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# DYNAMIC PARTICLE SYSTEMS FOR OBJECT STRUCTURE EXTRACTION

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## ABSTRACT

A new deformable model based on the use of a particle system is introduced. By defining local behaviors for each particle, the system behaves as an active contour model embedding variable topology and regularization properties. As a classical model, the model converges to the feature that is searched for. The interest of the particle system is illustrated through two applications: the first one concerns the use of the system as a skeleton extractor based on the propagation of the particles (or agents) inside tree-shaped objects. Using this method, it is possible to generate a cartography of structures such as veins or channels. In the second illustration, the system is used to avoid the problem of initialization of a piecewise cubic B-spline network used to straighten curved text lines.

Keywords: Deformable models, Snakes, Particle System, Structure Extraction.

## INTRODUCTION

Deformable models have been widely used since the original definition of snakes or active contours (Kass et al, 1988). Most developments of deformable models deal with boundary detection and feature extraction on 2-D or 3-D images (Cohen, 1991). Basically, the method consists in deforming and moving an initial curve toward a region of interest. The solution is obtained by minimizing the energy of the model.

The inability to change from a given topology to another is a major drawback of classical deformable models. For this reason several approaches were proposed. Topologically adaptable snakes analysed the topology of curves (Mc Inerney and Terzopoulos, 1995) or surfaces (Mc Inerney and Terzopoulos, 1997) and led to cutting or fusion steps during the evolution. Level set methods considered the curve or the surface as the zero values of a higher dimension object (Sethian, 1996). Implicit models acted the same way but were generated from predefined primitives (Yahia et al., 1998). A special class of deformable models is represented by particle systems (Szeliski and Tonnesen, 1992) which allow topology changes during the evolution and provide results which are also built using predefined primitives, as opposed to level set methods.

The next section presents a deformable model based on the definition of a particle system (Angella et al., 1998). This model has also been introduced to overcome the classical deformable model drawbacks. The particle system presents the ability to change its topology. In addition, unlike the classical models, this method does not require an initialization close to the desired solution.

Then, we will present two applications of this model. First, a classical application consists in propagating the particles inside a grayscale tree-shaped object. Our algorithm simulates the evolution of the system which is subject to internal and external forces. The simultaneous use of particle systems and tree generation allows us to extract a linear piecewise and connected skeleton of the object. The second application concerns the straightening of text lines

(Laviolle et al, 2001): the approach consists in considering a piecewise cubic B-spline system, defining each curve as a particle. The system propagates along the vertical direction from a central position. Internal forces control the expansion of the curve system whereas external forces drive each curve toward a text line to obtain the global structure of the distorted text.

## PARTICLE SYSTEM

### MODEL DEFINITION

Let us denote  $S$  a particle system. This system is a set of nodes (or particles)  $M_i$ . Each node can evolve in the x-y image plane in accordance with different exerted forces. These forces are divided into two groups: internal and external forces. Internal forces control and regulate the expansion of the system whereas external forces drive its evolution according to the image we are working on. All behaviours derive from the resulting force exerted on the agents.

Our goal is to encourage the particle system to explore the image. In order to complete this task, some interactions rules between particles have to be defined. A repulsion behavior allows the system to expand itself. In addition, an attractive behavior forces particles not to move away from other particles. Thus, the internal forces are defined through the introduction of an interaction potential function gathering a long-range attraction term and a short-range repulsion term. The Lennard-Jones potential function fulfils this definition (Heyes, 1998). Classically, the Lennard-Jones potential describes the interaction between pairs of atoms in a solid or a liquid. It has the following form:

$$\phi(r) = \varepsilon \left( \left( \frac{\sigma}{r} \right)^{12} - \left( \frac{\sigma}{r} \right)^6 \right) \quad (1)$$

where  $r$  is the distance from any atom to any other atom. The constants  $\varepsilon$  and  $\sigma$  are important to determine the strength and shape of the interaction and hence the properties of the solid or liquid. Then, the force between individual atoms is the negative of the gradient of the potential.

