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THEORETICAL APPLICATION OF THE TENSORIAL ANALYSIS OF NETWORK FOR EMC AT THE SYSTEM LEVEL

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ABSTRACT

To resolve the Electromagnetic compatibility (EMC) at the system level we need to compute physical phenomenon and to connect various patterns existing at various scales. With this intention we need a technique able to link schematics coming from various jobs: electronic, harnesses and structures. This paper presents a technique based on the topological tensorial analysis of network developed by Gabriel KRON to compute all the interactions encountered in EMC on large systems.

1. Introduction

The EMC control of a large system is closely linked to the EMC behavior of its components. These components are located on the printed circuit board (PCB) and are typically less than one centimeter long. On the board, tracks (~5cm) are connected to wires (1m). These wires exchange energy through the fields with their environment: the system structure (some meters long). This structure radiates or receives fields coming from the environment (kilometers).

Each system layer has its own physic. For the components, we use the Poisson's diffusion equation in the time domain. For PCB, we use line (telegraph's) modeling, PEEC techniques [1]. For the equipment, we can use cavity patterns, moment method software [2]. For harnesses, there are special techniques using guided waves formalism (Branin's pattern for example) [3]. The stationary part of the currents carried by the wires radiates frequency fields inside the system cavity. These fields can leave the system through openings [4]. Contrary to this process, the external fields create fields inside the system's structure. These fields give energy to the wires. This energy is conducted to the equipment, and through the PCB tracks, to the components. We see that

the EMC at the system level requires a multiscale, multiphysic and multidomain method [5][6][7].

2. Graph and topology

A complex electronic system is made of several simple elementary circuits. Each circuit is called a primitive circuit. All these circuits have interactions through lines (conducted interactions), near field or far field radiations. The system can be seen as a complex graph with charges located on nodes and currents on edges. A group of edges makes a mesh. And a group of meshes makes a network. We present here the various space properties (including movements) and dimensions before introducing how these various levels of description can be grouped to represent a large system EMC problem.

2.1 Nodes and edges spaces

A system made of N edges is represented by a N dimensional space. Each edge is a component of the contravariant vector ***I*** (flow). The instantaneous power of the whole system is the intrinsic scalar S. This scalar defines the covariant vector of the electromotive forces and voltage differences (efforts) ***e***. Using the mute index writing [8], we have:

$$e_k I^k = S \tag{1}$$

A particular metric ***Z*** acting like an operator transforms the currents in voltages. This metric (a non Riemann's one) can be a non linear operator (for example in the case of diodes, semi-conductors). We have in general in the edge space:

$$e_a - V_a = Z_{ab}\left(I^b\right) \tag{2}$$

V being the vector of the nodes pair voltage difference. This relation is a generalized Kirchhoff’s law in the edge space. The diagonal elements of ***Z*** are the edge impedance (we use the notion of impedance even in the time domain, as ***Z*** is an operator. For example for an inductance we have: Z_{11}=Ld/dt(.)). When there is a coupling process between edges, we create a string. This string suited with the extra diagonal elements of the twice covariant tensor ***Z***. This tensor is formally defined as a tensorial product of two voltage vectors in the edge dual space (noticed with a star):

$$\boldsymbol{Z} = \frac{1}{S}\left(e^* \otimes e^*\right) \tag{3}$$

The nodes pair voltages are linked to the local scalar potential in the nodes space by the gradient relation translated in a topological graph:

$$\psi_k = V_m B_k^m \tag{4}$$

B is the classical incident matrix usually employed in graph theory. By the capacitor metric this relation makes the link between the nodes space and the edge one. We have:

$$q^m = C^{mm} \psi_m \qquad I^N = C^{NN} \partial_t V_N \tag{5}$$

These relations are an opportunity for us to detail much further what will be a strong advantage of the tensorial analysis: the capability to make theoretical studies on the network electromagnetic behavior. The tensor C is a part of the general metric Z representing the potential energy of the network. It admits an inverse through the identity matrix Δ. We can write:

$$\frac{1}{C^{ab}} = \frac{D_{ab}}{\left(D_{ab} \; C^{ab}\right)} = \frac{D_{ba}}{\Delta_b^{\;b}} = D_{ba} \tag{6}$$

