



**HAL**  
open science

## **Fault tolerant control design for polytopic LPV system.**

Mickael Rodrigues, Didier Theilliol, Samir Aberkane, Dominique Sauter

► **To cite this version:**

Mickael Rodrigues, Didier Theilliol, Samir Aberkane, Dominique Sauter. Fault tolerant control design for polytopic LPV system.. International Journal of Applied Mathematics and Computer Science, 2007, 17 (1), pp.27-37. 10.2478/v10006-007-0004-5 . hal-00158382

**HAL Id: hal-00158382**

**<https://hal.science/hal-00158382>**

Submitted on 3 Mar 2009

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Fault Tolerant Control

## for Polytopic LPV Systems

Mickael RODRIGUES<sup>†</sup>, Didier THEILLIOL<sup>\*</sup>, Samir ABERKANE<sup>\*</sup> and Dominique SAUTER<sup>\*</sup>

<sup>†</sup> Laboratoire d'Automatique et de Génie des Procédés, LAGEP - UMR - CNRS 5007

Université Claude Bernard Lyon I - CPE Lyon - Bd du 11 Novembre 1918

F-69622 Villeurbanne Cedex France - Tel: +33 472 431 892, Fax: +33 472 431 682

`rodrigues@lagep.univ-lyon1.fr`

<sup>\*</sup> Centre de Recherche en Automatique de Nancy, CRAN - CNRS - INPL - UHP - UMR 7039

BP 239, F-54506 Vandœuvre-lès-Nancy Cedex, France

Tel: +33 3 83 68 44 80, Fax: +33 3 83 68 44 62

`didier.theilliol@cran.uhp-nancy.fr`

### Abstract

This paper deals with Fault Tolerant Control (FTC) strategy for polytopic Linear Parameter Varying (LPV) systems. The main contribution consists in the design of a Static Output Feedback (SOF) dedicated for such systems in the presence of multiple actuator faults/failures. The controllers are designed through Linear Matrix Inequality (LMI) both in fault-free and faulty cases in order to preserve the system closed-loop stability. Hence, this paper provides a new sufficient (but not necessary) condition for the solvability of the stabilizing output feedback control problem. An example illustrates the effectiveness and performances of the proposed FTC method.

**Keywords:** Fault Tolerant Control, Multiple Actuator Failures, Polytopic LPV Systems, LMI, Static Output Feedback, Stability.

# 1 Introduction

As performance requirements increase in advanced technological systems, their associated control systems are becoming more and more complex. At the same time, complicated systems could have various consequences in the event of component failures. Therefore, it is very important to consider the safety and fault tolerance of such systems at the design stage. For these safety-critical systems, Fault Tolerant Control Systems (FTCS) have been developed to meet these essential objectives. FTCS have been a subject of great practical importance, which has attracted a lot of interest for the last three decades. Bibliographical reviews on reconfigurable Fault Tolerant Control Systems can be found in [14], [29].

The objective of FTCS is to maintain current performances close to desirable ones and preserve stability conditions in the presence of component and/or instrument faults; in some circumstances reduced performances could be accepted as a trade-off. In fact, many FTC methods against actuator failures have been recently developed by [2], [13]. Almost all the methods can be categorized into two groups [29]: passive [7], [25] and active [23], [27], [28] approaches.

First of all, passive FTC deals with a presumed set of system component failures based on the actuator redundancies at the controller design stage. The resulting controller usually has a fixed structure and parameters. However, the main drawback of a passive FTC approach is that as the number of potential failures and the degree of system redundancy increase, controller design could become very complex and the performance of the resulting controller (if exists) could become significantly conservative. Moreover, if an un-anticipated failure occurs, passive FTC can not ensure system stability and can not reach again nominal performances. Controllers switching underlines the fact that many faulty system representations have to be identified so as to synthesize off-line pre-computed and stabilizing controllers. These identifications are sometimes difficult to obtain and it is restrictive to consider only pre-determined actuator faults and not all actuator faults.

On the other part, AFTC is characterized by an on-line Fault Detection and Isolation (FDI) scheme [17] and an automatic control reconfiguration mechanism. Active FTC strategy allows to consider more faults than passive one: some research works deal with it and underline the problem of closed-loop system stability in the presence of multiple actuator failures [11], [12], [18], [24], [27], [30]. Moreover, AFTC is often dedicated to linear systems or linearization of nonlinear systems but not for LPV systems.

On the other part, various system modeling techniques in fault-free case are presented in [9], [15], [26] which deal with Linear Parameter Varying (LPV) and/or polytopic representations. The main motivation of polytopic LPV or just LPV systems is for analysis and control of nonlinear systems. Moreover from our knowledge, there is no work for handling multiple actuator failures and polytopic LPV systems.

