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Localization of Bogolyubov Quasiparticles in Interacting Bose-Einstein Condensates Subjected to Correlated Random Potentials

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We study the Anderson localization of Bogolyubov quasiparticles in an interacting Bose-Einstein condensate (with healing length ξ) subjected to a random potential (with finite correlation length $\sigma_{\rm R}$). We derive analytically the Lyapunov exponent as a function of the quasiparticle momentum k and we study the localization maximum $k_{\rm max}$. For 1D speckle potentials, we find that $k_{\rm max} \propto 1/\xi$ when $\xi \gg \sigma_{\rm R}$ while $k_{\rm max} \propto 1/\sigma_{\rm R}$ when $\xi \ll \sigma_{\rm R}$, and that the localization is strongest when $\xi \sim \sigma_{\rm R}$. Numerical calculations support our analysis and our estimates indicate that the localization of the Bogolyubov quasiparticles is accessible in current experiments with ultracold atoms.

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One important issue in mesoscopic physics concerns the effects of disorder in systems where both quantum interferences and particle-particle interactions play crucial roles. It is known that multiple scattering of non-interacting quantum particles from a random potential leads to strong Anderson localization (AL) [1], for arbitrary weak disorder in 1D and 2D and for strong-enough disorder in 3D [2]. The problem is more involved in the presence of interactions. Strong disorder in repulsively interacting Bose gases induces novel insulating quantum states, such as the Bose [3] and Lifshits [4] glasses. For moderate disorder and interactions, the system forms a Bose-Einstein condensate (BEC) [4, 5, 6], and in this regime, the disorder induces the depletion of the condensed and superfluid fractions [7] and the damping of sound waves [8].

These studies have direct applications to experiments on liquid ^{4}He in porous media [9], in particular to understand the absence of superfluidity. Moreover, the realization of disordered gaseous BECs [10, 11, 12, 13, 14] has renewed this issue, due to an unprecedented control of the parameters in these systems. For instance, using optical speckle fields [15] one can control the amplitude and design the correlation function of the random potential almost at will. Speckle potentials with a correlation length of a fraction of a micron have been realized [14], opening possibilities for experimental studies of AL [16, 17].

Transport processes in repulsively interacting BECs can exhibit AL [17, 18]. However, for BECs at equilibrium, interaction-induced delocalizing effects dominate disorder-induced localization, except for very weak interactions [5, 6]. The ground state of an interacting BEC at equilibrium is thus extended. Beyond, one may wonder how the many-body (collective) excitations of the BEC behave in the presence of weak disorder. In dilute BECs, these excitations correspond to the generation of quasiparticles (particle-hole pairs) described by the Bogolyubov theory [19]. In this case, the interplay of interactions and disorder is subtle and strong arguments indicate that the Bogolyubov quasiparticles (BQP) are sensitive to a random potential *screened* by the BEC density [5]. This problem has been addressed in the idealized case of random potentials made of a series of impurities, i.e. for uncorrelated potentials [20].

In this Letter, we present a general quantitative treatment of the localization of the BQPs in an interacting BEC with healing length ξ in a weak random potential with arbitrary correlation length $\sigma_{\rm R}$. For weak disorder, we give an explicit transformation that allows us to map rigorously the many-body Bogolyubov equations onto the Schrödinger equation for a non-interacting particle in a screened random potential, which we derive analytically. We calculate the Lyapunov exponent as a function of the BQP wavenumber k for a 1D speckle potential. For a given ratio $\xi/\sigma_{\rm R}$, we determine the wavenumber $k_{\rm max}$ for which the localization is maximum: We find that for $\xi \gg \sigma_{\rm R}$, $k_{\rm max} \sim 1/\xi$ while for $\xi \ll \sigma_{\rm R}$, $k_{\rm max} \sim 1/\sigma_{\rm R}$. The absolute maximum of localization is found when $\xi \sim \sigma_{\rm R}$, a situation where the finite-range correlations of the disorder have to be taken into account. Numerical calculations support our analysis. Finally, possibilities to observe the AL of the BQPs in gaseous BECs are discussed.