In our case, we consider the following function:

$$\rho(d) = \frac{B}{d^m} - \frac{A}{d^n} \quad (2)$$

where  $d$  is the distance between two particles.  $A$ ,  $B$ ,  $m$  and  $n$  are positive coefficients used to set the equilibrium distance. In our application, we chose  $m=2$ ,  $n=1$ . Then the equilibrium distance is equal to  $2B/A$ .

Using  $\rho$ , we can compute a global interaction force  $f_{\text{int}}^{(t,i)}$  :

$$f_{\text{int}}^{(t,i)} = - \sum_{M_j \in \nu_i^{(t)}} \vec{\nabla} \rho(d_{ij}) \quad (3)$$

where  $\nu_i^{(t)}$  is the neighborhood of  $M_i$  at step  $t$ . The notion of neighborhood will be defined precisely in the next section.

The external forces depend on the considered application. We will describe these forces as well as additional internal forces in the next section for the two considered applications.

## SYSTEM EVOLUTION

One can control the behavior of each particle  $M_i$  by computing  $F_i$ , the total resulting force. Then, if we consider that each particle is a mechanical system, its evolution follows the classical Newtonian mechanical relation:

$$m_i \cdot \frac{dv_i(t)}{dt} = F_i \quad (4)$$

where  $m_i$  stands for the mass of the particle. For our purpose,  $m_i$  is a constant and can be included in  $F_i$ . The numerical integration of this differential equation is obtained using the Euler integration method. This leads to the choice of a time interval parameter, resulting from a precision/computation time compromise. Then, (4) becomes:

$$\begin{cases} v_i^t = v_i^{t-1} + \delta t \cdot (F_i - \alpha_f v_i^{t-1}) \\ x_i^t = x_i^{t-1} + \delta t \cdot v_i^t \end{cases} \quad (5)$$

where  $x_i^t$  and  $v_i^t$  are respectively the position and the velocity of a particle  $M_i$  at step  $t$ .  $\alpha_f$  is a friction coefficient added to ensure the stability of the system.

Our approach considers the evolution of the system as a solution of a Newtonian mechanical equation (4). Another equivalent approach can be proposed by generating all forces from a unique potential function, so that the evolution of the particle system  $S$  will be similar to the process involved when minimizing a global energy, as it is performed when dealing with classical active contour (Kass et al, 1988).

## APPLICATIONS OF THE MODEL

### OBJECT STRUCTURE EXTRACTION

The problem of skeleton extraction has found many solutions based on mathematical morphology algorithms (Serra, 1982). Here, we propose to extract structures on grayscale tree-like objects by using the particle system as described in the previous section (Angella et al., 1998).

The evolution of the system is obtained through the use of the Lennard-Jones potential as described above. In addition, two other internal forces were presented in (Angella et al, 1998) in the framework of object structure extraction.

The first one provides the system with learning abilities, forcing particles to move into relevant information areas according to the trajectory of other particles. This force, inspired by the evolutionary computation domain, acts like insect pheromones. Thus, it drive particles preferably on the paths already used by prior particles (Dorigo, Gambardella, 1997). To achieve this task, we build a velocity map  $I_{ph}$  using the velocity  $v_i^t$  of each particle  $M_i$  at each step  $t$  of the algorithm. This map saves the average of the velocity observed in each pixel  $p$  of the image. Then, we can define an elementary pheromonal force which is averaged around  $x_i^t$ :

$$f_{ph}^{(t,i)} = \frac{1}{card \Gamma_i^{(t)}} \cdot \sum_{p \in \Gamma_i^{(t)}} (\bar{\nabla} I_{ph})_p \quad (6)$$

where  $x_i^t$  is the position of  $M_i$  at step  $t$  and  $\Gamma_i^{(t)}$  is the given area around  $x_i^t$ .

The last internal force is a regularization force between particles. This force tends to align particles which are part of the same branch of a given object by attracting each particle to the location corresponding to the middle point between its two neighbors. Let us consider a particle  $M_i$  with a neighboring, at step  $t$ ,  $\mathcal{V}_i^t = \{M_j, M_k\}$ . The force is defined by:

$$f_{reg}^{(t,i)} = \left( \frac{x_j^{(t)} - x_k^{(t)}}{2} - x_i^{(t)} \right) \quad (7)$$

$x_j^t, x_k^t$  and  $x_i^t$  are respectively the position of  $M_j$ ,  $M_k$  and  $M_i$  at step  $t$ .

