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Connection between Effective Chiral Lagrangians and Landau-Migdal Fermi Liquid Theory of Nuclear Matter

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This lecture is based in part on work done in collaboration with G.E. Brown and on a recent paper co-authored with Bengt Friman. It deals with making a connection between effective chiral Lagrangians – low-energy effective theory of QCD – and Landau Fermi liquid theory extended by Migdal to nuclear matter. I discuss how to obtain a link between observables in relativistic heavy-ion processes and low-energy spectroscopic data, giving a new insight into how chiral dynamics manifests itself in nuclear systems.

I. INTRODUCTION

It is generally accepted that the physics of low-energy strong interactions relevant for nuclear physics is governed by effective theories based on chiral Lagrangians. At long-wavelength limit, the strategy of implementing chiral Lagrangian field theory is in principle known and has been successfully applied to low-energy interactions in the pseudo-Goldstone (i.e., pion) sector. Now how to implement such a strategy in nuclear dynamics that involves many-body interactions is an entirely different matter and has met with little success. This is an important issue however for ultimately understanding what happens in relativistic heavy-ion physics looking for extreme states of hadronic matter since the experiments looking for such states will sample all ranges of strong interactions, from low to high energy and hence would require highly nonperturbative to perturbative regimes of QCD. This means that the theory will have to explain correctly low-energy nuclear processes in order to make sense in the extreme conditions probed in relativistic heavy-ion collisions.

In this lecture I would like to describe some of the recent developments along this direction [1–4].

Let me start with what I consider to be an exciting new development in nuclear physics. It was shown in recent publications by Li, Ko and Brown [5] that the dilepton production data of CERES [6] and HELIOS-3 [7] can be simply and quantitatively understood if the mass of the vector mesons ρ and ω scales in dense and/or hot medium according to the scaling (BR scaling) proposed by Brown and Rho [1]. That the vector mesons “shed” their masses as the density (or temperature) of the matter increases is expected in an intuitive interpretation of the interplay of the condensation of quark-antiquark pairs and the dynamical generation of light-quark hadron masses and is

in fact corroborated by QCD sum rules [8,9] and model calculations [10]. Thus, the dilepton data are consistent with the most conspicuous prediction of BR scaling (see [11] for other mechanisms). The proposal of [1], however, goes further than this and makes a statement on the relation between the scaling of meson masses and that of baryon masses:

$$\frac{m_M^*}{m_M} \approx \sqrt{\frac{g_A}{g_A^*} \frac{m_B^*}{m_B}} \approx \frac{f_\pi^*}{f_\pi} \equiv \Phi(\rho) \quad (1)$$

where the subscript M stands for light-quark non-Goldstone mesons, B for light-quark baryons, g_A the axial-current coupling constant and f_π the pion decay constant. The star denotes an in-medium quantity. (Although temperature effects can also be discussed in a similar way, we will be primarily interested in density effects in this paper.)

Two immediate questions are raised in these developments: Firstly, is there evidence that the baryon mass scaling and the meson mass scaling are related as implied by the chiral Lagrangian? Secondly, we know from the Walecka model of nuclear matter [12] that the “scalar mass” of the nucleon drops as a function of density and that this reduction of the nucleon mass has significant consequences on nuclear spectroscopy and the static properties of nuclei. The question is: Is BR scaling related to the “conventional” mechanism for the reduction of the nucleon mass in nuclear matter and if so, how does it manifest itself in low-energy nuclear properties? Put differently, can a single chiral Lagrangian explain at the same time low-energy nuclear processes studied in a conventional way since a long time and the high-energy processes to be probed in heavy-ion collisions?

The aim of this lecture is to show, based on recent work [4,13,2], that the connection between the meson and baryon scalings can be made using the Landau-Migdal theory of nuclei and nuclear matter. See [14] for a related discussion. Our starting point is the effective chiral Lagrangian used in [1] where the scale anomaly of QCD is incorporated and baryons arise as skyrmions. This theory is mapped onto an effective meson-baryon chiral Lagrangian. We establish the relation between chiral and Walecka mean fields in medium as suggested in [2] and then invoke the Galilei invariance argument of Landau, which relates the nucleon effective mass to the Landau Fermi liquid parameters. Thus, we establish a relation between the parameters in eq. (1) and the Landau parameters. We discuss how this relation can be tested

with the effective g_A^* and the gyromagnetic ratios δg_l in nuclear matter. This then supplies a novel relation between the scaled masses, which may be reflected in the spectrum of dileptons produced in relativistic heavy-ion collisions, and low-energy spectroscopic information, g_A^* and δg_l . It also supplies an indirect and nontrivial connection between quantities figuring in chiral Lagrangians of QCD and those appearing in familiar many-body theory.

In order to avoid unnecessary complications we shall use the nonrelativistic approach to Landau Fermi liquid theory, referring to results obtained in the relativistic formulation [15,16] where appropriate.