Using (4), (5) and (6) and saying that $\boldsymbol{C}$ and B are not time dependent, we have:

$$\begin{aligned} \partial_t q^m &= C^{mk} B_k^m \partial_t V_m \Leftrightarrow \\ \partial_t q^m &= C^{mk} B_k^m D_{mm} I^m \end{aligned} \tag{7}$$

Between the capacitor at the nodes (connected between the nodes and a virtual node link with the zero potential at the infinity) and the capacitor of the edges we have:

$$C^{mk} B_k^m = C^{mm} \tag{8}$$

So:

$$\partial_t q^m = C^{mm} D_{mm} I^m = I^m \tag{9}$$

This is the charge conservation law. The diagonal elements of $\boldsymbol{Z}$ are electronic components, dielectrics or resistors, while the extra-diagonal

elements are cross-talked between edges through radiated fields. This interaction go through what we call a string: an electromotive force is created on an edge, coming from a current of another edge. But this interaction doesn't go from one node to another. This is not a classical edge. The matrix B can be seen as a connectivity matrix between the nodes and the edges spaces. These matrices allow going through the various spaces whatever their dimensions are. When the space bases are equal in dimensions, the connectivity matrix are square ones. But Kron has shown that these matrices allow making connections between spaces of different dimensions without loosing any information coming from one space to another (this is very important: don't think that the mesh space in the Kron's definition can be compared to the classical mesh circuit description). That's what we will show now by studying the mesh space level.

2.2 The mesh space

When one writes the Maxwell's equation of a closed loop of current density through various medium (dielectric and resistive ones) one obtains the equation [9][10]:

$$\begin{aligned} \oint_{\Gamma} d\vec{\Gamma}\cdot\vec{E} &= \left(\frac{i\Gamma_{\sigma}}{\sigma S}\right)+\left(\int_{t} dt\frac{i\Gamma_{\epsilon}}{\epsilon S}\right)-hE_0 \\ \oint_{\Gamma} d\vec{\Gamma}\cdot\vec{E} &= -L\frac{di}{dt} \end{aligned} \tag{10}$$

The closed circulation of the electric field is equal to the work of the losses in the resistive medium (Γ_{σ}, σ, S) plus the work in the dielectric medium (Γ_{ϵ}, ϵ, S) minus the electromotive force E_0 that creates the current. This closed electric circulation is equal to the magnetic flow through the loop, or to the self inductance L multiplied by the time derivative of the current. The Maxwell's equations in their integral form demonstrate that the magnetic induction and energy are located in a closed loop of edge: i.e. a mesh. The fact that the magnetic field appears in the mesh space is coherent with its two dimensions intrinsic property [11]. To solve a problem and take into account both electric and magnetic fields phenomenon, one must transform the edges impedances and all other objects in the mesh space. The base of the mesh space is the mesh currents. In the graph shown in figure 1 we illustrate a two meshes network with two nodes and three edges.

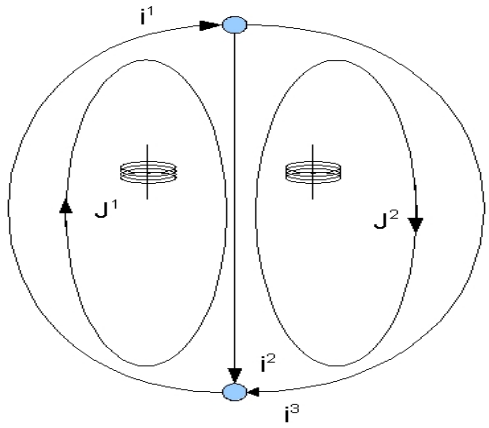

Fig. 1. An example of network with three edges and two meshes.

As the inductances are linked with the mesh space, we represent the inductance symbol at the center of the mesh.

Between the meshes current and the edge ones we can construct a group of relations. Being in the referential of one edge, for example edge 1, associated with edge current i^1, we can write that the current i^1 passes through the edge 1 in the edge space, and the current J^1 passes through the edge 1 in the mesh space. The current i^2 passes through the edge 2 in the edge space, and the currents J^1 minus the current J^2 passes through the edge 2 in the mesh space. Finally, calling L the connectivity between the edge and the mesh space (see example §3.2):

$$i^k = L^k_m J^m \tag{11}$$

This matrix L is not a square one because the mesh space is at least one dimension less than the edge one. The laws of transformation for $\boldsymbol{e}$ and $\boldsymbol{Z}$ are:

$$e_m = e_p L^p_m \qquad Z_{mn} = L_m{}^a Z_{ab} L^b_n \tag{12}$$

After this transformation, as the only interactions taken into account in the edge space are the resistive and dielectric ones (plus all the electronic non linear components and the self inductance of the edges) we must add to the new tensor $\boldsymbol{Z}$ the self inductances of the loops and

the mutual inductances between loops. This is done by adding a magnetic tensor ***M***:

$$\zeta_{mn}=Z_{mn}+M_{mn} \tag{13}$$

ζ is the metric tensor including all the magnetic field interactions at the network level. The mesh space is the space where the problems are finally solved. The metric in the mesh space includes the Lagrange's operator of energy, we will show that later. There is a last property particular to the mesh space: as each mesh is a closed loop of edges, the nodes pair voltage in this space is null. For this space Kirchhoff's law is reduced to: $e_m=Z_{mn}\left(J^n\right)$. This equation can be resolved in the frequency or in the time domain (through finite difference time domain method).

2.3 The moment space

As we have seen in the case of the nodes to edge and edge to mesh spaces, connectivity allows changing of scale value by going through one space dimension to another. When we want to compute the far field interaction between networks we have to change of scale. In our technique, the near electrostatic field is represented by a capacitance edge. I propose as a definition that two networks are in only far field interaction if they have no edge between any of their nodes and no mutual inductance between any of their meshes. Under these conditions, we need a new space of description to represent all the far field energy emitted by an active network. Each mesh creates a magnetic moment defined by [12]:

$$m^q=S_n^q J^n \tag{14}$$

Each moment creates a far magnetic field defined by:

$$B_\omega=\alpha_{\omega q} m^q \tag{15}$$

Projected in a local 3D space, each moment has its own coordinates:

$$m^x=n_q^x m^q \tag{16}$$

Here we have a connectivity to the fundamental geometric space. This allows us to define the global moment of the network:

$$\vec{m}=\vec{u}_x n_q^x m^q \tag{17}$$

This vector gives all the information to compute the far magnetic field emitted by the network. Classical relations of antennas between electric and magnetic field, propagation delay and so on give the context to end the computation. What is less evident is to transform the network information to the mesh one, as we want to resolve the problems in the mesh space. The moment of a first network creates in a second network an electromotive force E which depends on the mutual inductance with:

$$E_\omega=M_{\omega q}\partial_t J^q \tag{18}$$

Using the previous relations we can write:

$$M_{\omega q}\partial_t J^q=S_\omega^\omega \alpha_{\omega q}\partial_t m^q \tag{19}$$

With (14) we finally obtain:

$$M_{\omega q}=S_\omega^\omega \alpha_{\omega q} S_q^q \tag{20}$$

From the network space, the interactions are taken into account using (20) and adding to the tensor ***M*** the elements coming from these interactions defined through α.

2.4 Movements

When one of the two networks is in movement in comparison with the other one, inductions come both from the time variation of the field, and also from the changing direction of the reference attached to one network. One considers a first base vectors ***e*** attached to a first network and 3D local space, and a second one ***u*** attached to a second network and 3D local space. Between them a matrix to go from one base to the other exists:

$$u_k = e_q \Lambda_k^q \tag{21}$$

The electromotive force E induced on an edge N of a second network is created by the current of a first edge n of the first network. E is given by:

$$E = \partial_t \left(\int_{C(N)} d\vec{C} \cdot \vec{A}(N) \right) \tag{22}$$

The vector dC depends on the second base. A is defined in the first base. We can replace the vector by their coordinates and write:

$$E = \partial_t \left(\int_{C(N)} \vec{u}_x dC^x \cdot \vec{e}_x A^x \right) \tag{23}$$

With (21) we obtain:

$$E = \partial_t \left(\int_{C(N)} \vec{e}_y \Lambda_x^y dC^x \cdot \vec{e}_x A^x \right) \tag{24}$$

Finally, knowing the fundamental 3D metric $g_{yx} = \mathbf{e}_y . \mathbf{e}_x$:

$$E = \int_{C(N)} dC^x \left\{ \partial_t \left(\Lambda_x^y \right) g_{yx} A^x + \ldots \right.$$
$$\left. \ldots + \Lambda_x^y \partial_t \left(g_{yx} \right) A^x + \Lambda_x^y g_{yx} \partial_t \left(A^x \right) \right\} \tag{25}$$

The last term in this integral is the classical e.m.f. derived from the time variation of the vector potential. The first two terms are included in the Christoffel's symbols when one referential follows curvilinear trajectory. This is the case in the electrical machine where the rotor is a moving network in comparison with the stator. The Kron's machine [13] uses this development which is the more general that can be imagined. Finally when there is a movement between the networks, one transformation must be applied added to the others seen before:

$$Z'_{ab} = Z_{ab} + \Gamma_{ab,c} \dot{x}^{c} \tag{26}$$

Where Γ is the Christoffel's coefficient matrix extracted from (25).