We consider a d-dimensional Bose gas in a potential $V(\mathbf{r})$ with weak repulsive short-range atom-atom interactions, characterized by the coupling constant g. Its physics is governed by the many-body Hamiltonian

$$\hat{H} = \int \mathrm{d}\mathbf{r}\big\{(\hbar^2/2m)[(\nabla\hat{\theta})^2\hat{n} + (\nabla\sqrt{\hat{n}})^2] + V(\mathbf{r})\hat{n} + (g/2)\hat{n}^2 - \mu\hat{n}\big\} \qquad (1)$$

where m is the atomic mass, μ is the chemical potential and $\hat{\theta}$ and $\hat{n}$ are the phase and density operators which fulfill the commutation relation $[\hat{n}(\mathbf{r}), \hat{\theta}(\mathbf{r}')] = \mathrm{i}\delta(\mathbf{r}-\mathbf{r}')$. According to the Bogolyubov-Popov theory [19, 21, 22], for small phase gradients ($\hbar^2|\nabla\theta|^2/2m \ll \mu$) and small density fluctuations ($\delta\hat{n} \ll n_{\rm c}$, where $n_{\rm c} = \langle\hat{n}\rangle$ and $\delta\hat{n} = \hat{n} - n_{\rm c}$), Hamiltonian (1) can be diagonalized up to second order in the form $\hat{H} = E_0 + \sum_\nu \epsilon_\nu\, \hat{b}_\nu^\dagger \hat{b}_\nu$, where $\hat{b}_\nu$ is the annihilation operator of the excitation (BQP) of energy ϵ_ν. The many-body ground state of the Bose gas corresponds to a BEC with a uniform phase and a density governed by the Gross-Pitaevskii equa-

tion (GPE):

$$\mu = -\hbar^2\nabla^2(\sqrt{n_{\rm c}})/2m\sqrt{n_{\rm c}} + V(\mathbf{r}) + gn_{\rm c}(\mathbf{r}). \qquad (2)$$

Expanding $\hat{\theta}$ and $\delta\hat{n}$ in the basis of the excitations:

$$\hat{\theta}(\mathbf{r}) = \frac{-i}{2\sqrt{n_{\rm c}(\mathbf{r})}}\sum_\nu \left[f_\nu^+(\mathbf{r})\,\hat{b}_\nu^\dagger - h.c.\right] \qquad (3)$$

$$\delta\hat{n}(\mathbf{r}) = \sqrt{n_{\rm c}(\mathbf{r})}\sum_\nu \left[f_\nu^-(\mathbf{r})\,\hat{b}_\nu + h.c.\right], \qquad (4)$$

and inserting these expressions into Eq. (1), the Hamiltonian reduces to the diagonal form $\hat{H} = E_0 + \sum_\nu \epsilon_\nu\,\hat{b}_\nu^\dagger\hat{b}_\nu$, provided that the functions $f_\nu^\pm$ fulfill the Bogolyubov-de Gennes equations (BdGE) [23]:

$$\left[-(\hbar^2/2m)\nabla^2 + V + \ gn_{\rm c} - \mu\right] f_\nu^+ = \epsilon_\nu f_\nu^- \qquad (5)$$

$$\left[-(\hbar^2/2m)\nabla^2 + V + 3gn_{\rm c} - \mu\right] f_\nu^- = \epsilon_\nu f_\nu^+, \qquad (6)$$

with the normalization condition $\int d\mathbf{r}\left[f_\nu^+(\mathbf{r})f_{\nu'}^-(\mathbf{r})^* + f_\nu^-(\mathbf{r})f_{\nu'}^+(\mathbf{r})^*\right] = 2\delta_{\nu,\nu'}$. Equations (2,5-6) form a complete set to calculate the ground state (BEC) and excitations (BQPs) of the Bose gas. Then, using Eqs. (3,4), one can compute all properties of finite temperature or time-dependent BECs.

Here, we analyze the properties of the BQPs in the presence of weak disorder. According to Eqs. (5,6), these are determined by the interplay of the disorder (V) and the BEC density background ($n_{\rm c}$). Let $V(\mathbf{r})$ be a weak random potential ($\widetilde{V} \ll \mu$, see below) with a vanishing average ($\langle V\rangle = 0$) and a finite-range correlation function, $C(\mathbf{r}) = V_{\rm R}^2 c(\mathbf{r}/\sigma_{\rm R})$, where $V_{\rm R} = \sqrt{\langle V^2\rangle}$ is the standard deviation, and $\sigma_{\rm R}$ the correlation length of V. It has been shown in Refs. [5, 6] that the BEC density is extended for strong-enough repulsive interactions (i.e. for $\xi \ll L$, where $\xi = \hbar/\sqrt{4m\mu}$ is the healing length and L the size of the BEC). More precisely, up to first order in $V_{\rm R}/\mu$, the solution of the GPE (2) reads