II. RENORMALIZATION GROUP FIXED POINTS AND CHIRAL LAGRANGIANS

Before I enter into the main topic of this lecture, let me mention one important issue here. Given an effective Lagrangian that describes QCD in the low energy nonperturbative regime, how does one use it to describe nuclear many-body processes? The answer to this question is not known at the moment. In fact it is not even clear whether it is meaningful or necessary to ask such a question. Nonetheless some of us have been asking this question since some time.

Let me describe briefly how I understand this problem.

Let me start by assuming that Walecka mean field theory of nuclear matter [12] is correct. Now to go from chiral Lagrangians to Walecka theory, thus far, two approaches have been proposed. One approach is to look at nuclear matter as a “chiral liquid” resulting as a solitonic matter from an effective chiral action [17]. I do not have much to add to this approach; as it stands, no quantitative insight can be gained from it. Instead I will focus on the second approach which is to use a chiral Lagrangian endowed with four-Fermi interactions having the quantum numbers of an isoscalar vector (say, ω) and isoscalar scalar (say, σ) exchanges treated *in mean field with BR scaling* [2,18,19]. For $N = Z$ nuclear matter, pions average out (see later however for their important role). I will take this as an (heuristic) evidence that effective chiral Lagrangians can be mapped to Walecka theory. The mean fields provide the Fermi surface in the usual way. As was shown in [4] and will be discussed shortly, this theory can then be connected to Landau Fermi liquid theory of nuclear matter [21,22].

Now given the effective Lagrangian with the Fermi surface, one can then apply the renormalization group arguments developed in condensed matter physics [23] in the following way. In the presence of a Fermi surface, the four-Fermi interactions yield the Landau parameters of quasiparticle interactions as renormalization group fixed points in the sense that the four-Fermi interactions are

marginal, i.e., with vanishing β functions. As in condensed matter systems, one might have “BCS”-type four-Fermi interactions that can become marginally relevant (e.g., kaon condensation) but here I will not deal with this possibility.

There is one basic difference in nuclear matter from condensed matter systems and that is that in QCD at quantum level, there is a scale anomaly. This scale anomaly associated with the fact that the energy-momentum tensor has a nonvanishing trace in QCD at quantum level will give rise to an anomalous dimension to the scalar field associated with the trace anomaly. This suggests that an effective chiral Lagrangian with the scale anomaly incorporated (as used in BR scaling) can be mapped to Walecka-type theory with the scalar field σ having an anomalous dimension. I believe that this is the mechanism behind the successful mean-field chiral Lagrangian theory of nuclei and nuclear matter by Furnstahl et al [20] who find phenomenologically a large anomalous dimension $d_a \approx 1.7$ for the σ field for which the incompressibility modulus comes out to be $K \sim 200$ MeV and many-body forces are strongly suppressed at the saturation point [24]. This turns out to be what is needed for a mean field theory of the chiral phase transition proposed in [25].

In the rest of the lecture, I discuss how Landau theory can be incorporated into this chiral Lagrangian scheme.

III. BR SCALING

The BR scaling relation (1) that relates the dropping of light-quark non-Goldstone-boson masses to that of the nucleon mass which in turn is related to that of the pion decay constant was first derived by incorporating the trace anomaly of QCD into an effective chiral Lagrangian. The basic idea can be summarized as follows. We wish to write an effective chiral Lagrangian which at mean-field level reproduces the quantum trace anomaly while including higher chiral order effects relevant for nuclear dynamics. To do this, we write the effective Lagrangian in two parts

$$\mathcal{L} = \mathcal{L}_{inv} + \mathcal{L}_{sb} \quad (2)$$

where \mathcal{L}_{inv} is the scale-invariant part and \mathcal{L}_{sb} the scale-breaking part of the effective Lagrangian. We introduce the *chiral-singlet* scalar field χ , as an interpolating field for $\text{Tr } G^2$,

$$\theta_\mu^\mu = \frac{\beta(g)}{2g} \text{Tr } G_{\mu\nu} G^{\mu\nu} \equiv \chi^4, \quad (3)$$

where we have dropped the quark mass term (here we consider the chiral limit). The simplest possible invariant piece of the Lagrangian then takes the form

$$\mathcal{L}_{inv} = \frac{f_\pi}{4} \left(\frac{\chi^2}{\chi_0^2} \right) \text{Tr} (\partial_\mu U \partial^\mu U^\dagger) + \frac{1}{32g^2} \text{Tr} [U^\dagger \partial_\mu U, U^\dagger \partial_\nu U]^2 + \dots \quad (4)$$

where χ_0 is a number which we define to be the expectation value of χ in matter-free vacuum and the ellipsis stands for other-scale invariant terms including the kinetic energy term for the χ field. Note that this is the simplest possible form based on the most economical assumption. One could perhaps write much more complicated and yet scale-invariant forms using the same set of fields but invoking different assumptions, and thus obtain a different type of scaling. Experiments will tell us which one is the right form.