Starting our research on FTC and polytopic systems, we can note that Multi-models often use polytopic representations. In [5], the authors develop an output feedback through  $\mathcal{LMI}$  in multi-models context but only in fault-free case. A solution has been proposed in the same multi-models context in [19] where the authors design a static state feedback which take into account multiple actuator failures. From a practical point of view, a state feedback need to use an estimator if all the states are not measurable. It can be difficult to design such state estimators in the same time the system is reconfigured. So, we propose to develop a solution to handle FTC and polytopic LPV systems with a SOF design. An output feedback design is less restrictive than state feedback design and it can bring solutions for practical FTC problems where only system outputs are available. Output Feedback design is also developed in [8] with a sufficient condition for the solvability of stabilizing SOF control problem and in [10] with structured uncertainty. Also, [20] develop a robust SOF for linear discrete-time systems with polytopic uncertainties through  $\mathcal{LMI}$  synthesis. However, all these studies do not take into account any actuator failures, dealing with linear systems and not with LPV systems.

In this paper, an Active FTC strategy is developed to avoid actuator fault/failure effects on

polytopic LPV systems. In many of research works, Feedback design is only used for polytopic LPV system in fault-free case [1] and [3], but not consider actuator failures: this paper deals with a SOF synthesis in the presence of multiple actuator failures. Under the assumption that the fault is detected, isolated and estimated, the developed method preserves the system performances through an appropriate controller re-design in faulty case. Multiple controllers are designed such that any controller can maintain the closed-loop stability for any combination of multiple actuator failures.

The paper is organized as follows. Section II defines the polytopic LPV systems representation under multiple actuator failures. In Section III, we develop a controller synthesis for each actuator and we generate an output feedback control law for polytopic LPV systems both in fault-free and faulty cases. FTC philosophy is carried out under accurate FDI information. An illustrative example is given in Section IV to underline the synthesis. Finally, concluding remarks are given in the last Section.

## 2 Polytopic LPV systems with multiple actuator failures

Let consider the discrete LPV representation in fault-free case:

$$\begin{aligned}x_{k+1} &= \tilde{A}(\theta)x_k + \tilde{B}(\theta)u_k \\y_k &= \tilde{C}(\theta)x_k + \tilde{D}(\theta)u_k\end{aligned}\tag{1}$$

where  $x \in \mathbb{R}^n$  represents the state vector,  $u \in \mathbb{R}^p$  is the input vector,  $y \in \mathbb{R}^m$  is the output vector. The system (1) considers affine parameters dependence such  $\tilde{M}(\theta) = \tilde{M}_0 + \sum_{j=1}^v \theta_j \tilde{M}_j$  with the following notation:

$$\widetilde{M} = \begin{bmatrix} \widetilde{A} & \widetilde{B} \\ \widetilde{C} & \widetilde{D} \end{bmatrix} \quad (2)$$

The affine LPV system (1) with bounded parameters  $\underline{\theta}_j \leq \theta_j(k) \leq \bar{\theta}_j$ , (where  $\bar{\theta}_j$  and  $\underline{\theta}_j$  represents the maximum and the minimum value of  $\theta_j$  respectively) can be represented by a polytopic form (see [3], [16]) when the varying parameter  $\theta(k)$  evolves in a polytopic domain  $\Theta$  of vertices  $[\theta_1, \theta_2, \dots, \theta_v]$  (where the vertices are the extreme values of the parameter  $\theta$ ). In the following, we consider only systems strictly proper such that  $D = 0$ . The system can be defined via a matrix polytope with summits  $S_j := [A_j, B_j, C_j], \forall j \in [1, \dots, N]$  and barycentric combination where  $N = 2^v$ . Consequently, under a multiplicative actuator fault representation [18], system (1) can be rewritten as a polytopic representation:

$$\begin{aligned} x_{k+1} &= \sum_{j=1}^N \alpha_k^j(\theta) [A_j x_k + B_j (I_p - \gamma) u_k] \\ y_k &= \sum_{j=1}^N \alpha_k^j(\theta) [C_j x_k] \end{aligned} \quad (3)$$