$$n_{\rm c}(\mathbf{r}) = [\mu - \widetilde{V}(\mathbf{r})]/g \qquad (7)$$

where $\widetilde{V}(\mathbf{r}) = \int d\mathbf{r}' G_\xi(\mathbf{r}-\mathbf{r}')V(\mathbf{r}')$ with G_ξ, the Green function of the linearized GPE [4, 6]. The function G_ξ is easily written in Fourier space [24]: $G_\xi(\mathbf{q}) = (2\pi)^{-d/2}/[1 + (|\mathbf{q}|\xi)^2]$. Then, one finds

$$\widetilde{V}(\mathbf{q}) = V(\mathbf{q})/[1 + (|\mathbf{q}|\xi)^2] \qquad (8)$$

so that, roughly speaking, $\widetilde{V}(\mathbf{r})$ is obtained from $V(\mathbf{r})$ by suppressing the short-wavelength components (i.e. the components with $|\mathbf{q}|$ larger than ξ^{-1}). The potential $\widetilde{V}(\mathbf{r})$ is thus a smoothed potential [6]. Then, if V is a homogeneous random potential, so is $\widetilde{V}$, and according to Eq. (7), the BEC density profile $n_{\rm c}$ is random but extended [4, 6].

In order to study the localization properties of the BQPs, we have to solve the BdGEs (5,6), which is difficult in general because they are strongly coupled. We show however that, for a weak (possibly random) potential $V(\mathbf{r})$, this difficulty can be overcome by using appropriate linear combinations $g_\mathbf{k}^\pm$ of the $f_\mathbf{k}^\pm$ functions. The right choice turns out to be $g_\mathbf{k}^\pm = \pm\rho_k^{\pm 1/2} f_\mathbf{k}^+ + \rho_k^{\mp 1/2} f_\mathbf{k}^-$ with $\rho_k = \sqrt{1 + 1/(k\xi)^2}$ and $k = |\mathbf{k}|$. Then, for $V = 0$, the equations for $g_\mathbf{k}^\pm$ are uncoupled [see Eqs. (9,10)] and we recover the well-known plane-wave solutions of wavevector $\mathbf{k}$ and energy $\epsilon_\mathbf{k} = \rho_k(\hbar^2k^2/2m)$ [19]. For weak but finite V, inserting Eq. (7) into Eqs. (5,6), we find

$$\frac{\hbar^2k^2}{2m}g_\mathbf{k}^+ = -\frac{\hbar^2}{2m}\nabla^2 g_\mathbf{k}^+ - \frac{2\rho_k\widetilde{V}}{1+\rho_k^2}g_\mathbf{k}^- + \left[V - \frac{3+\rho_k^2}{1+\rho_k^2}\widetilde{V}\right]g_\mathbf{k}^+ \qquad (9)$$

$$-\rho_k^2\frac{\hbar^2k^2}{2m}g_\mathbf{k}^- = -\frac{\hbar^2}{2m}\nabla^2 g_\mathbf{k}^- - \frac{2\rho_k\widetilde{V}}{1+\rho_k^2}g_\mathbf{k}^+ + \left[V - \frac{1+3\rho_k^2}{1+\rho_k^2}\widetilde{V}\right]g_\mathbf{k}^-. \qquad (10)$$

Equations (9,10) are coupled at most by a term of the order of V [since $|\widetilde{V}| \le |V|$ and $2\rho_k/(1+\rho_k^2) \le 1$]. This formulation is thus suitable for perturbative approaches. It should be noted that the functions $g_\mathbf{k}^+$ and $g_\mathbf{k}^-$ have very different behaviors owing to the different signs in the left-hand-side terms in Eqs. (9) and (10). Equation (10) can be solved to the lowest order in $\widetilde{V}_{\rm R}/\mu$ by a Green function method similar to that used in Ref. [6]: we find $g_\mathbf{k}^-(\mathbf{r}) \simeq \frac{2/\epsilon_k}{1+\rho_k^2}\int d\mathbf{r}' G_{\xi_k}(\mathbf{r}-\mathbf{r}')\widetilde{V}(\mathbf{r}')g_\mathbf{k}^+(\mathbf{r}')$, where $\xi_k = \xi/\sqrt{1+(k\xi)^2}$. Equation (9) cannot be solved using the same method because the perturbation series diverges. Nevertheless, from the solution for $g_\mathbf{k}^-$, we find that $|g_\mathbf{k}^-/g_\mathbf{k}^+| \lesssim \frac{2/\epsilon_\mathbf{k}}{1+(k\xi)^2}|\widetilde{V}| < |\widetilde{V}|/\mu \ll 1$. The coupling term in Eq. (9) can thus be neglected to first order in $\widetilde{V}_{\rm R}/\mu$ and we are left with the closed equation