As for the scale-breaking term \mathcal{L}_{sb} , we assume that it contains just the terms needed to reproduce the full trace anomaly. We add other scale-invariant terms representing higher chiral order terms to assure the correct vacuum potential which we shall call $V(\chi, U)$. Fortunately all we need to know about the potential V is that it contains a source for the χ mass term and that, for a given density, it attains its minimum at $\chi^* = \langle \chi \rangle^*$ in the sense of the Coleman-Weinberg mechanism [26]. (We will return later to what this quantity χ^* represents physically.)

The fact that the vacuum expectation value is obtained by minimizing the potential, which contains a scale-breaking term, implies that we are treating the breaking of the scale invariance as a spontaneous symmetry breaking. It is well-known that the spontaneous breaking of the scale symmetry occurs only if it is explicitly broken, since otherwise the potential would be flat [27]. Given the ground state characterized by χ^* which is fixed by the anomaly, we then shift the field in (2)

$$\chi(x) = \chi'(x) + \chi^*. \quad (5)$$

After shifting, we still have the scale-invariant and scale-breaking pieces although the manifest invariance is lost as is the case with *all* spontaneously broken symmetries. The low-energy physics for the scaling we are interested in is lodged in the former. Since the theory contains two parameters, f_π and g , we define

$$\begin{aligned} f_\pi^* &= f_\pi \frac{\chi^*}{\chi_0}, \\ g^* &= g. \end{aligned} \quad (6)$$

The second relation follows since the Skyrme quartic term in (4) is scale-invariant by itself. I will argue later that in the baryon sector there is an important radiative correction – absent in the meson sector – which modifies this scaling behavior. This allows us to redefine the parameters that appear in the chiral Lagrangian in terms of the “starred” parameters f_π^* and g^* . Since the KSRF relation [28] is an exact low-energy theorem as shown

by Harada, Kugo and Yamawaki [29], it is reasonable to assume that it holds also in medium. This leads to

$$m_V^*/m_V \approx \frac{f_\pi^* g^*}{f_\pi g} \approx \frac{f_\pi^*}{f_\pi} \equiv \Phi(\rho) < 1 \quad \text{for } \rho \neq 0 \quad (7)$$

where the subscript V stands for ρ or ω meson. Similarly the mass of the scalar field is reduced

$$m_\sigma^*/m_\sigma \approx \Phi(\rho). \quad (8)$$

Here we denote the relevant scalar field by the usual notation σ for reasons given below.

Now in order to find the scaling behavior of the nucleon mass, we use the fact that the nucleon arises as a soliton (skyrmion) from the effective chiral Lagrangian as in the free-space. The soliton mass goes like

$$m_S \sim f_\pi/g. \quad (9)$$

If one assumes that by the same token the coupling constant g in the soliton sector is not modified in the medium, eq. (9) implies that the nucleon mass is also proportional to f_π^* ,

$$m_N^*/m_N \sim \Phi(\rho). \quad (10)$$

However there is a caveat to this. When it comes to the nucleon effective mass, there is one important non-mean-field effect of short range that is known to be important. This is an intrinsically quantum effect that cannot be accounted for in low orders of the chiral expansion, namely the mechanism that quenches the axial-current coupling constant g_A in nuclear matter. This effect is closely related to the Landau-Migdal interaction in the spin-isospin channel g'_0 (involving Δ -hole excitations) as discussed in [3,30]. The axial-vector coupling constant of the skyrmion is governed by coefficient g of the Skyrme quartic term. This implies that in the baryon sector, the mean-field argument, which is valid in the mesonic sector, needs to be modified. This is reminiscent of the deviation in the nucleon electromagnetic form factor from the vector dominance model which works very well for non-anomalous processes involving mesons. These two phenomena may be related.

As shown in [1,31], a more accurate expression, at least for densities up to $\rho \sim \rho_0$, is¹

$$m_N^*/m_N \approx \Phi(\rho) \sqrt{\frac{g_A^*}{g_A}}. \quad (11)$$

¹This was derived using the scaling behavior of the Skyrme quartic Lagrangian and the relation between g_A and the coefficient g . Although this relation is justified strictly at the large N_c limit (where N_c is the number of colors), we think that it is generic and will emerge in any chiral model that has the correct symmetries.