As we want our system to be located inside objects, we use external forces directly depending on the gradient of the image  $I$ , defined as follows:

$$f_{ext} = -k_{ext} \cdot \vec{\nabla}(\|\nabla G * I\|) \quad (8)$$

$\|\nabla G * I\|$  is the modulus of a filtered gradient of the image.  $G$  is a low pass filter used to extend the influence area of objects boundaries in  $I$ .  $k_{ext}$  is used to control this influence.

Our particle system will evolve inside objects from an initial location. Particles are generated at location  $x_{init}$  with the initial velocity  $v_{init}$ . Those parameters are chosen so that the particles will move inside the tree-shaped object. The generation is T-periodic. In fact, a test is run before generating a particle in order to avoid overpopulation in the generating area.

The process described above allows to simulate the evolution of a deformable expandible tree. This tree is created and modified by linking neighboring particles and cutting some branches according to the information provided by the image. The evolution of the tree  $T$  can be viewed as the joint evolution of the particle system  $S$  and a set of links  $L$  between the elements of  $S$ .

Updating the tree provides it with the capability of topology changes. In addition,  $L$  is useful for the determination of the neighborhoods  $\mathcal{V}_i^t$  and then for the computation of  $f_{int}^{(t,i)}$  and  $f_{reg}^{(t,i)}$ . In fact, these neighborhoods are necessary to list particles which influence a given particle. That is the reason why the algorithm does not rely on the shape of objects but on their structure reflected by  $L$ .

We define the set of links  $L$  with the binary function:

$$l^{(t)} : (i, j) \mapsto \begin{cases} 1 & \text{if } M_i \text{ and } M_j \text{ are linked at step } t \\ 0 & \text{if not} \end{cases} \quad (9)$$

with  $l^{(t)}(i, i) = 0, \forall i$

(9) leads to the definition of the neighborhood of  $M_i$  at step  $t$ :

$$\mathcal{V}_i^t = \{M_j / l^{(t)}(i, j) = 1\} \quad (10)$$

Finding a neighbor for  $M_i$  according to its trajectory is done in the following senses:

- in time, which means that it must be close to the previous location of  $M_i$ ,
- in space, which means that it must be close to  $M_i$  too.

In order to find this specific agent, we propose to scan the entire trajectory of  $M_i$  beginning from step  $t$  to step 1 using a restricted area around the trajectory. The selected agent is  $M_j$  with:

$$j = \arg \min_{k / M_k \in S} \left( \|x_k^t - x_i^m\| \mid \|x_k^t - x_i^m\| \leq D_\varepsilon \right) \quad (11)$$

$D_\varepsilon$  is used to restrict the search process around the trajectory of  $M_i$  and  $m$  stands for the current observed iteration varying from  $t$  down to 1. The search process is stopped at a step as soon as a minimum is found.

The tree update process modifies the link structure at each step of the evolution of  $S$ . Links between agents are dynamically changed according to the geometric relations between them and taking advantage of the previous trajectories of agents. We analyze different cases corresponding to the different configurations for each agent: additions and convergences, deletions, divergences and shifts:

- Additions and Convergences: if a particle has an empty neighborhood, a neighbor must, however, be selected among the other particles using formula (11). This situation happens for a newly generated particle or for a particle whose links were cut because it moved away from the object structure. In addition, the system can hold optional convergence cases, leading to cycle generations, so that the tree becomes a graph including loops.

- Deletions: if the agent  $M_i$  moves too far away from its neighbors, its links must be cut off. The deletion process is just based on the definition of a deletion distance.