where  $\alpha_k^j(\theta) = \alpha(\bar{\theta}_j, \underline{\theta}_j, \theta_j(k), k)$ :  $\theta_j(k)$  is the value of  $\theta_j$  at sample  $k$  (see [16] and [22] for more details about LPV to polytopic representation).  $A_j \in \mathbb{R}^{n \times n}$ ,  $B_j \in \mathbb{R}^{n \times p}$ ,  $C_j \in \mathbb{R}^{m \times n}$  are time-invariant matrices defined for the  $j^{\text{th}}$  models. The polytopic system is scheduled through functions designed as the following:  $\alpha_k^j(\theta), \forall j \in [1, \dots, N]$  lie in a convex set  $\Omega = \{\alpha_k^j(\theta) \in \mathbb{R}^N, \alpha_k(\theta) = [\alpha_k^1(\theta), \dots, \alpha_k^N(\theta)]^T, \alpha_k^j(\theta) \geq 0, \forall j, \sum_{j=1}^N \alpha_k^j(\theta) = 1\}$ . These functions are assumed to be available in real time depending on fault-free parameters measurement [4]. The matrix  $\gamma$  is defined as follows:

$$\gamma \triangleq \text{diag}[\gamma^1, \gamma^2, \dots, \gamma^p], \quad 0 \leq \gamma^i \leq 1 \text{ such that for extreme values}$$

$$\begin{cases} \gamma^i = 1 \rightarrow \text{represents a total failure of the } i\text{th actuator } i \in [1, \dots, p] \\ \gamma^i = 0 \rightarrow \text{denotes the healthy } i\text{th actuator} \end{cases} \quad (4)$$

Note:  $\gamma^i$  can take any value between 0 and 1. It represents the loss of effectiveness of  $i$ th actuator, i.e. for example a loss of effectiveness 70% of 1st actuator will be represented by  $\gamma^1 = 0.7$ . When an actuator fault appears on the system, if the controller is not designed by taking into account such problem, the closed-loop system stability can not be ensured obviously. So, we propose to develop a SOF for polytopic systems with multiple actuator failures.

### 3 Fault Tolerant Control for polytopic LPV Systems

#### 3.1 Nominal control law design

Let us recall the multiplicative actuator fault representation on a polytopic system as follows:

$$x_{k+1} = \sum_{j=1}^N \alpha_k^j [A_j x_k + \sum_{i=1}^p B_j^i (I_p - \gamma) u_k]$$

$$y_k = C x_k \quad (5)$$

where  $\alpha_k^j$  represents  $\alpha_k^j(\theta)$  for simplicity and matrices  $B_j^i$  represent a total failure in all actuators except the  $i$ -th such that:

$$B_j^i = [0, \dots, 0, b_j^i, 0, \dots, 0] \quad (6)$$

and  $B_j = [b_j^1, b_j^2, \dots, b_j^p]$  with  $b_j^i \in \mathbb{R}^{n \times 1}$ . It is assumed that each column of  $B_j$  is full column rank whatever the model  $j$ . The following assumptions are considered:

*Assumption 1:* The pairs  $(A_j, b_j^i), \forall i = [1, \dots, p]$  are assumed to be controllable  $\forall j \in [1, \dots, N]$   $\square$

*Assumption 2:* The matrix  $C = C_j, \forall j \in [1, \dots, N]$ .  $\square$

*Assumption 3:* The matrix  $C$  is full row rank.  $\square$

*Assumption 4:* At every time instant there is at least one fault-free actuator which means that the situation  $\gamma^1 = \dots = \gamma^p = 1$  is excluded.  $\square$

In the nominal case, the SOF can be expressed such as:

$$u_k = -Fy_k \quad (7)$$

with  $y_k = Cx_k$  and  $F \in \mathbb{R}^{p \times m}$  is the output feedback controller gain. In fault-free case ( $\gamma = 0$ ), the system (5) with a nominal control law  $u_k = -Fy_k$  is equivalent to:

$$\begin{aligned} x_{k+1} &= \sum_{j=1}^N \alpha_k^j [A_j x_k + B_j (I - \gamma)(-Fy_k)] \\ &= \sum_{j=1}^N \alpha_k^j (A_j - B_j FC) x_k \end{aligned} \quad (8)$$

The stability of the closed-loop system is established with a  $\mathcal{LMI}$  pole placement. In order to achieve some desired transient performance, a pole placement should be considered. For many problems, exact pole assignment may not be necessary, it suffices to locate the pole of the closed-loop system in a sub-region of the complex left half plane [6], [18].

So, let define a disk region  $\mathcal{LMI} \mathcal{D}$  included in the unit circle with an affix  $(-q, 0)$  and a radius  $r$  such that  $(q + r) < 1$ . These two scalars  $q$  and  $r$  are used to determine a specific region included in the unit circle so as to place closed-loop system eigenvalues. The pole placement of the closed-loop system (8) for all the models  $j \in [1 \dots N]$  in a  $\mathcal{LMI}$  region, can be expressed as the following:

$$\begin{pmatrix} -rX & qX + (A_jX - B_jFCX)^T \\ qX + (A_jX - B_jFCX) & -rX \end{pmatrix} < 0 \quad (9)$$

However these inequalities are no longer linear with regard to the unknown matrices  $X = X^T > 0$  and  $F, \forall j \in [1 \dots N]$ . So, the solution is not guaranteed to belong to a convex domain and the classical tools for solving sets of matrix inequalities cannot be used. It constitutes the major difficulty of output feedback design.

