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Electrical Field Modified by Injected Space Charge in Blade–Plate Configuration

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This paper presents the numerical resolution of the problem defined by corona discharge in a gas at the edge of a blade facing a plate. We use the finite element method to solve the Poisson equation and the method of characteristics to determine the distribution of charge density (charge conservation equation). The main point is the use of a structured mesh which is redefined at each iteration step to avoid artificial numerical diffusion when solving the charge conservation equation. This algorithm is applied using an injection law giving the charge density at the injector as a function of the local electric field.

Index Terms—Finite elements, injection through corona discharge, method of characteristics (MOC), space charge effect.

I. INTRODUCTION

THE electric corona discharge in gas is commonly used in many engineering devices and processes. It is generated, for example, in order to electrically charge small particles or drops or surfaces of larger objects, to initiate chemical reactions, etc. The injected space charge of density ρ modifies the distribution of electrical field \mathbf{E} which, in turn, controls the distribution of space charge. In some applications, like electrostatic precipitation, several electrodes are injecting charge carriers in an insulating medium, and, often, these injecting electrodes are not wires. The discharges then occur in restricted zones of the electrodes where the electric field takes high values.

There are not yet reliable and accurate numerical models for the computer simulation of this phenomenon. Previous attempts to obtain numerical solutions are based on the classical methods to solve the equations of Poisson and of charge conservation. These attempts using structured or unstructured mesh were limited to two-dimensional (2-D) configurations where the injecting electrodes are wires characterized by an approximately constant electric field at their surface. The problem is much more difficult when the injecting electrode is sharp (for example, a blade or a needle). For a high enough voltage applied between the electrodes, a corona discharge is created near the needle tip or the blade edge. In these cases, it is not possible to prescribe *a priori* the value of the electric field at the injecting electrode as it is commonly done for injecting wires. Recent attempts [1], [2] propose to use some sort of injection law to account for the resulting effect of the corona discharge, i.e., of the complex ionizing phenomena occurring in the bi-ionized layer of small extent close to the zone of high curvature of the electrode. In our previous study [9], the injected charge was fixed *a priori*. We focus here on the use of such an injection law combined with the redefinition of the

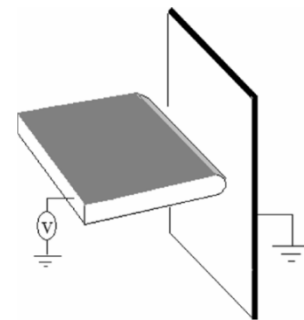


Fig. 1. Schematic view of the two electrodes configuration blade–plane.

mesh at each iteration step to obtain a satisfactory solution of the problem in the 2-D blade–plate configuration.

II. MATHEMATICAL FORMULATION

Most numerical studies to determine the space charge density and the electrical field were performed in 2-D electrodes configurations, for instance, for wire-duct electrostatic precipitators [3]–[5] and axi-symmetric point-plane configuration [1] (a few works considered real three-dimensional (3-D) configurations [6], [7]). We consider here the two-dimensional (2-D) problem of a blade facing a plate as sketched in Fig. 1, with the radius of curvature r of the blade being small compared with the distance d between blade and plate. In practice, the domain will be bounded by a field line of the harmonic field at a distance high enough from the blade edge (Fig. 2). The distributions of electric potential V and charge density ρ between the two electrodes are governed by the coupled Poisson equation (1) and charge conservation (2)

$$\nabla^2 V = -\rho/\varepsilon \quad (1)$$

$$\nabla \cdot \mathbf{J} = 0 \quad (2)$$

where the current density is $\mathbf{J} = \sigma \mathbf{E} + \rho(\mathbf{u} + K\mathbf{E}) - D\nabla\rho$; here, ε denotes the medium permittivity, \mathbf{E} denotes the electrical field $\mathbf{E} = -\nabla V$, σ the medium conductivity, K denotes the