- Divergences and Shifts: the divergence, or shift behavior, occurs around a junction on the object structure. It is a local reorganization of the set of links. In order to understand how the tree is updated, let us analyze Fig 1. In the vicinity of a junction (step a), the particle  $M_2$  does not follow the particle  $M_1$ , so that the structure of the links in  $L$  does not reflect the structure of the object at step b. At step c, we decide to cut off all links to  $M_2$  and suitably reconnect the tree at step d. This topology change becomes possible if one can find a criterion to decide when cutting the skeleton at step c. The rule adopted is based on angle comparisons considering that the angle between two links associated to a given particle has to be greater than a fixed minimal angle  $\theta_{\text{junction}}$ . If it is not the case, a junction is detected, the two links are removed and new links have to be found leading to a topology change.

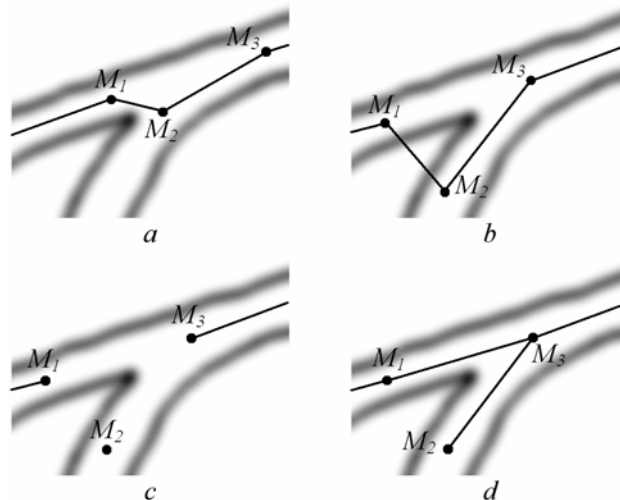


Fig. 1. Links update for the divergence-shift case (particles are moving from right to left)

This simultaneous evolution of  $S$  and  $L$  allows us to hierarchically explore objects and to overcome the topology changes problem. Finally, this approach returns a hierarchical, connected and piecewise linear “skeleton” of objects without any post-processing.

Figure 2 illustrates the evolution of the system on a synthesized tree-like object. In this case, all particles are generated at the same location with the same initial velocity vector  $v_{init}$ . The tree update process consists in adding, deleting or shifting links according to the geometric relations between the particles and taking the trajectories into account (Angella et al, 1998). Finally, the evolution stops because of the initialization vicinity test.

Figure 3 shows a result obtained on a more complex image. The simultaneous use of a particle system and tree generation allows us to extract the skeleton of the river on an aerial image. The particles were first generated from the top-left of the structure and were then allowed to explore the object. Fig 3a and Fig 3b show respectively the trajectories of the particles and the obtained structure. We can note that some ramifications are not explored. Some areas on the image plane do not provide the particle with relevant external forces because of the low contrast between the object and its environment. In addition the object structure might become too thin at the end of a ramification to allow the extension of the system.