$$-(\hbar^2/2m)\nabla^2 g_\mathbf{k}^+ + \mathcal{V}(\mathbf{r})g_\mathbf{k}^+ \simeq (\hbar^2k^2/2m)g_\mathbf{k}^+, \qquad (11)$$

where

$$\mathcal{V}(\mathbf{r}) = V(\mathbf{r}) - \frac{1+4(k\xi)^2}{1+2(k\xi)^2}\widetilde{V}(\mathbf{r}). \qquad (12)$$

Interestingly, Eq. (11) is similar to the Schrödinger equation for non-interacting bare particles with wavevector $\mathbf{k}$, in a random potential $\mathcal{V}(\mathbf{r})$. This mapping allows us to determine the localization properties of the BQPs using standard methods for bare particles in 1D, 2D or 3D [25]. However, since $\mathcal{V}(\mathbf{r})$ depends on the wavevector $\mathbf{k}$ itself, the localization of the BQPs is dramatically different than that of bare particles as discussed below.

In the remainder of the Letter, we restrict ourselves to the 1D case, for simplicity, but also because AL is expected to be stronger in lower dimensions [2]. It is also worth noting that the Lyapunov exponent Γ_k is a self-averaging quantity in 1D which can be computed in the Born approximation using the phase formalism approach [25] (see also Ref. [17]). We find $\Gamma_k = (\sqrt{2\pi}/8)(2m/\hbar^2k)^2\mathcal{C}(2k)$, where $\mathcal{C}(q)$ is the Fourier

transform of the correlation function of $\mathcal{V}(z)$, provided that $\Gamma_k \ll k$ [16, 17, 25]. Since $\mathcal{C}(q) \propto \langle|\mathcal{V}(q)|^2\rangle$, the component of $\mathcal{V}$ relevant for the calculation of Γ_k is $\mathcal{V}(q = 2k)$. From Eqs. (8,12), we find that

$$\mathcal{V}(2k) = \mathcal{S}(k\xi)V(2k); \quad \mathcal{S}(k\xi) = 2(k\xi)^2/[1 + 2(k\xi)^2] \tag{13}$$

and the Lyapunov exponent of the BQP reads

$$\Gamma_k = [\mathcal{S}(k\xi)]^2\gamma_k \tag{14}$$

where $\gamma_k = (\sqrt{2\pi}/32)(V_{\rm R}/\mu)^2(\sigma_{\rm R}/k^2\xi^4)c(2k\sigma_{\rm R})$ is the Lyapunov exponent for a bare particle with the same wavenumber k [17, 25].

Let us summarize the validity conditions of the perturbative approach presented here. It requires: (i) the smoothing solution (7) to be valid (i.e. $\widetilde{V}_{\rm R} \ll \mu$); (ii) the coupling term proportional to g_k^- in Eq. (9) to be negligible (which is valid if $\widetilde{V}_{\rm R} \ll \mu$); and (iii) the phase formalism to be applicable. The latter requires $\Gamma_k \ll k$, i.e. $(V_{\rm R}/\mu)(\sigma_{\rm R}/\xi)^{1/2} \ll (k\xi)^{3/2}[1 + 1/2(k\xi)^2]$, which is valid for any k if $(V_{\rm R}/\mu)(\sigma_{\rm R}/\xi)^{1/2} \ll 1$.

Applying Eq. (14) to uncorrelated random potentials [$C(z) = 2D\delta(z)$ with $\sigma_{\rm R} \to 0$, $V_{\rm R} \to \infty$ and $2D = V_{\rm R}^2\sigma_{\rm R}\int {\rm d}x c(x) = cst$], one recovers the formula for Γ_k found in Ref. [20]. Our approach generalizes this result to potentials with finite-range correlations, which proves useful since uncorrelated random potentials are usually crude approximations of realistic disorder, for which $\sigma_{\rm R}$ can be significantly large. For instance, we will show below that if $\xi \lesssim \sigma_{\rm R}$, as e.g. in the experiments reported in Refs. [10, 11, 12, 13, 14], the behavior of Γ_k versus k is dramatically affected by the finite-range correlations of the disorder.