This relation will be used later to deduce a formula for g_A^* in nuclear matter. Beyond $\rho = \rho_0$, we expect that g_A^* remains constant ($g_A^* = 1$) and that Φ scaling takes over except near the chiral phase transition at which the coupling constant g will fall according to the “vector limit.” [3]

A. What is χ^* ?

The χ field interpolating as χ^4 for the dimension-4 field $\text{Tr}G_{\mu\nu}^2$ may be dominated by a scalar glueball field, which perhaps could be identified with the $f_J(1710)$ seen in lattice calculations [32]. However, for the scaling we are discussing which is an intrinsically low-energy property, this is too high in energy scale. In the effective Lagrangian (4), such a heavy degree of freedom should not appear explicitly. The only reasonable interpretation is that the χ field has two components,

$$\chi = \chi_h + \chi_l \quad (12)$$

corresponding to high (h) and low (l) mass excitations, and that the high mass (glueball) component χ_h is integrated out. The “vacuum” expectation value we are interested in is therefore $\langle \chi_l \rangle^*$. The corresponding fluctuation must interpolate 2π , 4π etc. excitations as discussed in [3] and it is this field denoted by σ that becomes the dilaton degenerate with the pion at the chiral phase transition as suggested by Weinberg’s mended symmetry [33]. It is also this component which plays an essential role in the relation between chiral Lagrangians and the Walecka model [18,19,2]. This procedure may also be justified by a phenomenological instanton model anchored in QCD [25].

For a more physical interpretation and a detailed discussion on the separation (12), see Adami and Brown [34]. A somewhat different separation is advocated by Furnstahl et al. in [20].

B. Four-Fermi interactions

In order to make contact with many-body theory of nuclear matter, we reinterpret the BR scaling in terms of a baryon chiral Lagrangian in the relativistic baryon formalism. There is a problem with chiral counting in this formalism² but our argument will be made at mean-field order as in [2].

²As we know from the work of Gasser, Sainio and Svarc [35], the relativistic formulation of baryon chiral perturbation theory requires a special care in assuring a correct chiral counting. What we will find below is that in order to get to the correct formulation from the point of view of Landau

The Lagrangian contains the usual pionic piece \mathcal{L}_π , the pion-baryon interaction $\mathcal{L}_{N\pi}$ and the four-Fermi contact interactions

$$\mathcal{L}_4 = \sum_\alpha \frac{C_\alpha^2}{2} (\bar{N}\Gamma_\alpha N)(\bar{N}\Gamma^\alpha N) \quad (13)$$

where the Γ^α ’s are Lorentz covariant quantities – including derivatives – that have the correct chiral properties. The leading chiral order four-Fermi contact interactions relevant for the scaling masses are of the form

$$\mathcal{L}_4^{(\delta)} = \frac{C_\sigma^2}{2} (\bar{N}N\bar{N}N) - \frac{C_\omega^2}{2} (\bar{N}\gamma_\mu N\bar{N}\gamma^\mu N). \quad (14)$$

As indicated by our choice of notation, the first term can be thought of as arising when a massive isoscalar scalar meson (say, σ) is integrated out and similarly for the second term involving a massive isoscalar vector meson (say, ω). Consequently, we can make the identification

$$C_\sigma^2 = \frac{g_\sigma^2}{m_\sigma^2}, \quad C_\omega^2 = \frac{g_\omega^2}{m_\omega^2}. \quad (15)$$

The four-Fermi interaction involving the ρ meson quantum number will be introduced below, when we consider the electromagnetic currents. As is well known [18,2], the first four-Fermi interaction in (14) shifts the nucleon mass in matter,

$$m_N^\sigma = m_N - C_\sigma^2 \langle \bar{N}N \rangle. \quad (16)$$

In [2] it was shown that this shifted nucleon mass scales the same way as the vector and scalar mesons

$$\frac{m_V^*}{m_V} \approx \frac{m_\sigma^*}{m_\sigma} \approx \frac{m_N^\sigma}{m_N} \approx \Phi(\rho). \quad (17)$$

This relation was referred to in [1] as “universal scaling.” There are two points to note here: First as argued in [2], the vector-meson mass scaling applies also to the masses in (15). Thus, in medium the meson mass should be replaced by $m_{\sigma,\omega}^*$. Consequently, the coupling strengths C_σ and C_ω are density-dependent.³ Second, the scaling can

Fermi liquid theory of normal nuclear matter and making contact with Walecka theory at mean-field order, it is essential to keep relativistic corrections from the start. This probably has to do with the presence of the Fermi sea in the effective chiral Lagrangian approach. This seems to suggest that the usual chiral counting valid in free space needs to be modified in medium.

³I should point out that for the purpose of the ensuing discussion, neither the detailed knowledge of the “heavy” degrees of freedom that give rise to the four-Fermi interactions nor the specific form of the density dependence will be needed. What really matters are the quantum numbers involved. The latter is invoked in reducing various density-dependent parameters to the universal one, $\Phi(\rho)$.

be understood in terms of effects due to the four-Fermi interactions, which for nucleons on the Fermi surface correspond to the fixed-point interactions of Landau Fermi liquid theory according to Shankar and Polchinski [23]. We shall establish a direct connection to the Landau parameters of the quasiparticle-interaction.