We propose to transform  $\mathcal{BMT}$  conditions (9) in  $X$  and  $F, \forall j \in [1 \dots N]$ , in  $\mathcal{LMT}$  conditions which will be used to synthesize directly a stabilizing SOF. We will synthesize controllers  $F_i$  for each actuator in order to define a SOF control law.

**Theorem 1** *Consider the system (5) in fault-free case ( $\gamma = 0$ ), defined  $\forall j \in [1 \dots N]$ . Let assume that it is possible to find matrices  $X_i = X_i^T > 0$ ,  $M$  and  $V_i \forall i = [1, \dots, p]$  such that  $\forall i = [1, \dots, p], \forall j = [1, \dots, N]$ :*

$$\begin{pmatrix} -rX_i & qX_i + (A_jX_i - B_j^iV_iC)^T \\ qX_i + A_jX_i - B_j^iV_iC & -rX_i \end{pmatrix} < 0 \quad (10)$$

with

$$CX_i = M_iC \quad (11)$$

The control law with the SOF  $u_k = -Fy_k$  allows to place the eigenvalues of the closed-loop system (5) in a predetermined  $\mathcal{LMT}$ -region with  $FM = V$ ,  $F = \sum_{i=1}^p G_iV_i(C C^T (C \sum_{i=1}^p X_i C^T)^{-1})$  or  $F = V C C^T (C X C^T)^{-1}$ , with  $G_i \in \mathbb{R}^{p \times p}$  is a matrix equals to zero except in the diagonal entry

$$(i, i) \text{ where there is a one such as: } G_i = \begin{bmatrix} 0 & \dots & 0 \\ \vdots & 1 & \vdots \\ 0 & \dots & 0 \end{bmatrix} \quad \blacksquare$$

*Proof:*

As proposed in [18], summation of (10) under the actuators set  $i \in [1, \dots, p]$  of the system (5)

$i = [1, \dots, p]$  gives for one model  $j, \forall j = [1, \dots, N]$ :

$$\sum_{i=1}^p \begin{pmatrix} -rX_i & qX_i + (A_jX_i - B_j^iV_iC)^T \\ qX_i + A_jX_i - B_j^iV_iC & -rX_i \end{pmatrix} < 0 \quad (12)$$

Let denote  $X = \sum_{i=1}^p X_i$  (with  $X = X^T > 0$ ) to obtain

$$\begin{pmatrix} -rX & qX + (A_jX - \sum_{i=1}^p B_j^iV_iC)^T \\ qX + (A_jX - \sum_{i=1}^p B_j^iV_iC) & -rX \end{pmatrix} < 0 \quad (13)$$

$\forall i = [1, \dots, p], \forall j = [1, \dots, N]$ . Now, denote the  $l$ -th row of the matrix  $V_i$  as  $V_i^l, i = [1, \dots, p]$

and  $l = 1, \dots, p$  which can be calculated from:

$$V_i^l = G_l V_i \quad (14)$$

Therefore,

$$\begin{aligned} \sum_{i=1}^p B_j^i V_i C &= \sum_{i=1}^p [0, \dots, 0, b_j^i, 0, \dots, 0] V_i^i C \\ &= B_j \sum_{i=1}^p V_i^i C = B_j \left( \sum_{i=1}^p G_i V_i C \right) = B_j V C \end{aligned} \quad (15)$$

with  $V = \sum_{i=1}^p G_i V_i$ .

Moreover, we get  $\forall i = [1, \dots, p], \forall j = [1, \dots, N]$

$$\begin{pmatrix} -rX & qX + (A_jX - B_jVC)^T \\ qX + (A_jX - B_jVC) & -rX \end{pmatrix} < 0 \quad (16)$$

With the changes of variables  $V = FM$  and  $CX = MC$  which substituted in  $\mathcal{LM}\mathcal{I}$  (16), lead to

$$\begin{pmatrix} -rX & qX + (A_j X - B_j FCX)^T \\ qX + (A_j X - B_j FCX) & -rX \end{pmatrix} < 0 \quad (17)$$