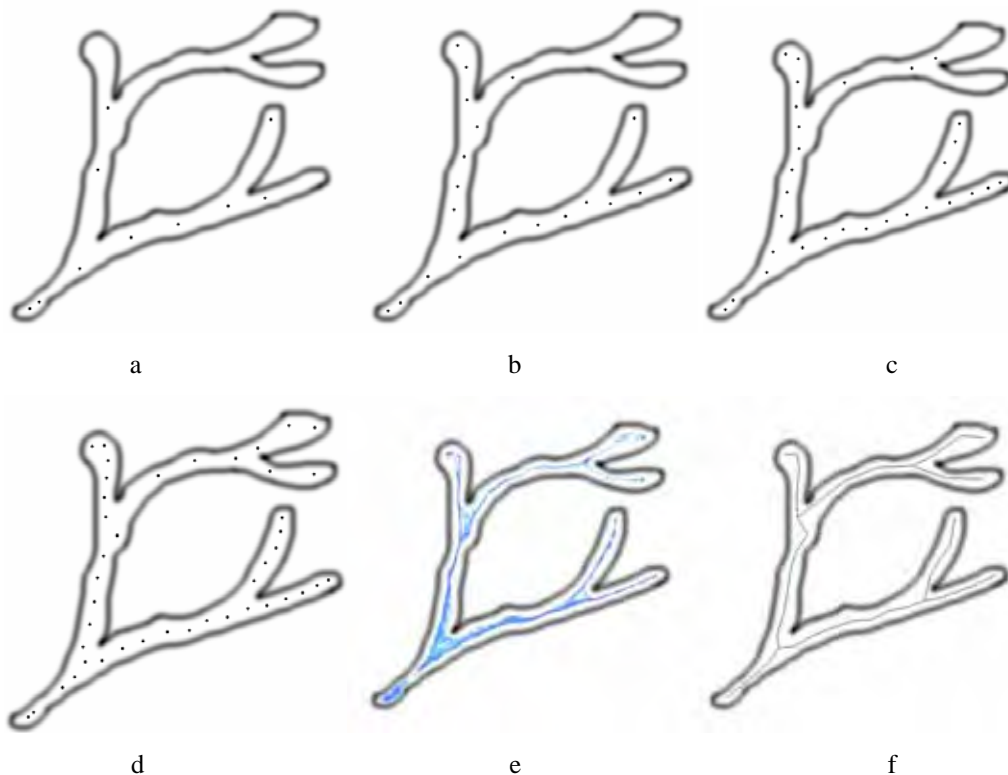


Fig. 2. Synthesized tree-like object. 2a, 2b, 2c and 2d: Evolution of the particle system.  
 Fig 2e: Particle trajectories. Fig 2f: final tree

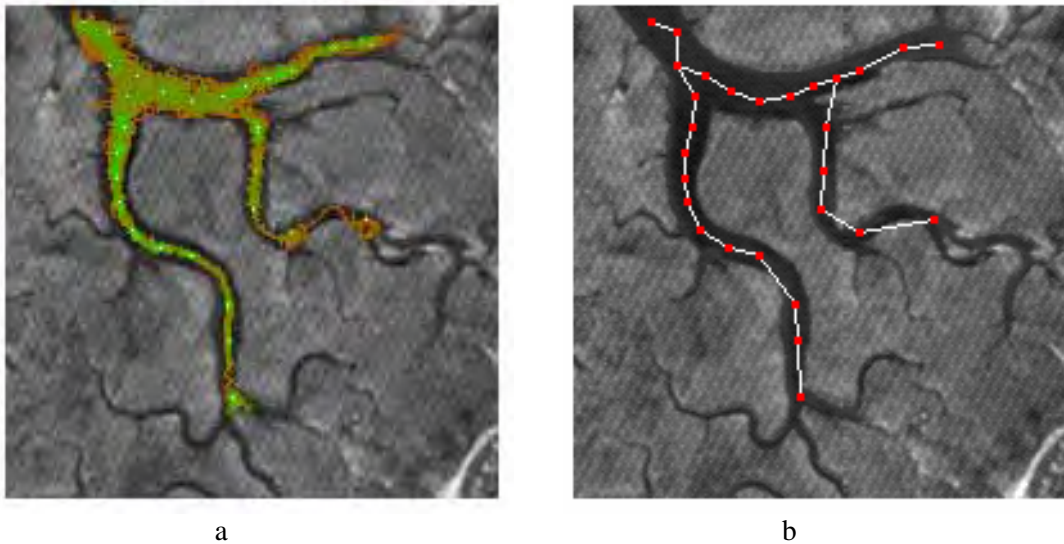


Fig. 3. Real tree-like object structure extraction. Fig 2a: Particle trajectories. Fig 2b: main ramifications of the skeleton for a satellite image. Particles were generated at the top left of the picture.

#### ACTIVE CONTOURS NETWORK TO STRAIGHTEN DISTORTED TEXT LINES

We proposed a 2D extension of the system to straighten distorted text (Lavialle et al., 2001). For such a purpose, the system of particles is transformed into a system of curves. Our purpose is to straighten the kind of curved text given in Fig. 4a. The method consists in associating a curve to each line of the text. We note  $a_i^k$  the  $k^{\text{th}}$  parameter of the  $i^{\text{th}}$  curve  $C_i$ . Our goal is then to define an energy function depending on  $a_i^k$ . Using a classical approach, each curve can be modeled by Basis splines (B-splines) (Lehmann et al., 1999). In our case we propose to use piecewise cubic B-splines (i.e. each curve is a set of connected cubic B-splines).