2.5 Fundamental topological relation

Between the edge B, nodes N and mesh M space, for R sub network we have the fundamental topological relation:

$$M = B - N + R \tag{27}$$

This relation must be verified at any time we compute a problem using our technique.

2.6 Graphs and equations

An EMC problem is first represented by the engineer who's responsible for resolving the problem. This representation is made using a graph. This graph can include classical electronic symbol (R, L, C, transistors, etc.) with special lines to represent the field radiation. Between the graph and the $\boldsymbol{Z}$ tensor organization there is a direct link in the edge space [10]. For the mesh space the difficulty comes from the fact that there is not only one mesh base. But an engineer does not have in general difficulties to choose the best configuration of meshes on a graph, because he knows the physic of the phenomenon he wants to compute. For example, if we consider the schematic figure 2 attached to a simple capacitor: it includes dielectric losses (conductance g) and the capacitor C itself (eventually a self inductance too).

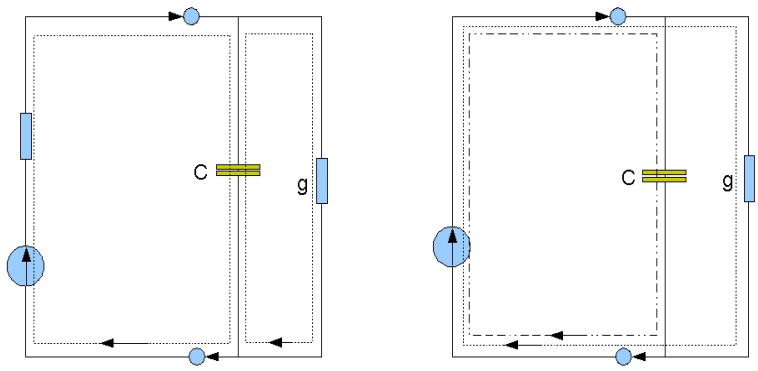

Fig. 2. A network with two various choices of mesh currents.

The first proposal on the left is not the good one. There is no physical flux between g and C in the capacitor. They are separated only by the equivalent electrical schematic of the capacitor, but in fact dielectric and losses properties are detailed ones of the same crystal. So the second choice is the good one, where two meshes (in order to respect the relation 27) start from the generator and go to the capacitor through the dielectric and resistor edges. The same idea must be considered when there is a non linearity in the circuit. A first solution consists in making equality between the edge current and the mesh one at the non linearity level. To reach that, the only condition is to have only one mesh current flowing through this edge. The non linear function is kept and we have only to replace the edge current by the mesh one to resolve the problem. Once the graph is found, the tensorial equation linked to this graph is automatically obtained. And so, after resolving these equations using SCILAB [14] for example, all the mesh currents are resolved and as a consequence, the edge currents and voltage are resolved too. Many publications showing concrete examples were made, these publications are listed in the next paragraph.

The equations are constructed from a graph which translates the engineer's mind. Following the Feynman's idea and diagrams for quantum electrodynamics, the symbolic graph is the input work of this technique. As an example let us imagine a microcontroller source of a radiated field. This field is proportional to the circuit consumption and to its activity. The field creates an emf in an antenna of an embedded receiver. A graph of the problem is presented figure 3.

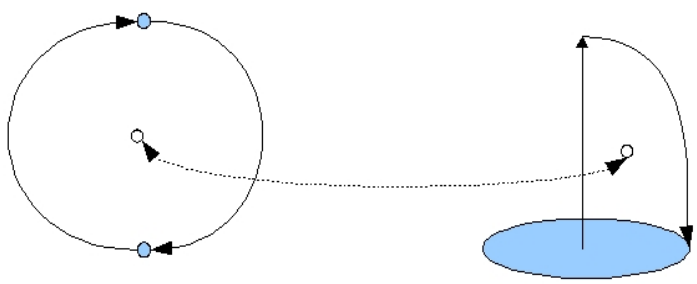

Fig. 3. An example of graph

The simple view of this graph implies an edge space of four dimensions, one string between two meshes (a moment interaction), no charge description at the nodes level. So, two connectivities will be used, one to go form the edge space to the mesh one, and another one to go from the meshes to the moments. We see how a simple graph is immediately translated into tensors organizations. And this graph is in a first step a translation of the engineers understanding of the problem.