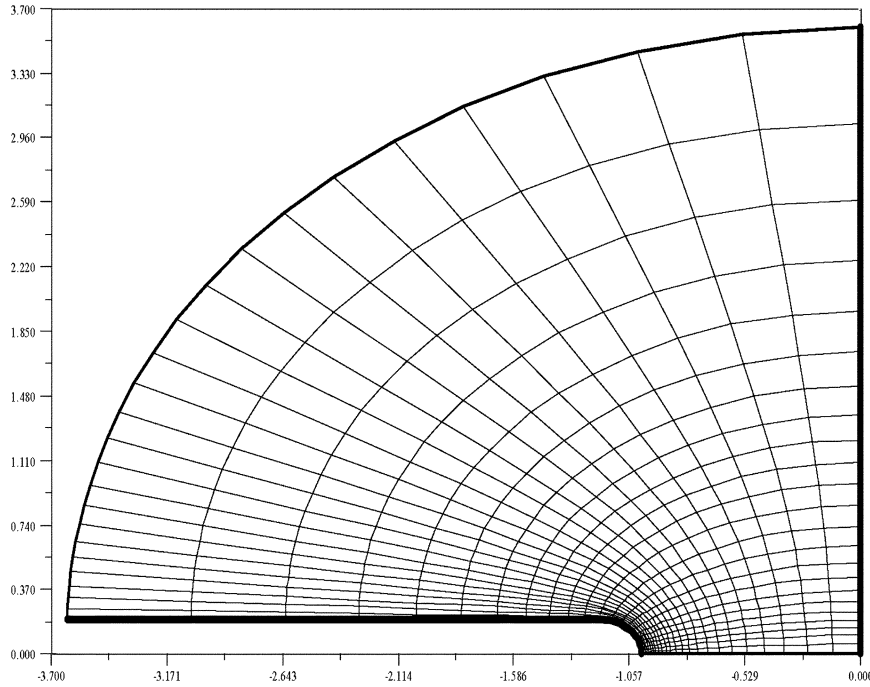


Fig. 2. Structured mesh for the finite element method (FEM) and method of characteristics (MOC) .

mobility of charge carriers, and \mathbf{u} denotes the velocity field of the medium. Assuming the medium conductivity to be zero and the diffusion and convection currents to be negligible compared with the drift current, the expression of \mathbf{J} simplifies into

$$\mathbf{J} = K\rho\mathbf{E}.$$

The boundary conditions associated with the Poisson equation (1) are of Dirichlet type ($V = Cst$) on the electrodes and of Neumann type ($\partial V/\partial n = 0$) on the axis of symmetry and on the outer boundary (field line). From the mathematical viewpoint, only one boundary condition concerning ρ is associated with (3), and it has been shown that the correct condition consists in prescribing the charge density at the injector [2]. Such a condition was not retained in most of the past works, with the authors using a condition on the electrical field at the injecting electrode. These authors followed the assumption of Kaptzov stating that, above the inception voltage V_{th} of corona discharge, the field distribution near the injecting electrode remains unchanged and equal to the one existing at threshold V_{th} . In practice, this proposition is good only for wires as injecting electrodes, the constant electrical field at the injector being given by Peek law [7].

In the case of blades and needles, it is not possible to prescribe *a priori* the field distribution on the injecting electrode. In a first attempt [2], we worked with an imposed charge distribution at the blade, near the edge (rectangular and Gaussian distributions). The corona discharge occurs at the edge of the blade where the field takes high values and generates a bi-ionized region. This region can be taken into account in an indirect way by introducing an injection law (the intricate physical phenomena in this region being ignored) giving the value of the space charge density which leaves the bi-ionized zone. In this

approximation, the density of injected charge ρ_{inj} at a point M of the injector is related to the local value of the electrical field $E(M)$ at this point [2]

$$\rho_{inj} \cdot (M) = f(E(M), E_{th}) \quad (3)$$

where E_{th} is the threshold field for corona inception and the function $f \neq 0$ only if $E(M) > E_{th}$. This function f increases very sharply with $E(M)$ so that the field resulting from the injection will remain close to E_{th} , thus approximately fulfilling the Kaptzov approximation. In practice, we used the linearized expression

$$\rho_{inj} \cdot (M) = A * [E(M) - E_{th}] \quad (4)$$

where A takes large values.

Taking the applied voltage V_{appl} as the reference for the electric potential and V_{appl}/d , $\varepsilon V_{appl}/d^2$, and $K\varepsilon V_{appl}^2/d^3$ for the field, the charge and the current densities respectively, (1) and (2) lead to the following nondimensional equations:

$$\nabla^2 V = -\rho \quad (5)$$

$$\mathbf{E} \cdot \nabla \rho = -\rho^2. \quad (6)$$

The boundary conditions then become

$$V = 1, \quad \rho_{inj} \cdot (M) = A * [E(M) - E_{th}] \text{ on the blade} \quad (7)$$

$$V = 0, \quad \text{on the plate} \quad (8)$$

$$\partial V/\partial n = 0, \quad \text{on the axis and on the outer boundary.} \quad (9)$$

With these conditions for the potential and the charge density on the electrodes, the problem is mathematically well posed.