$\forall i = [1, \dots, p], \forall j = [1, \dots, N]$ . We should note that inequalities (17) are  $\mathcal{BM}\mathcal{I}$ s which could not be solve with classical tools but recall the definition of the disk  $\mathcal{LM}\mathcal{I}$  region into unit circle (9). By multiplying each  $\mathcal{LM}\mathcal{I}$  (16) by  $\alpha_k^j$  and summing all of them, we obtain

$$\begin{pmatrix} -rX & qX + \sum_{j=1}^N \alpha_k^j (A_j X - B_j VC)^T \\ qX + \sum_{j=1}^N \alpha_k^j (A_j X - B_j VC) & -rX \end{pmatrix} < 0 \quad (18)$$

it is equivalent to

$$\begin{pmatrix} -rX & qX + (A(\alpha)X - B(\alpha)VC)^T \\ qX + (A(\alpha)X - B(\alpha)VC) & -rX \end{pmatrix} < 0 \quad (19)$$

with  $A(\alpha) = \sum_{j=1}^N \alpha_k^j A_j$  and  $B(\alpha) = \sum_{j=1}^N \alpha_k^j B_j$ . Due to the fact that matrix  $C$  is supposed to be full row rank, we deduce from (11) there exists a non-singular matrix  $M = CX C^T (C C^T)^{-1}$  and then after variables changes  $F = VM^{-1} = \sum_{i=1}^p G_i V_i (C C^T (C \sum_{i=1}^p X_i C^T)^{-1})$ . So, quadratic  $\mathcal{D}$ -stability is ensured by solving (18) with a SOF  $u_k = -F y_k$ .  $\square$

In nominal case, we do not really need Assumption 1 in the sense that the proposed SOF is sufficient by solving  $\mathcal{LM}\mathcal{I}$  (10-11). However, in faulty case, as the proposed FTC method considers actuators which are out of order, we have to assume that each pairs  $(A_j, b_j^i)$  are controllable because the lost of one actuator can make the system unstable if the Assumption 1 is not considered. Moreover, if Assumption 1 is untrue, try to find a solution of (10-11) will be not possible i.e. pole placement is obviously not possible for each separate controller.

### 3.2 Principles of Fault Tolerant Control Strategy

We present a reconfiguration strategy which is able to design a reconfigured controller from the nominal one with an exact fault estimation coming from FDI scheme i.e.  $\hat{\gamma} = \gamma$ . Without loss of generality, it is assumed that when actuator fault occurs on the system, the matrix  $\gamma$  in system (5) can be decomposed as follows:

$$\gamma = \begin{bmatrix} \gamma_{p-h} & 0 \\ 0 & I_h \end{bmatrix} \quad (20)$$

Thus,  $\gamma$  is a diagonal matrix:  $\gamma_{p-h}$  a diagonal matrix where its elements  $\gamma_{p-h}^i, i \in [1, \dots, p]$  are different from 1 which represent the number of actuators not out of order ( $\gamma^i \neq 1$ ) and  $I_h$  represents the number  $h$  of actuators totally failed. By recalling  $\gamma$  in (20), let define  $\Gamma$  such that

$$\Gamma \triangleq \begin{bmatrix} I_{p-h} - \gamma_{p-h} & 0 \\ 0 & 0_h \end{bmatrix} \begin{bmatrix} (I_{p-h} - \gamma_{p-h})^{-1} & 0 \\ 0 & 0_h \end{bmatrix} = \begin{bmatrix} I_{p-h} & 0 \\ 0 & 0_h \end{bmatrix} \quad (21)$$

where  $0_h$  represents actuators which are out of order and  $I_{p-h}$  represents governable ones. The corresponding matrix partitions of  $B$  is defined as:

$$B = [B_{p-h} \quad B_h] \quad (22)$$

$B_{p-h} \in \mathbb{R}^{n \times (p-h)}$  and  $B_h \in \mathbb{R}^{n \times h}$ . We will present a control law which is able to vanish actuator faults into the state space representation (3) and ensure closed-loop stability despite of multiple actuator failures. Based on a multiplicative fault representation (5), we propose to use the following control law  $u_{FTC}$  that must vanish all actuator faults on the system (5) such that:

$$u_{FTC} = \begin{bmatrix} (I_{p-h} - \gamma_{p-h})^{-1} & 0 \\ 0 & 0_h \end{bmatrix} u_{nom} = \begin{bmatrix} I_{p-h} \\ 0_{h \times (p-h)} \end{bmatrix} [I_{p-h} - \gamma_{p-h}]^{-1} \begin{bmatrix} I_{p-h} & 0_{(p-h) \times h} \end{bmatrix} u_{nom} \quad (23)$$

Let introduce the set of indexes of all actuators that are not out of order [16], i.e.

