows the evolution of the F measure as a function of K evaluated for the crudest approximation map given by the *MR* (with $\beta=8$ and $\rho=.1, .2, .3, .4, .5, .6, .7, .8$ and $.9$), *Merge_L2* and *Douglas Peucker*. *MR* has a much better F for all ρ values. Furthermore, the higher ρ , the better F in general.

**** *FIG. 1 around here* ****

FIG.2 compares the experimental time complexity of the optimal solution (*FSDP*), of the full search dynamic programming solution (*FSDP*), of the multiresolution algorithm (*MR*), of the Douglas-Peucker algorithm (*DP*) and of the *Merge_L2* algorithm. The time complexity is measured as ‘user clock ticks’ spent as N increases while K remains fixed on a Pentium 4 processor running Linux. The scale used in *FIG. 2* is logarithmic, so that all curves are linear with different slopes. The figure shows that the Douglas-Peucker complexity curve has the same slope as the *MR* complexity curve. As the Douglas-Peucker algorithm is known to be $O(N)$, these two algorithms have the same linear complexity even though *MR* is more expensive, since the *MR* curve is above the *DP* curve. *DPFS* curve has a higher slope, and as such exhibits a polynomial complexity as expected (*FSDP* complexity is in $O(KN^2)$). The *Merge_L2* complexity curve has a slope in between *FSDP* and Douglas-Peucker curve as expected (*Merge_L2* complexity is known to be in $O(N.log(N))$).

**** *FIG.2 around here* ****

4. Conclusion

We have explored multiresolution (*MR*) applied to the problem of simplifying a discrete curve using nested polygonal approximations. It consists in iteratively applying a simplification algorithm to get the successive nested approximations, from the finest (the original curve) to the coarsest. We have shown both theoretically and practically that, when the simplification is based on a *Reduced Search Dynamic Programming (RSDP)* algorithm, *MR* has a linear time complexity ($O(N)$), whatever the chosen number of resolution levels.

This solution is suboptimal but maintains partial optimality between each resolution levels. It offers good approximating solutions when real time and storage space are issues, namely each time the optimal solution cannot be calculated due to the size of N . For all tests we have performed, the quality of the resulting approximation is significantly better than the quality of well known heuristic approaches (the Douglas-Peucker splitting approach or the merge approach that have respectively a complexity in $O(N)$ and $O(N\text{Log}(N))$): the gain on the quality measure F varies from 30% to 50 % according to the tuning of parameters. To our knowledge, MR associated to $RSDP$ provides the best polygonal approximating algorithm having complexity in $O(N)$. The experimental results give some highlights for the choice of the parameters, i.e. ρ and β , that could be tuned according to the task. Furthermore, the multiresolution aspect of the method allows managing simultaneously various resolution levels, a functionality that could be very useful to speed up time series information retrieval tasks.

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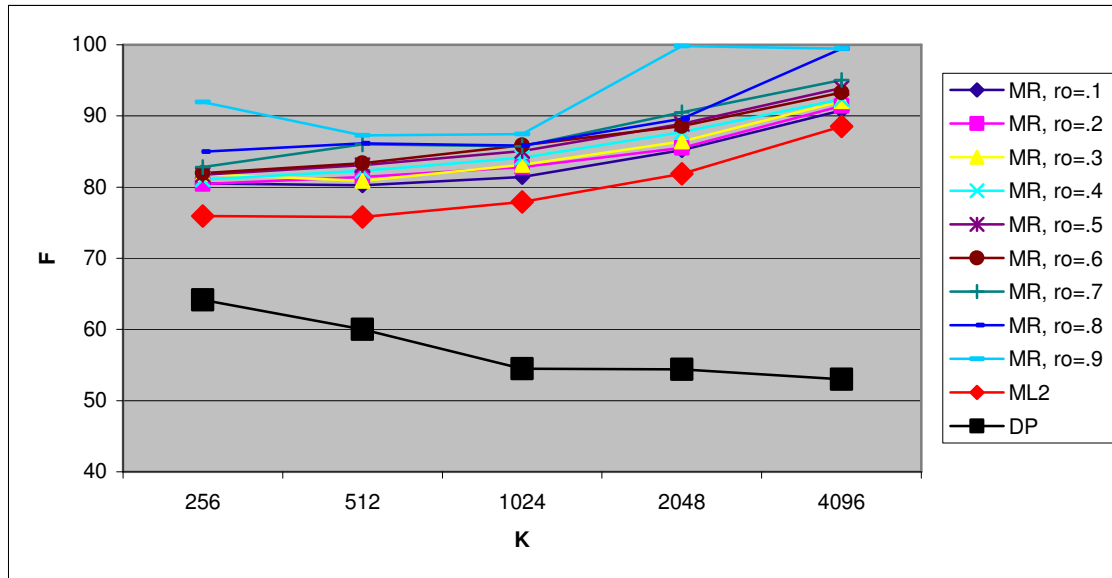