The technique is sufficiently intelligent to give to the engineers a method to translate their expertise.

3. Concrete Applications

3.1 Published examples

The first application was made obviously by Gabriel KRON himself, mainly for the simulation of electrical machinery [15]. But Kron had viewed all the possibilities of the method. In the numerical computation of differential equations [16], one can say that Kron was at the origin of modern numerical method for electromagnetism (TLM, etc.). He had shown the multiphysic aspect of the tensorial analysis also [17]. In EMC the technique can be used for problem at the component level [18][19]. The technique allows taking into account macro models developed from experimentations coupled with analytical models extracted from microelectronics. Non linear models for power electronic were developed too. These models give fast and accurate simulation in the time domain to predict the emission of power cells for power supply or electrical machines [20]. But the formalism gives also an efficient context to make theoretical studies on electromagnetism and EMC [21]. Using the capability of the method to reuse functions coming from simulation and experiments, it allows developing complex and hybrid models using previous works made with more classical approaches [22]. And of course, the technique gives many opportunities to compute EMC problem at the system level [23][24][25][26][27][28].

3.2 A simple example

Just to enlighten the readers, we detail a simple example. We consider two loops. The graph of the problem is given in figure 4.

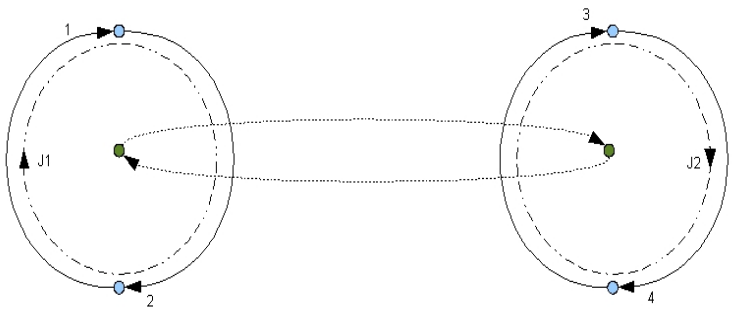

Fig. 4. A simple problem

The first action consists always in drawing a graph. This graph must be an image of the engineer's thinking. The second action is to give numbers to the edges and to choose mesh numbers and circulation. It is possible now to define the edge to mesh connectivity:

$$L = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix} \tag{28}$$

Here we don't have any coupling process between edges. So, the ***Z*** tensor has this organization:

$$Z = \begin{bmatrix} Z1 & 0 & 0 & 0 \\ 0 & Z2 & 0 & 0 \\ 0 & 0 & Z3 & 0 \\ 0 & 0 & 0 & Z4 \end{bmatrix} \tag{29}$$

As can be seen the tensor does not have any extra-diagonal element because no interaction exists between edges. The source vector is a covariant one with the following organization:

$$e = \begin{bmatrix} e_1 & e_2 & e_3 & e_4 \end{bmatrix} \tag{30}$$

The next action consists in applying the edge to mesh transformation making relation (12). We find here:

$$Z' = \begin{bmatrix} Z_1 + Z_2 & 0 \\ 0 & Z_3 + Z_4 \end{bmatrix} \tag{31}$$

To this new tensor ***Z'*** we must add the inductance one which includes the interaction we have here through strings (the interaction is symmetric) between the two meshes. This tensor ***M*** is equal to:

$$M = \begin{bmatrix} sL_1 & -sM_{12} \\ -sM_{21} & sL_2 \end{bmatrix} \tag{32}$$

s is the Laplace operator. The equations of the problem are given by:

$$e_m = \zeta_{mn} J^n \tag{33}$$

With:

$$e_m = e_a L_m^a = \begin{bmatrix} e_1+e_2 & e_3+e_4 \end{bmatrix}$$
$$\zeta_{mn} = Z'_{mn} + M_{mn} \tag{34}$$

Developing (33) gives:

$$\begin{cases} e_1+e_2=\zeta_{11} J^1+\zeta_{12} J^2 \\ e_3+e_4=\zeta_{21} J^1+\zeta_{22} J^2 \end{cases} \tag{35}$$

Computing the inverse of ς gives the solution of the problem, i.e. all the mesh currents. Now if we want to extract from these results the voltage across *Z4* we write:

$$V_4=Z_{44} L_m^4 J^m \tag{36}$$

Z_{44}=Z4. Here we see that no information is lost at all from the edge space.