$$\Phi \triangleq \{i : i \in (1, \dots, p), \gamma^i \neq 1\} \quad (24)$$

and note that  $u_{FTC} = \begin{bmatrix} (I_{p-h} - \gamma_{p-h})^{-1} & 0 \\ 0 & 0_h \end{bmatrix} u_{nom} = - \begin{bmatrix} (I_{p-h} - \gamma_{p-h})^{-1} & 0 \\ 0 & 0_h \end{bmatrix} F_{nom} y_k = -F_{FTC} y_k$  where  $F_{nom}$  is a nominal controller and  $F_{FTC}$  the new controller. So, this specific control law in the state space representation (5) leads to:

$$\begin{aligned} B_j(I - \gamma)u_{FTC} &= B_j \begin{bmatrix} I_{p-h} - \gamma_{p-h} & 0 \\ 0 & 0_h \end{bmatrix} \begin{bmatrix} (I_{p-h} - \gamma_{p-h})^{-1} & 0 \\ 0 & 0_h \end{bmatrix} u_{nom} \\ &= B_j \Gamma u_{nom} = \sum_{i \in \Phi} B_j^i u_{nom}^i \end{aligned} \quad (25)$$

which avoids actuator fault effect and where  $\sum_{i \in \Phi} B_j^i$  represents the actuators not out of order, i.e.  $\sum_{i \in \Phi} B_j^i = B_{p-h}$  and  $u_{nom}^i$  the  $i$ -th element of  $u_{nom}$ . From Assumption 1, due to the fact that each pair  $(A_j, b_j^i), \forall i = [1, \dots, p]$  are assumed to be controllable  $\forall j = [1, \dots, N]$ , the system remains still controllable in spite of actuator failures.

Remark:

We have assumed for simplicity that matrix  $\gamma$  can be decomposed as in (20) in order to consider the two distinguished cases which are  $\gamma^i = 1$  for actuators out of order and  $\gamma^i \neq 1$  actuators still valid: it is directly indicated by the FDI scheme. Of course, not only the first actuators are always valid and the last ones are not: Assumption 4 indicates that any actuator can fails but

at least one is still governable. So by generalizing, let's recall that each element  $\gamma^i, i \in [1, \dots, p]$  (of diagonal matrix  $\gamma$ ) can take any value in  $[0, \dots, 1]$  and let denote

$$u_{FTC} = \begin{bmatrix} u_{FTC}^1 \\ \vdots \\ u_{FTC}^p \end{bmatrix} \quad (26)$$

then each element  $u_{FTC}^i$  of  $u_{FTC}$  can be calculated as follows:

$$\text{If } \gamma^i \neq 1 \text{ then } u_{FTC}^i = (1 - \gamma^i)^{-1} u_{nom}^i \quad (27)$$

$$\text{If } \gamma^i = 1 \text{ then } u_{FTC}^i = 0$$

So, expression (25):  $B_j(I-\gamma)u_{FTC} = \sum_{i \in \Phi} B_j^i u_{nom}^i$  remains unchanged and the system still remains controllable under Assumption 1. In the following and without loss of generality, we will consider the case with  $\gamma$  defined in (20).

### 3.3 Faulty control law design

By considering the system (5) and based on the previous synthesis control law in section (3.1), the FTC method will be developed in this section under assumption that actuator fault estimation  $\hat{\gamma}$  is exactly known, i.e.  $\hat{\gamma} = \gamma$ .

**Theorem 2** *Consider the system (5) with multiple actuator failures ( $\gamma^i \neq 0$ ) under the Assumption 4  $\forall j, j = [1, \dots, N]$  and the set of indexes of the actuators which are not out of order (24). Let the matrices  $M, X_i$  and  $V_i$  be determined as in Theorem 1, then the following control law (28)*

$$\begin{aligned}
u_{FTC} &= - \begin{bmatrix} (I_{p-h} - \gamma_{p-h})^{-1} & 0 \\ 0 & 0_h \end{bmatrix} \left( \sum_{i \in \Phi} G_i V_i (C C^T (C \sum_{i \in \Phi} X_i C^T)^{-1}) \right) y_k \\
&= - \begin{bmatrix} (I_{p-h} - \gamma_{p-h})^{-1} & 0 \\ 0 & 0_h \end{bmatrix} F_{rec} y_k = -F_{FTC} y_k
\end{aligned} \tag{28}$$

with  $G_i \in \mathbb{R}^{p \times p}$  (a matrix equals to zero except in the diagonal entry  $(i, i)$  where there is a one), stabilizes the closed-loop system and places the closed-loop poles in the following  $\mathcal{LMI}$  stability region

$$\begin{pmatrix} -rX & qX + (A_j X - B_j F_{rec} C X)^T \\ qX + (A_j X - B_j F_{rec} C X) & -rX \end{pmatrix} < 0 \tag{29}$$