Let us note  $C_i$  the B-Spline.  $a_i^{k,s}$  is the  $k^{\text{th}}$  control point of the  $s^{\text{th}}$  spline of  $C_i$ . We force the continuity of the model by fixing some constraints on control points in order to obtain smoothed curves (Lavialle et al., 2001).

The horizontal coordinate of each parameter  $a_i^{k,s}$  is fixed and the value of  $a_i^{k,s}$  yields the vertical position of the curve  $C_i$ . Then, we define the internal energy by assuming that the text is locally uniform (i.e. the line spacing is constant). Thus, our purpose is to move each curve toward the centre of its two neighbours:

$$E_{\text{int}}^{k,s}(i) = \left[ a_i^{k,s} - \frac{1}{2}(a_{i-1}^{k,s} + a_{i+1}^{k,s}) \right]^2 \quad (12)$$

We propose to define the set of curves as a particle system. The system evolves along the vertical direction from an initial location. In addition to the internal energy defined above, we introduce an interaction force based on a Lennard-Jones potential between the control points of neighboring curves to force them to move from their initial location.

The minimization of the external energy allows to attract the curves toward the text. Although top of the text is easy to detect, it may be full of interference (more particularly for letters such as f,h,k,l and t, accents in some languages and all upper-cases). When considering the bottom of the lines, such a phenomena may occur but less often (q,p,g,y and j). We choose to focus our attention on the bottom of the lines. The computation of the external energy is



based on a mathematical morphology transformation: we first use an opening on the original image to fill in the spaces between letters. Figure 4b shows the result of such an opening with a  $11 \times 5$  pixels sized structuring element.

Then, considering the bottom of the lines, the external energy is computed by using a vertical Deriche gradient (Deriche, 1987). The smoothing obtained through the use of this operator is necessary to extend the influence of the lines and then to attract the curves toward the solution (see Fig. 4c).

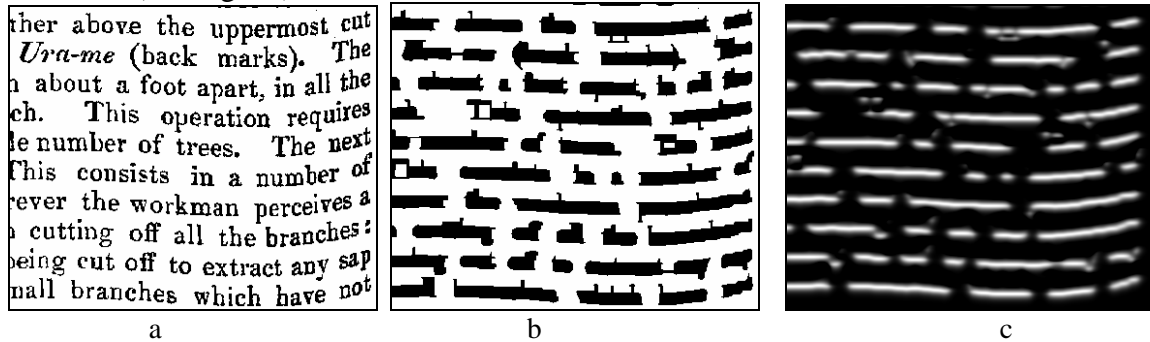


Fig. 4a: Curved text. Fig 4b: result of an opening. Fig. 4c: External energy computed from 4a.

The derivative function of the external energy (considered as a function of  $a_i^{k,s}$ ) is computed through:

$$\frac{\partial E_{ext}}{\partial a_i^{k,s}} = \frac{\partial E_{ext}}{\partial y} \frac{\partial C_i}{\partial a_i^{k,s}} \quad (13)$$

The derivative  $\partial E_{ext} / \partial y$  is computed using the finite differences on a discrete grid. The term  $\partial C_i / \partial a_i^{k,s}$  is obtained considering the model of curves used.