FIG. 1 – Fidelity measure (F) as a function of K evaluated for the crudest approximation map given by the MR (with $\beta=4$ and $\rho=.1, .2, .3, .4, .5, .6, .7, .8$ and $.9$), $Merge_L2$ and $Douglas Peucker$.

Log10(nbTicks) vs. N

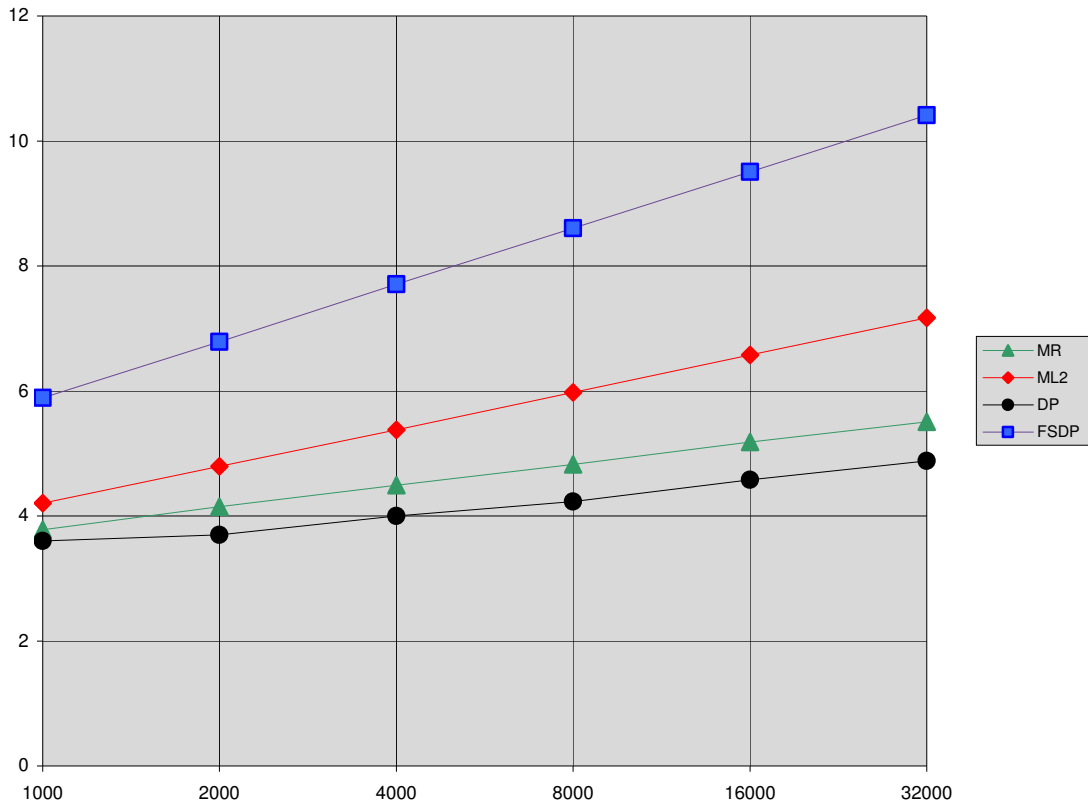


FIG. 2 –Comparison of experimental time complexity (expressed in user clock ticks) on a logarithmic scale for *Douglas-Peucker* algorithm (DP, circles, black), *MR* algorithm (triangles, green), *Merge_L2* (rhombuses, red) and “*Full Search*” dynamic programming procedure (*FSDP*, squares, blue). Here $K = 10$ for all methods, and $\rho=.5$, $\beta=4$ for *MR*.