The SOF control law  $u_k = -F_{FTC} y_k$  is computed with  $F_{rec} M = V$ ,

$$F_{rec} = \sum_{i \in \Phi} G_i V_i (C C^T (C \sum_{i \in \Phi} X_i C^T)^{-1}) = V C C^T (C X C^T)^{-1}. \quad \blacksquare$$

*Proof:*

Applying the new control law (28) to the faulty system (5), leads to the following equation

$$B_j (I - \gamma) u_{FTC} = -B_j \Gamma \left( \sum_{i \in \Phi} G_i V_i (C C^T (C \sum_{i \in \Phi} X_i C^T)^{-1}) \right) y_k \tag{30}$$

with  $\Gamma$  calculated in (21) and defined as

$$\Gamma = \begin{bmatrix} I_{p-h} & 0 \\ 0 & 0_h \end{bmatrix} \tag{31}$$

$\Gamma$  is a diagonal matrix which contains only entries zero (representing total faults) and one (no fault), see section 3.2. Since  $B_j \Gamma = \sum_{i \in \Phi} B_i^j$  models only the actuators that are not out of order, then performing the summations in the proof of Theorem 1 over the elements of  $\Phi$

shows that  $\sum_{i \in \Phi} G_i V_i (C C^T (C \sum_{i \in \Phi} X_i C^T)^{-1})$  is the output feedback gain matrix for the faulty system  $(A_j, \sum_{i \in \Phi} B_i^j, C)$ .  $\square$

We need to assume the pairs  $(A_j, b_j^i), \forall i = [1, \dots, p]$  are controllable  $\forall j = [1, \dots, N]$  because we consider the case of actuators which are out of order: the system has to be controllable with at least one actuator. Moreover, if there is a solution for each  $\mathcal{LMI}$  (10-11) it means that each pair  $(A_j, b_j^i)$  is controllable. However, the assumption 1 does not guarantee the feasibility of (10-11) i.e. the proposed SOF solution is only sufficient for computing the controller, but not necessary.

## 4 Illustrative Example

The feature of the proposed scheme and effectiveness of the Fault-Tolerant Control System are developed using an illustrative example with a SOF for polytopic LPV systems. We present the case of two actuator faults which make unstable the closed-loop system. Let consider a system described by  $N = 4$  unstable models. These four models can be adapted from a LPV model, where each of these ones represents a vertex, as it is done in [9] or in [22] where an aluminum cantilever beam is considered under parametric uncertainties. The discrete state space representation (5) consists of the following matrices:

$$A_1 = \begin{bmatrix} 0.75 & 0 & 0 & 0 \\ 0 & 0.85 & 0 & 0 \\ 0 & 0 & 1.25 & 0 \\ 0 & 0 & 0 & 1.5 \end{bmatrix}, A_4 = \begin{bmatrix} 0.6375 & 0 & 0 & 0 \\ 0 & 0.7225 & 0 & 0 \\ 0 & 0 & 1.0625 & 0 \\ 0 & 0 & 0 & 1.275 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 0.525 & 0 & 0 & 0 \\ 0 & 0.595 & 0 & 0 \\ 0 & 0 & 0.875 & 0 \\ 0 & 0 & 0 & 1.05 \end{bmatrix}, A_2 = \begin{bmatrix} 0.6 & 0 & 0 & 0 \\ 0 & 0.68 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1.2 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, B_1 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix}$$

The other matrices are  $B_2 = 0.8B_1$ ,  $B_3 = 0.7B_1$  and  $B_4 = 0.85B_1$ . The system is in closed-loop way with a SOF  $u_k = - \begin{bmatrix} (I_{p-h} - \gamma_{p-h})^{-1} & 0 \\ 0 & 0_h \end{bmatrix} Fy_k$  (with  $y_k = Cx_k$ ) which is synthesized by Theorem 1 and Theorem 2. The following matrices are directly issued from the first Theorem (with Tklmitool version 2.2 which is a Matlab-based graphical user interface to semidefinite programming (SeDuMi) developed by R. Nikoukhah, F. Delebecque, J.-L. Commeau and L. El Ghaoui, and later upgraded by L. Paolopoli, see <http://www.eecs.berkeley.edu/~elghaoui/links.htm>) with parameters  $q = -0.05$ ,  $r = 0.93$  arbitrarily chosen for stabilizing the closed-loop system:

$$V_1 = \begin{bmatrix} -0.157 & -0.153 & -0.132 \\ 0 & 0 & 0 \end{bmatrix}, V_2 = \begin{bmatrix} 0 & 0 & 0 \\ -0.157 & -0.153 & -0.132 \end{bmatrix}$$