Let us discuss the physical content of Eqs. (11,12). According to Eqs. (5,6), the properties of the BQPs are determined by both the bare random potential V and the BEC density $n_{\rm c}$ in a non-trivial way. Equation (11) makes their roles more transparent. Having in mind that the appearance of the smoothed potential $\widetilde{V}(z)$ in Eq. (12) is reminiscent of the presence of the meanfield interaction $gn_{\rm c}$ in the BdGEs (5,6), it appears that the random potential $\mathcal{V}(z)$, relevant for the BQPs, results from the *screening* of the random potential $V(z)$ by the BEC density background [5]. More precisely, the expression (13) for the Fourier component $\mathcal{V}(2k)$, relevant for the Lyapunov exponent of a BQP, shows that the screening strength depends on the wavenumber k. In the *free-particle regime* ($k \gg 1/\xi$), we find that the Lyapunov exponent of a BQP equals that of a bare particle with the same wavenumber ($\Gamma_k \simeq \gamma_k$), as expected. In the *phonon regime* ($k \ll 1/\xi$), the disorder is strongly screened and we find $\Gamma_k \ll \gamma_k$, i.e. the localization of a BQP is much weaker than that of a bare particle. These findings agree with and generalize the results obtained from the transfer matrix method, which applies to random potentials made of a 1D random series of δ-scatterers [20].

Our approach applies to any weak random potential with a finite correlation length. As an important application, we examine now in more details the case of 1D speckle potentials used in quantum gases [10, 11, 12, 13, 14]. Inserting the corresponding reduced correlation function, $c(\kappa) = \sqrt{\pi/2}(1 - \kappa/2)\Theta(1 - \kappa/2)$ where Θ is the Heaviside function [17], into Eq. (14), we find

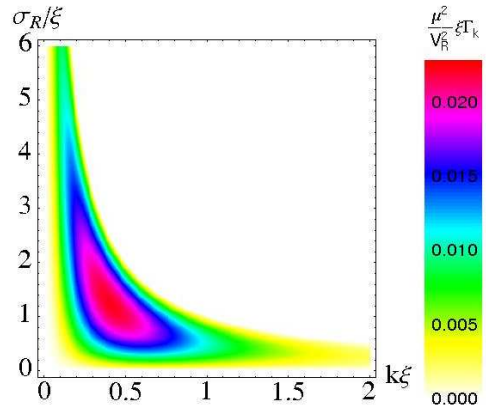

Figure 1: (color online) Density plot of the Lyapunov exponent of the BQPs for a 1D speckle potential.

$$\Gamma_k = \frac{\pi}{8}\left(\frac{V_{\rm R}}{\mu}\right)^2 \frac{\sigma_{\rm R}k^2(1 - k\sigma_{\rm R})}{[1 + 2(k\xi)^2]^2}\Theta(1 - k\sigma_{\rm R}) \tag{15}$$

which is plotted in Fig. 1. In order to test our general approach on the basis of this example, we have performed numerical calculations using a direct integration of the BdGEs (5,6) in a finite but large box of size L. The Lyapunov exponents are extracted from the asymptotic behavior of $\log[r_k(z)/r_k(z_k)]/|z - z_k|$, where z_k is the localization center and $r_k(z)$ is the envelope of the function g_k^+, obtained numerically. The numerical data, averaged over 40 realizations of the disorder, are in excellent agreement with formula (15) as shown in Fig. 2. These results validate our approach. It should be noted however that the numerical calculations return BQP wavefunctions that can be strongly localized for very small momenta k. This will be discussed in more details in a forthcoming publication [26].

Of special interest are the maxima of Γ_k, which correspond to the situations where the localization of a BQP is maximum. It is straightforward to show that, for a fixed set of parameters $(V_{\rm R}/\mu, \xi, \sigma_{\rm R})$, Γ_k is non-monotonic and has a single maximum, $k_{\rm max}$, in the range $[0, 1/\sigma_{\rm R}]$ (see Fig. 2). This contrasts with the case of bare particles, for which the Lyapunov exponent γ_k decreases monotonically as a function of k, provided that $C(2k\sigma_{\rm R})$ decreases versus k (which is valid for a broad class of random potentials [25]). The existence of a localization maximum versus the wavenumber k is thus specific to the BQPs and results from the strong screening of the disorder in the phonon regime. In general, the value of $k_{\rm max}$, shown in the inset of Fig. 2 versus the correlation length of the disorder, depends on both ξ and $\sigma_{\rm R}$. In the limiting case where $\sigma_{\rm R} \ll \xi$, we find $k_{\rm max} \simeq \frac{1}{\sqrt{2}\xi}\left(1 - \frac{\sigma_{\rm R}/\xi}{2\sqrt{2}}\right)$, so that the localization is maximum near the crossover from the phonon regime to the free-particle regime as for uncorrelated potentials [20].