IV. LANDAU'S EFFECTIVE MASS OF THE NUCLEON

In the Landau-Migdal Fermi liquid theory of nuclear matter [21,22], the interaction between two quasiparticles on the Fermi surface is of the form (neglecting tensor interactions)

$$\mathcal{F}(\vec{p}, \vec{p}') = F(\cos \theta) + F'(\cos \theta)(\vec{\tau} \cdot \vec{\tau}') + G(\cos \theta)(\vec{\sigma} \cdot \vec{\sigma}') + G'(\cos \theta)(\vec{\tau} \cdot \vec{\tau}')(\vec{\sigma} \cdot \vec{\sigma}'), \quad (18)$$

where θ is the angle between \vec{p} and \vec{p}' . The function $F(\cos \theta)$ can be expanded in Legendre polynomials,

$$F(\cos \theta) = \sum_l F_l P_l(\cos \theta), \quad (19)$$

with analogous expansions for the spin- and isospin-dependent interactions. The coefficients F_l etc. are the Landau Fermi liquid parameters. Some of the parameters can be related to physical properties of the system. The relation between the effective mass and the Landau parameter F_1 (eq. (23)) is crucial for our discussion.⁴

An important point of this paper is that one must distinguish between the effective mass m_N^σ , which is of the same form as Walecka's effective mass, and the Landau effective mass, which is more directly related to nuclear observables. To see what the precise relation is, we include the non-local four-Fermi interaction due to the one-pion exchange term, $L_4^{(\pi)}$.

The total four-Fermi interaction that enters in the renormalization-group flow consideration à la Shankar-Polchinski is then the sum

$$\mathcal{L}_4 = \mathcal{L}_4^{(\pi)} + \mathcal{L}_4^{(\delta)}. \quad (20)$$

The point here is that the non-local one-pion-exchange term brings additional contributions to the effective nucleon mass on top of the universal scaling mass discussed above. We now compute the nucleon effective mass with the chiral Lagrangian and make contact with the results

⁴In this lecture, we make no use of the Landau parameters G and G' . The G' in particular is believed to figure in the response to the weak axial current by nuclei. We shall see that one can actually obtain a similar result using only the F and F' parameters in combination with the pionic interaction.

of Fermi liquid theory [13]. We start with the single-nucleon energy in the non-relativistic approximation⁵

$$\epsilon(p) = \frac{p^2}{2m_N^\sigma} + C_\omega^2 \langle N^\dagger N \rangle + \Sigma_\pi(p) \quad (21)$$

where $\Sigma_\pi(p)$ is the self-energy from the pion-exchange Fock term. The self-energy contribution from the vector meson (second term on the right hand side of (21)) comes from an ω tadpole (or Hartree) graph. The Landau effective mass m_L^* is related to the quasiparticle velocity at the Fermi surface

$$\frac{d}{dp} \epsilon(p)|_{p=p_F} = \frac{p_F}{m_L^*} = \frac{p_F}{m_N^\sigma} + \frac{d}{dp} \Sigma_\pi(p)|_{p=p_F}. \quad (22)$$

Using Galilean invariance, Landau [21] derived a relation between the effective mass of the quasi-particles and the velocity dependence of the effective interaction described by the Fermi-liquid parameter F_1 :

$$\frac{m_L^*}{m_N} = 1 + \frac{F_1}{3} = \left(1 - \frac{\tilde{F}_1}{3}\right)^{-1}, \quad (23)$$

where $\tilde{F}_1 = (m_N/m_L^*)F_1$. The corresponding relation for relativistic systems follows from Lorentz invariance and has been derived by Baym and Chin [15].

With the four-Fermi interaction (20), there are two distinct velocity-dependent terms in the quasiparticle interaction, namely the spatial part of the current-current interaction and the exchange (or Fock) term of the one-pion-exchange. In the nonrelativistic approximation, their contributions to \tilde{F}_1 are ($\tilde{F}_1 = \tilde{F}_1^\omega + \tilde{F}_1^\pi$)

$$\tilde{F}_1^\omega = \frac{m_N}{m_L^*} F_1^\omega = -C_\omega^2 \frac{2p_F^3}{\pi^2 m_N^\sigma}, \quad (24)$$

$$\tilde{F}_1^\pi = -3 \frac{m_N}{p_F} \frac{d}{dp} \Sigma_\pi(p)|_{p=p_F}, \quad (25)$$

respectively.

Using eq. (22) we find

$$\left(\frac{m_L^*}{m_N}\right)^{-1} = \frac{m_N}{m_N^\sigma} + \frac{m_N}{p_F} \frac{d}{dp} \Sigma_\pi(p)|_{p=p_F} = 1 - \frac{1}{3} \tilde{F}_1, \quad (26)$$

which implies that

$$\frac{m_N}{m_N^\sigma} = 1 - \frac{1}{3} \tilde{F}_1^\omega. \quad (27)$$

This formula gives a relation between the σ -nucleon interaction (eq. (16)) and the ω -nucleon coupling (eq. (24)). The ω -exchange contribution to the Landau parameter

⁵We treat the scalar and vector fields self-consistently and the self-energy from the pion exchange graph as a perturbation.