III. NUMERICAL TECHNIQUE

The Poisson equation can be solved by the finite difference method FDM [5] or the charge simulation method CSM [8]. As in most works [2]–[4], [6], [7], this equation is solved here by the finite element method (FEM). To solve the equation of conservation of charge, the method of characteristics (MOC) was used in a lot of studies [2], [5], [8], [9], [11], but some authors [4], [7] used the finite volume method FVM (donor cell). Apart from a few works using unstructured mesh [4], most authors worked with a structured mesh [6], [10], [11]. In a first study [9], we used a structured mesh and Poisson equation was solved by the FEM. We tested the two numerical methods FVM and MOC to determine the distribution of space charge between blade and plate. This study showed that the MOC is better than the FVM, because FVM gives a local minimum in the current density on the plate collector; therefore, only the MOC is used in this work with a structured mesh for solving the problem.

The two coupled equations are solved by successive approximations until convergence. The space charge ρ being known, the FEM with triangular elements of first order is used to determine the potential distribution. Then, with the new field distribution, the MOC is used to determine the charge density ρ at the nodes of the structured mesh. For strong injections, it is necessary to introduce an under-relaxation between successive approximations to have convergence of the iterative process [9]. The structured mesh (Fig. 2) is redefined at each approximation in order to integrate (6) along the field lines and, therefore, to avoid numerical diffusion. The nodes of this mesh are the points of intersection between electric field lines and the equipotential lines; the initial structured mesh is defined from the harmonic solution of Laplace [9].

IV. MESH REDEFINITION

When the nodes remain the same during the whole iteration, independently of the structured or unstructured character of the mesh, the solution for a discontinuous distribution of the charge density on the injector (blade) gives a continuous distribution of current density on the plate. This does not arise from the diffusion of ions [discarded in the models (5) and (6)], but from the so-called numerical diffusion. If the different sets of nodes lay on field lines, integration of (6) along these lines then saves the discontinuous nature of charge density. This led us to redefine the mesh at each iteration step.

With a new distribution of electric potential calculated by FEM, new field lines are determined and the new nodes are defined on them from prescribed values of the potential. To determine these new field lines issuing from points on the blade, several techniques have been tested; the most satisfactory one consists in approximating the local new distribution of potential by a linear function defined through a least square procedure (with weights) involving the six nodes of the two neighboring quadrangles (this gives far better results than the linear form on the triangular elements of the mesh).

The crucial step is to select the nodes on the blade which are the initial points of the new field lines. In the case where these points are fixed and distributed smoothly on the blade, the lines are found to be packed in the charge free region just

outside the charged zone. This is due to the contrast between the strong divergence of field lines in the region of strong charge density and the zero divergence in the charge free zone (Fig. 3). Therefore, the nodes must be selected very close to each other in the injecting part of the acute extremity of the blade in order to have a rather regular spacing of the field lines in the major part of the domain (Fig. 3).

We investigated two techniques to determine the nodes on the blade which act as the starting points of the new field lines. The first technique is a sort of shooting method: The node location on the blade is modified in order to have the position at which the field line intersects the plate as close as possible to predetermined points (defined by the Laplacian field). In order to minimize the computation time, the field lines are determined only once at each iteration step and the difference between the new intersection on the plate and the target is used to redefine the starting node of the next iteration. Extra constraints on the starting nodes displacement and an under-relaxation damp the oscillations of these nodes and lead to the convergence of the iterative process.

In the second technique, one determines the starting nodes on the blade in a global way. The width of the injecting zone of the blade is determined by the condition $E(M) = E_{th}$ deduced from the injection law (7). The number of starting nodes is kept constant in each of the charged and charge free zones and their relative distribution on each part of the blade is always the same. Unfortunately, this technique of starting nodes redefinition leads to oscillations of important amplitude during the iterative process which does not converge. These oscillations have a simple “physical” origin: If the electric field on the blade takes values higher at iteration step k than at step $k - 1$, the width of injecting zone on the blade is increased as well as the injected space charge ρ_{inj} . Then, the field lines will diverge more, the space charge effect will be stronger and will induce a reduction of the field on the blade; this, in turn, will tend to reduce the space charge. The introduction of an under-relaxation in the redefinition of starting nodes has not led to convergence of the iteration.