4. From the mesh space to the Lagrange's operators

In any system it is possible to write the potential energy created by each pair of charges. We have:

$$W_E = \sum_i \frac{1}{2C^{ii}} \left(q^i\right)^2 + \sum_{i,j} \frac{1}{\gamma^{ij}} \left(q^{ij}\right)^2 \tag{37}$$

These energies are stored across each capacitor of the system. q^{ij} is a charge on an edge supplied by two currents *i* and *j*. By definition:

$$q^{ij}=q^i-q^j \tag{38}$$

By replacement of q^{ij} in (37) we obtain:

$$W_E=\sum_i \left\{ \frac{1}{2}\left(\frac{1}{C^{ii}}+\frac{1}{\gamma^{ij}}\right)\left(q^i\right)^2 - \sum_j \frac{1}{2\gamma^{ij}}\left(q^i q^j\right)\right\} \tag{39}$$

From this expression we can construct the U matrix operator of the potential energy (here for 4 elements):

$$U=\begin{bmatrix} \frac{1}{C^{11}}+\frac{1}{\gamma^{12}} & \frac{-1}{\gamma^{12}} \\ \frac{-1}{\gamma^{21}} & \frac{1}{C^{22}}+\frac{1}{\gamma^{21}} \end{bmatrix} \tag{40}$$

The same development can be made for the kinetic energy T ($L_{ij}\dot{I}^i\dot{I}^j$ terms) [29]. The operators obtained like U are the Lagrange's operators, but they are strictly equals to the $\boldsymbol{Z}$ tensor capacitance and inductance parts when it's written in the mesh space. The self modes of the system are given by the equation:

$$\det\left| T^{-1}U-\omega^2\Delta \right| \tag{41}$$

The ω are the pulsation of the modes.

5. A theoretical study

Another advantage of having an equation that represents the system behavior is the possibility to make all kinds of studies that can be made on the base of an equation: experience plans, studies of behaviors, etc. I give here an example [30] of theoretical study that these equations allow to do.

The total power developed on all the system is given by the relation:

$$P_T = V_k I^k \tag{42}$$

V_k can be replaced by its expression and the edge currents by their expression in the mesh currents. One obtains:

$$P_T = e_k K^k_\nu J^\nu + J^\nu L_\nu^{\ k} Z_{kk} L^k_{\ \nu} J^\nu \tag{43}$$

The meshes currents are created by disturbances sources with:

$$J^{\nu} = Y^{\nu\omega} E_{\omega} \tag{44}$$

So:

$$P_T = \left(e_k L^{k}_{\nu} Y^{\nu\omega} + E_{\omega} Y^{\omega\nu} L_{\nu}^{k} Z_{kk} L^{k}_{\nu} Y^{\nu\omega} \right) E_{\omega} \tag{45}$$

One can make in factor the connectivity, the sources and the inverse of the metric:

$$P_T = \left\{ 1 + \frac{E_{\omega}}{e_k} Y^{\omega\nu} L_{\nu}^{k} Z_{kk} \right\} e_k L^{k}_{\nu} Y^{\nu\omega} E_{\omega} \tag{46}$$

The quadruple product ***e**L**YE*** is the power delivered from the sources to the edges: P_b. Finally:

$$\frac{P_T}{P_b} = \left\{ 1 + \frac{E_{\omega}}{e_k} Y^{\omega\nu} L_{\nu}^{k} Z_{kk} \right\} \tag{47}$$

The triple product ***YLZ*** has the dimension of a transfer function which transports the energy from the external layer to the component. Writing:

$$S_k^{\omega} = Y^{\omega\nu} L_{\nu}^{k} Z_{kk} \tag{48}$$

One obtains:

$$\frac{P_T}{P_b} = \left\{ 1 + \frac{E_{\omega}}{e_k} S_k^{\omega} \right\} \tag{49}$$

The added energy that is an image of the EMC risk to disturb the system is the term:

$$R = \frac{E_{\omega}}{e_k} S_k^{\omega} \tag{50}$$

This equation is fundamental for the system EMC. It implies that if the sources ***E*** are higher, the risk is higher. If the transfer function ***S*** is higher, the risk is higher too. And the lower the susceptibility level ***e*** of the component, the higher the risk *R* of disturbances. This equation

demonstrates that to reduce the risk of disturbances of the system, it's better to (in the order): reduce the source, then if it is not possible, to reduce *S*, and finally to increase ***e***. The study of equation (50) shows how the system stability changes when these parameters change.