F_1 is due to the velocity-dependent part of the potential, $\sim \vec{p}_1 \cdot \vec{p}_2/m_N^2$. This is an $\mathcal{O}(p^2)$ term, and consequently suppressed in naive chiral counting. Nonetheless it is this chirally non-leading term in the four-Fermi interaction (14) that appears on the same footing with the chirally leading terms in the ω and σ tadpole graphs. This shows that there must be subtlety in the chiral counting in the presence of a Fermi sea.

The pion contribution to F_1 can be evaluated explicitly [36]

$$\frac{1}{3}\tilde{F}_1^\pi = -\frac{3f_{\pi NN}^2 m_N}{8\pi^2 p_F} \left[\frac{m_\pi^2 + 2p_F^2}{2p_F^2} \ln \frac{m_\pi^2 + 4p_F^2}{m_\pi^2} - 2 \right] \approx -0.153. \quad (28)$$

Here $f_{\pi NN} \approx 1$ is the non-relativistic πN coupling constant. The numerical value of \tilde{F}_1^π is obtained at nuclear matter density, where $p_F \approx 2m_\pi$.

One of the important results of this paper is that eq. (27) relates the only unknown parameter \tilde{F}_1^ω to the universal scaling factor Φ . Note that in the absence of the one-pion-exchange interaction – and in the nonrelativistic approximation – m_N^σ can be identified with the Landau effective mass m_L^* . In its presence, however, the two masses are different due to the pionic Fock term. We propose to identify the scaling nucleon mass defined in eq. (11) with the Landau effective mass:

$$m_L^* = m_N^*. \quad (29)$$

We note that the Landau mass is defined at the Fermi surface, while the scaling mass refers to a nucleon propagating in a “vacuum” modified by the nuclear medium. Although the two definitions are closely related, their precise connection is not understood at present. Nevertheless, eq. (29) is expected to be a good approximation (see also section 5.2).

V. ORBITAL GYROMAGNETIC RATIOS IN NUCLEI

Given the effective Lagrangian with the BR scaling and its relation to Landau Fermi liquid theory, how can one describe nuclear magnetic moments and axial charge transitions? This is an important question because these nuclear processes are sensitive to both the scaling properties and exchange currents. Here we consider the gyromagnetic ratios $g_l^{(p,n)}$ of the proton and the neutron in heavy nuclei, deferring the issue of the nuclear axial-charge transitions [37] to a later publication [13]. We start with the Fermi liquid theory result for the gyromagnetic ratio.⁶

⁶This quantity has been extensively analyzed in terms of standard exchange currents and their relations, via vector-

A. Migdal’s formula

The response to a slowly-varying electromagnetic field of an odd nucleon with momentum \vec{p} added to a closed Fermi sea can, in Landau theory, be represented by the current [22,16]

$$\vec{J} = \frac{\vec{p}}{m_N} \left(\frac{1 + \tau_3}{2} + \frac{1}{6} \frac{F_1' - F_1}{1 + F_1/3} \tau_3 \right) \quad (30)$$

where m_N is the nucleon mass in medium-free space. The long-wavelength limit of the current is not unique. The physically relevant one corresponds to the limit $q \rightarrow 0, \omega \rightarrow 0$ with $q/\omega \rightarrow 0$, where (ω, q) is the four-momentum transfer. The current (30) defines the gyromagnetic ratio

$$g_l = \frac{1 + \tau_3}{2} + \delta g_l \quad (31)$$

where

$$\delta g_l = \frac{1}{6} \frac{F_1' - F_1}{1 + F_1/3} \tau_3 = \frac{1}{6} (\tilde{F}_1' - \tilde{F}_1) \tau_3. \quad (32)$$

B. Chiral Lagrangian results

In this section we compute the gyromagnetic ratio using the chiral Lagrangian and demonstrate that Migdal’s result (32) is reproduced. The derivation will be made in terms of Feynman diagrams. The single-particle current $\vec{J}_1 = \vec{p}/m_N^\sigma$ is given by a diagram with the external nucleon lines dressed by the scalar and vector fields. Note that it is the universally scaled mass m_N^σ that enters, not the Landau mass. This leads to a gyromagnetic ratio

$$(g_l)_{sp} = \frac{m_N}{m_N^\sigma} \frac{1 + \tau_3}{2}. \quad (33)$$

At first glance this result seems to imply the enhancement of the single quasiparticle gyromagnetic ratio by the factor $1/\Phi$ (for $\Phi < 1$) over the free space value. However this interpretation, often made in the literature, is not correct. We have to take into account the corrections carefully.