V. RESULTS

By adequately choosing the various parameters influencing the rate of convergence, a good convergence of the iterative process is obtained even for rather large values of the injection parameter (A up to 100): For a mesh with $\cong 2500$ nodes, after about 90 iterations, the average error in the charge calculation between two successive iterations is of the order 10^{-7} . By using at each iteration step an extra interpolation between the previous mesh and the one determined from the new potential distribution, the computation time could be reduced (70 iterations instead of 90), but this implies an adequate choice of the parameters (factors of interpolation). The MOC technique with redefinition of the structured mesh gives satisfactory results: The current flowing from the blade to the plate is constant to within 0.4%. Figs. 3–5 are relative to a blade of hyperbolic cross-section with a nondimensional radius of curvature $r = 0.02$ at the edge. Fig. 3 shows the field lines in the central zone for a strong charge injection (the nondimensional charge density at the blade $\rho_{inj} \leq 24$). The coulombic repulsion results

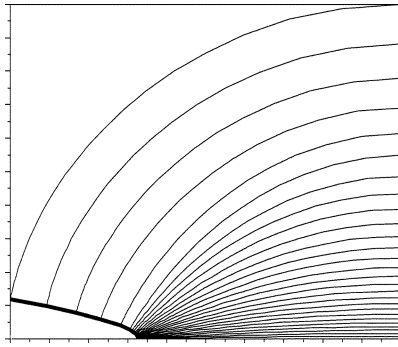


Fig. 3. Field lines in the case $V/V_{th} = 5$ corresponding to a strong injection (blade of hyperbolic cross section with a nondimensional radius of curvature $r = 0.02$).

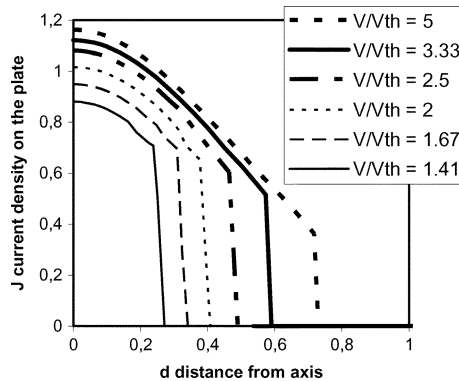


Fig. 4. Current density on the plate versus distance from the symmetry axis for various values of the applied voltage (the injection law (4) is used with $A = 50$) for hyperbolic blade–plate configuration $r = 0.02$.

in a strong spreading of the field lines near the blade edge (note that the starting nodes of the field lines are very close).

The existence of a threshold field for corona discharge induces a quasidiscontinuity in the charge density injected by the blade [despite the fact that (4) mathematically implies a continuity]. This explains the sudden drop of current density on the collecting plate (Fig. 4) consistent with the experimental observations. Fig. 4 shows that the magnitude of the injected space charge and its spreading are increasing with the applied voltage V . The current–voltage characteristic curve is presented in Fig. 5 using the nondimensional total current. Below the inception voltage V_{th} , there is no corona effect and $I = 0$. For high applied voltages, there is a tendency to saturation of the nondimensional current corresponding to an asymptotic law $I \cong 1.5K\epsilon V^2/d^2$ for the total current per unit length.

The same algorithm has been applied successfully to the configuration of planar blade with cylindrical edge. Retaining the same injection law, we get results for the current density on the plate fully similar to the ones of Fig. 4. The only difference between the application of the algorithm in the two cases (planar and hyperbolic blades) is the choice of the parameters of nodes distributions and of the under-relaxation factors.

VI. CONCLUSION

We obtain the numerical solution of the problem of injected space charge in the blade–plate configuration by the application of the FEM to determine the potential distribution and of the MOC to determine the distribution of charge density and by redefining

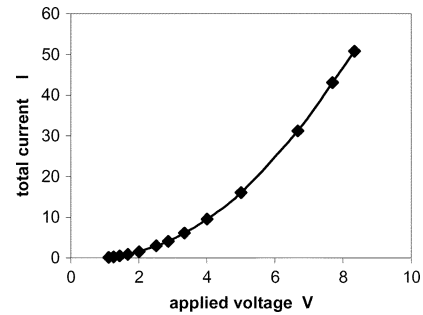


Fig. 5. Current–voltage characteristic using for hyperbolic blade–plate configuration $r = 0.02$.

the mesh at each step of the successive approximations scheme. This numerical solution, obtained by using an injection law, very satisfactorily accounts for the physical observations, particularly for the quasidiscontinuity of the current density on the plate. The technique of successive approximations converges rapidly when the charge density distribution ρ_{inj} is given on the blade. The convergence is slower when imposing an injection law which modifies ρ_{inj} at each iteration step, combined with individually redefining the starting nodes on the blade.

The algorithm developed with a blade of hyperbolic shape has been successfully applied for a planar blade with cylindrical edge. The most difficult point in the mesh redefinition is to determine on the blade the nodes which are the initial points of the field lines. The global redefinition of the starting nodes has to be refined in order to have a convergence of the iteration and, maybe, a decrease of the computation time.

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