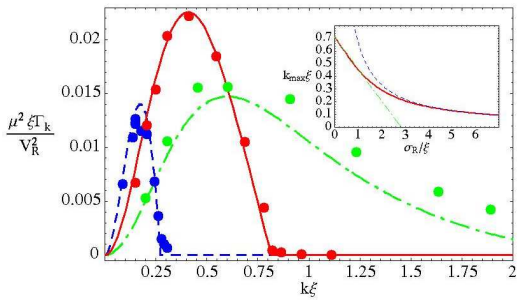

Figure 2: (color online) Lyapunov exponent of the BQPs for a 1D speckle potential. The lines correspond to Eq. (15) and the points to numerical results for $\xi = 1.25 \times 10^{-5} L$, $V_{\rm R} = 0.075\mu$ and $\sigma_{\rm R} = \sqrt{3/2}\xi$ (solid red line), $\sigma_{\rm R} = 3.7\xi$ (dashed blue line), $\sigma_{\rm R} = 0.4\xi$ (dash-dotted green line). Inset: localization maximum versus the ratio of the correlation length of the disorder to the healing length of the BEC (the dash-dotted and dashed lines correspond to the limits $\sigma_{\rm R} \ll \xi$ and $\sigma_{\rm R} \gg \xi$ respectively; see text).

For $\sigma_{\rm R} \gg \xi$ however, we find $k_{\rm max} \simeq 2/3\sigma_{\rm R}$, so that $k_{\rm max}$ is no longer determined by the healing length but rather by the correlation length of the disorder and lies deep in the phonon regime. For $k > 1/\sigma_{\rm R}$, we find that Γ_k vanishes. This defines an *effective mobility edge* due to the special long-range correlations in speckle potentials which also appears for bare particles [17, 27].

Finally, let us determine the absolute localization maximum. The Lyapunov exponent Γ_k decreases monotonically versus ξ and $V_{\rm R}/\mu$. However, for fixed values of $V_{\rm R}/\mu$ and ξ, Γ_k has a maximum at $\sigma_{\rm R} = \sqrt{3/2}\,\xi$ and $k = \xi^{-1}/\sqrt{6}$ (see Fig. 1) and we find the corresponding localization length ($L_{\rm max} = 1/\Gamma_{\rm max}$):

$$L_{\rm max}(\xi) = (512\sqrt{6}/9\pi)(\mu/V_{\rm R})^2\xi. \qquad (16)$$

Note that, at the localization maximum, we have $\sigma_{\rm R} \sim \xi$, so that the disorder cannot be approximated by a uncorrelated random potential and the long-range correlations must be accounted for as in our approach. For $\sigma_{\rm R} = 0.3\mu$m [14] and $V_{\rm R} = 0.2\mu$, we find $L_{\rm max} \simeq 280\mu$m, which can be smaller than the system size in disordered, ultracold Bose gases [11, 14, 28].

In conclusion, we have presented a general treatment for the AL of BQPs in an interacting BEC subjected to a random potential with finite-range correlations. We have calculated the Lyapunov exponents for a 1D speckle potential and we have shown that the localization is strongest when $\sigma_{\rm R} \sim \xi$. We have found that the localization length can be smaller than the size of the BEC for experimentally accessible parameters. We expect that the AL of BQPs could be observed directly, for instance as a broadening of the resonance lines in Bragg spectroscopy, a well mastered technique in gaseous BECs [29].

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[27] Beyond the Born approximation, Γ_k and γ_k may not strictly vanish for $k > 1/\sigma_{\rm R}$. However, the localization length can several orders of magnitude larger than the system size in studies of quantum gases [17].
[28] One-millimeter-long disordered BECs have been produced at Rice University; R. Hulet and Y. Chen, private communication.
[29] D.M. Stamper-Kurn *et al.*, Phys. Rev. Lett. **83**, 2876 (1999); J. Steinhauer *et al.*, *ibid.* **90**, 060404 (2003); S. Richard *et al.*, *ibid.* **91**, 010405 (2003).