8. To connect primitive networks

We reach here the ultimate process of our technique: the method to connect various patterns coming from various tools to compute the complete graph of a system. The connections can start from the nodes space to end at the moment one. One shows here the conducted connection of four primitive networks. The simplest primitive network that we can imagine is a simple edge with two nodes. One considers four primitive networks shown figure 5.

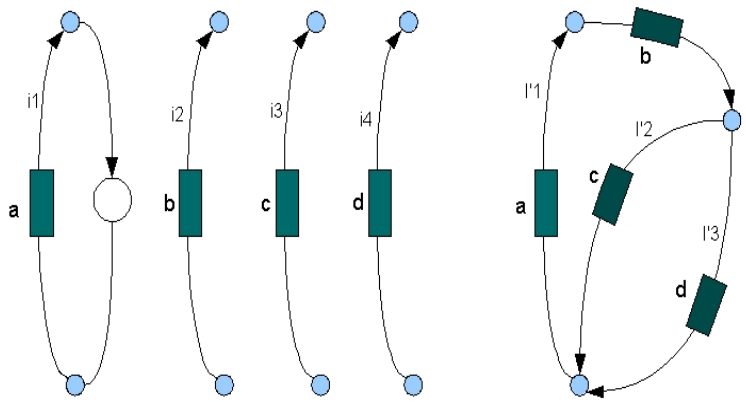

Fig. 5. Connection of primitive networks

Each primitive network is characterized by its own current I^k that can be activated using an emf applied on the primitive network (see first primitive network figure 5).

When the primitive networks are connected, the new network has its own edge currents. The transformation matrix is defined by writing the relations between the primitive network currents and the new network currents. Here we have:

$$\begin{cases} i^1 = i'^1 \\ i^2 = i'^1 \\ i^3 = i'^2 \\ i^4 = i'^3 \end{cases} \tag{51}$$

From these relations we obtain a transformation matrix:

$$F = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{52}$$

Computing the next equation:

$$Z_{cd} = F_c^{\ a} Z_{ab} F^b_{\ d} \tag{53}$$

Where Z_{ab} is the tensor made with the four primitive networks (each diagonal element is one primitive network impedance), and Z_{cd} the impedance tensor of the network constituted. This is an example of direct connection involving conduction interaction. Another possibility to connect various networks (it always means various patterns modeling various elements of a system) is to create magnetic coupling between meshes of two different networks. In this case, the interaction creates extra-diagonal elements in the M tensor. The same approach to connect various networks using the moment vectors of these networks can be used. In this case, equation (20) is used to create the special “mutual” interaction added in the tensor M of the global system constructed.

9. Future works

Immunity at the component level still has to be improved and clarified. Connectivity between the fields and its boundary conditions (currents, voltages) can be explored in a better way. These two thematics are part of two thesis in progress. Another research orientation for future works will concern coupling between physics, mechanics and thermal effects.

10. Conclusion

I have tried to show the entire possibilities offered by the tensorial analysis of network applied to the large system EMC problems. In comparison with the original Kron’s method, I propose to add the moment space which allows reducing the problem where there is many networks present. The definition of $\boldsymbol{Z}$ as a generalized non linear operator allows merging time and frequency domain in the same problem using Z_{ii} applied to a current like a convolution product of an impulse response (obtained using a Fourier transform of the frequency transfer function computed with a moment method for example) with an

unknown current element that has to be solved [10]. It is possible to extend the tensorial equations for a multiphysic problem, coupling mechanics, electronics and fluid physics [17]. The link with statistical models needed for the component susceptibility was shown too [10] in order to complete the chain from the structures to the components. The method has proven its efficiency by the past for electrical machines and electro mechanics problems. It can give the same efficiency for the exhaustive problem of EMC on large systems.

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