All curves are generated at the initial location  $y_{init}$  with the initial velocity  $v_{init}$ . The evolution of the system follows the classical mechanical relation as described in the previous section.

Fig. 5 shows an example of distorted text. Fig. 6 illustrates the evolution of the curve system on a detail of Fig. 5. The curves are initialized at the middle of the page. After 150 iterations, each curve system converges toward a text line and the straightening is possible (Fig 7).

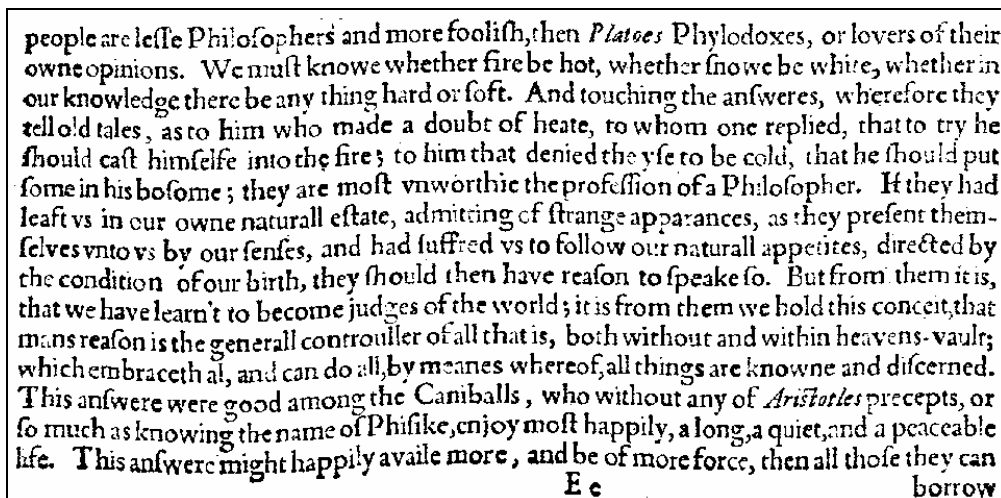


Fig. 5. Example of distorted text

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Fig. 6. Evolution of a curve system.

people are lesse Philosophers and more foolish, then *Platoes* Phylodoxes, or lovers of their  
 owne opinions. We must knowe whether fire be hot, whether snowe be white, whether in  
 our knowledge there be any thing hard or soft. And touching the answeres, wherefore they  
 tell old tales, as to him who made a doubt of heate, to whom one replied, that to try he  
 should cast himselfe into the fire; to him that denied the yse to be cold, that he should put  
 some in his bosome; they are most vnworthie the profession of a Philosopher. If they had  
 least vs in our owne naturall estate, admitting of strange apparances, as they present them-  
 selves vnto vs by our senses, and had suffred vs to follow our naturall appetites, directed by  
 the condition of our birth, they should then have reason to speake so. But from them it is,  
 that we have learn't to become judges of the world; it is from them we hold this conceit, that  
 mans reason is the generall controuller of all that is, both without and within heavens-vault;  
 which embraceth all, and can do all, by meanes whereof, all things are knowne and discerned.  
 This answer were good among the Caniballs, who without any of *Aristotles* precepts, or  
 so much as knowing the name of Phisike, enjoy most happily, a long, a quiet, and a peaceable  
 life. This answer might happily availe more, and be of more force, then all those they can  
 E e borrow

Fig. 7. Straightened text

## DISCUSSION

In this paper, we presented the notion of particle system which acts like a deformable model with robustness with respect to topology changes and regularization properties. When applied to the extraction of an object structure, the system returns a continuous and piecewise connected skeleton. As the particles first move through large ramifications, the method can give additional information on the hierarchy of structures. That is the reason why it can be used as a simple burring method in the case of complex shapes. We can notice that the 3-D extension of this method is obvious: it has been tested on synthesized 3-D images yet and proved to be efficient.

Through the second application, we demonstrated the interest of our method to simplify the initialization step in the case of a system of cubic B-splines used to straighten text lines.

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