The first correction to (33) is the contribution from short-ranged high-energy isoscalar vibrations corresponding to an ω meson. This contribution has been computed by several authors [39,40]. In the nonrelativistic approximation one finds

$$g_l^\omega = -\frac{1}{6} C_\omega^2 \frac{2p_F^3}{\pi^2} \frac{1}{m_N^\sigma} = \frac{1}{6} \tilde{F}_1^\omega. \quad (34)$$

current Ward identities, to nuclear forces [38].

Now using (27), we obtain the second principal result of this paper,

$$g_l^\omega = \frac{1}{6} \tilde{F}_1^\omega = \frac{1}{2} (1 - \Phi(\rho))^{-1}. \quad (35)$$

The corresponding contribution with a ρ exchange in the graph yields an isovector term

$$g_l^\rho = -\frac{1}{6} C_\rho^2 \frac{2p_F^3}{\pi^2} \frac{1}{m_N^\sigma} \tau_3 = \frac{1}{6} (\tilde{F}_1^\rho)' \tau_3 \quad (36)$$

where the constant C_ρ is the coupling strength of the four-Fermi interaction

$$\delta\mathcal{L} = -\frac{C_\rho^2}{2} (\bar{N} \gamma_\mu \tau^a N \bar{N} \gamma^\mu \tau^a N). \quad (37)$$

In analogy with the isoscalar channel, we may consider this as arising when the ρ is integrated out from the Lagrangian, and consequently identify

$$C_\rho^2 = g_\rho^2 / m_\rho^2. \quad (38)$$

Again in medium, m_ρ should be replaced by m_ρ^* . The results (34) and (36) can be interpreted in the language of chiral perturbation theory as arising from four-Fermi interaction counterterms in the presence of electromagnetic field, with the counter terms saturated by the ω and ρ mesons respectively (see eq. (92) of [19]).

The next correction is the pionic exchange current (known as Miyazawa term) which yields [36]

$$g_l^\pi = \frac{1}{6} ((\tilde{F}_1^\pi)' - \tilde{F}_1^\pi) \tau_3 = -\frac{2}{9} \tilde{F}_1^\pi \tau_3, \quad (39)$$

where the last equality follows from $(\tilde{F}_1^\pi)' = -(1/3) \tilde{F}_1^\pi$. Thus, the sum of all contributions is

$$\begin{aligned} g_l &= \frac{m_N}{m_N^\sigma} \frac{1 + \tau_3}{2} + \frac{1}{6} (\tilde{F}_1^\omega + (\tilde{F}_1^\rho)' \tau_3) + \frac{1}{6} ((\tilde{F}_1^\pi)' - \tilde{F}_1^\pi) \tau_3 \\ &= \frac{1 + \tau_3}{2} + \frac{1}{6} (\tilde{F}_1' - \tilde{F}_1) \tau_3 \end{aligned} \quad (40)$$

where eq. (27) was used with

$$\tilde{F}_1 = \tilde{F}_1^\omega + \tilde{F}_1^\pi, \quad (41)$$

$$\tilde{F}_1' = (\tilde{F}_1^\pi)' + (\tilde{F}_1^\rho)'. \quad (42)$$

Thus, when the corrections are suitably calculated, we do recover the familiar single-particle gyromagnetic ratio $(1 + \tau_3)/2$ and reproduce the Fermi-liquid theory result for δg_l (32)

$$\delta g_l = \frac{1}{6} (\tilde{F}_1' - \tilde{F}_1) \tau_3 \quad (43)$$

with \tilde{F} and \tilde{F}' in the theory given entirely by (41) and (42), respectively. Equation (40) shows that the isoscalar gyromagnetic ratio is not renormalized by the medium (other than binding effect implicit in the matrix elements) while the isovector one is. *It should be emphasized that contrary to naive expectations, BR scaling is not in conflict with the observed nuclear magnetic moments.* We will show below that the theory agrees quantitatively with experimental data.

VI. COMPARISON WITH EXPERIMENTS

A. Information from QCD sum rules

It is possible to extract the scaling factor $\Phi(\rho)$ from QCD sum rules – as well as from an in-medium Gell-Mann-Oakes-Renner relation [2] – and compare with our theory. In particular, the key information is available from the calculations of the masses of the ρ meson [8,9] and the nucleon [41,42] in medium. In their recent work, Jin and collaborators find (for $\rho = \rho_0$) [9,42]

$$\frac{m_\rho^*}{m_\rho} = 0.78 \pm 0.08, \quad (44)$$

$$\frac{m_N^*}{m_N} = 0.67 \pm 0.05. \quad (45)$$

We identify the ρ -meson scaling with the universal scaling factor,

$$\Phi(\rho_0) = 0.78. \quad (46)$$

This is remarkably close to the result that follows from the GMOR relation in medium [3,43]

$$\Phi^2(\rho_0) \approx \frac{m_\pi^{*2}}{m_\pi^2} \left(1 - \frac{\Sigma_{\pi N} \rho_0}{f_\pi^2 m_\pi^2} + \dots\right) \approx 0.6, \quad (47)$$

where the pion-nucleon sigma term $\Sigma_{\pi N} \approx 45$ MeV is used. In fact, in previous papers by Brown and Rho, the scaling factor Φ was inferred from the in-medium GMOR relation.

B. Prediction by chiral Lagrangian

Our theory has only one quantity that is not fixed by the theory, namely the scaling factor $\Phi(\rho)$ (\tilde{F}_1^π is of course fixed for any density by the chiral Lagrangian.). Since this is given by QCD sum rules for $\rho = \rho_0$, we use this information to make quantitative prediction.

1. Effective nucleon mass

The first quantity is the Landau effective mass of the nucleon (26),

$$\begin{aligned} \frac{m_N^*}{m_N} &= \Phi \left(1 + \frac{1}{3} F_1^\pi\right) \\ &= \left(\Phi^{-1} - \frac{1}{3} \tilde{F}_1^\pi\right)^{-1} \\ &= (1/0.78 + 0.153)^{-1} = 0.69(7) \end{aligned} \quad (48)$$

where we used (28) and (46). The agreement with the QCD sum-rule result (45) is both surprising and intriguing since as mentioned above, the Landau mass is “measured” at the Fermi momentum $p = p_f$ while the QCD

sum-rule mass is defined in the rest frame, so the direct connection remains to be established.

2. Effective axial-vector coupling constant

The next quantity of interest is the axial-vector coupling constant in medium, g_A^* , which can be obtained from the Landau mass (26) and the chiral mass (11) as

$$\frac{g_A^*}{g_A} = \left(1 + \frac{1}{3}F_1^\pi\right)^2 = \left(1 - \frac{1}{3}\Phi\tilde{F}_1^\pi\right)^{-2}, \quad (49)$$

which at $\rho = \rho_0$ gives

$$g_A^* = 1.0(0). \quad (50)$$

This agrees well with the observations in heavy nuclei [44]. Again this is an intriguing result. While it is not understood how this relation is related to the old one in terms of the Landau-Migdal parameter g'_0 in $NN \leftrightarrow N\Delta$ channel [30], it is clearly a short-distance effect in the ‘‘pionic channel’’ involving the factor Φ . This supports the argument [37] that the renormalization of the axial-vector coupling constant in medium cannot be described in low-order chiral perturbation theory.⁷

3. Orbital gyromagnetic ratio

Finally, the correction to the single-particle gyromagnetic ratio can be rewritten as

$$\delta g_l = \frac{4}{9} \left[\Phi^{-1} - 1 - \frac{1}{2}\tilde{F}_1^\pi \right] \tau_3 \quad (51)$$

where we have used (39) and the assumption that the nonet relation $C_\rho^2 = C_\omega^2/9$ holds. The nonet assumption would be justified if the constants C_ω and C_ρ were saturated by the ω and ρ mesons, respectively. At $\rho = \rho_0$, we find

$$\delta g_l = 0.22(7)\tau_3. \quad (52)$$

This is in agreement with the result [45] for protons extracted from the dipole sum rule in ^{209}Bi using the Fujita-Hirata relation [46]:

$$\delta g_l^{\text{proton}} = \kappa/2 = 0.23 \pm 0.03. \quad (53)$$

Here κ is the enhancement factor in the giant dipole sum rule. Given that this is extracted from the sum rule in the giant dipole resonance region, this is a bulk property, so our theory is directly relevant.

Direct comparison with magnetic moment measurements is difficult since BR scaling is expected to quench the tensor force which is crucial for the calculation of contributions from high-excitation states needed to extract the δg_l . Calculations with this effect taken into account are not available at present. Modulo this caveat, our prediction (52) compares well with Yamazaki’s analysis [47] of magnetic moments in the ^{208}Pb region

$$\begin{aligned} \delta g_l^{\text{proton}} &\approx 0.33, \\ \delta g_l^{\text{neutron}} &\approx -0.22 \end{aligned} \quad (54)$$

and also with the result of Arima et al. [48,47]

$$\delta g_l \approx 0.25\tau_3. \quad (55)$$

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⁷The g_A^* calculated here is for a quasiparticle sitting on top of the Fermi sea and is presumably a fixed-point quantity as one scales down in the sense of renormalization group flow. As such, it should be applicable within a configuration space restricted to near the Fermi surface. I think this is the reason why $g_A^* = 1$ was required in the $0\hbar\omega$ Monte Carlo shell-model calculation of Langanke et al [44]. The consequence of this result is that if one were to calculate core-polarization contributions involving multiparticle-multihole configurations mediated by tensor forces, one should obtain only a minor correction. As Gerry Brown has been arguing for some time, this can happen because of the suppression of tensor forces in the presence of BR scaling. Note also that the effective g_A^* obtained here has nothing to do with the so-called ‘‘missing Gamow-Teller strength’’ often discussed in the literature.

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