$$X_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.9680 & 0.1074 & 0.1079 \\ 0 & 0.1074 & 0.1738 & 0.1341 \\ 0 & 0.1079 & 0.1341 & 0.1071 \end{bmatrix}, M_1 = \begin{bmatrix} 0.9680 & 0.1074 & 0.1079 \\ 0.1074 & 0.1738 & 0.1341 \\ 0.1079 & 0.1341 & 0.1071 \end{bmatrix}$$

with  $X_1 = X_2$ ,  $M_1 = M_2$  and  $F = VM^{-1} = \sum_{i=1}^p G_i V_i (CC^T (C \sum_{i=1}^p X_i C^T)^{-1})$  :

$$F = \begin{bmatrix} -0.0253 & -1.2221 & 2.1734 \\ -0.0253 & -1.2221 & 2.1734 \end{bmatrix}, G_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, G_2 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

Parameters  $q$  and  $r$  are chosen in regards to system eigenvalues in the complex plane without FTC strategy. A  $\mathcal{LM}\mathcal{I}$ -region is defined into the unit circle (see section 3.1) with an affix  $(-q, 0)$  and

a radius  $r$ : for a same example we can define different combinations of parameters i.e. different  $\mathcal{LMI}$ -regions. This  $\mathcal{LMI}$ -region allows to place system eigenvalues in a stable region in spite of actuator failures: it is represented in Figure (5) with a dashed circle.

Figure (1) represents the parameters evolution of the system in nominal case: the system outputs(a), second actuator (b), first actuator (c) and parameters evolution  $\alpha_k^j$  (d). The closed-loop system is stable without any fault. At sample  $k = 2$ , the first actuator is out of order and also an actuator fault of 60% loss of effectiveness appears on the second actuator. The matrix  $\gamma$  is equal to

$$\gamma = \begin{bmatrix} 1 & 0 \\ 0 & 0.6 \end{bmatrix}, k \geq 2$$

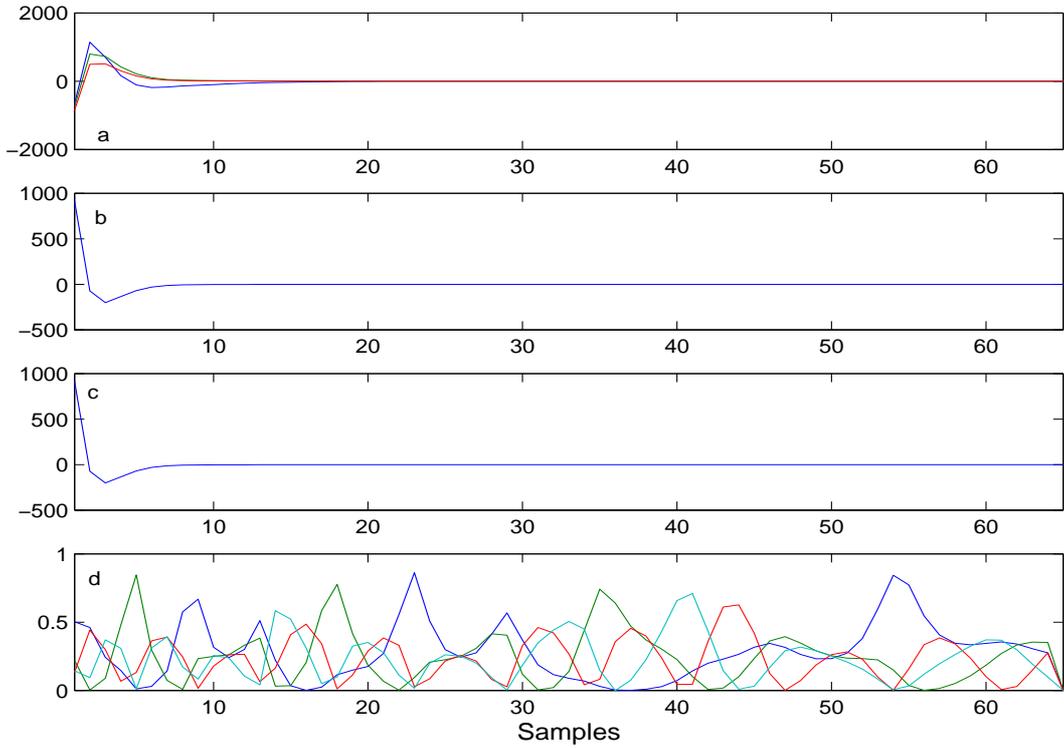


Figure 1: Nominal case: (a) the system outputs, (b) 2nd actuator, (c) 1st actuator and (d) parameters evolution  $\alpha_k^j$

Figure (2) represents the outputs in different situations: *a*) represents the nominal case, *b*) represents the faulty case with a failure on the 1st actuator and a fault on the second actuator

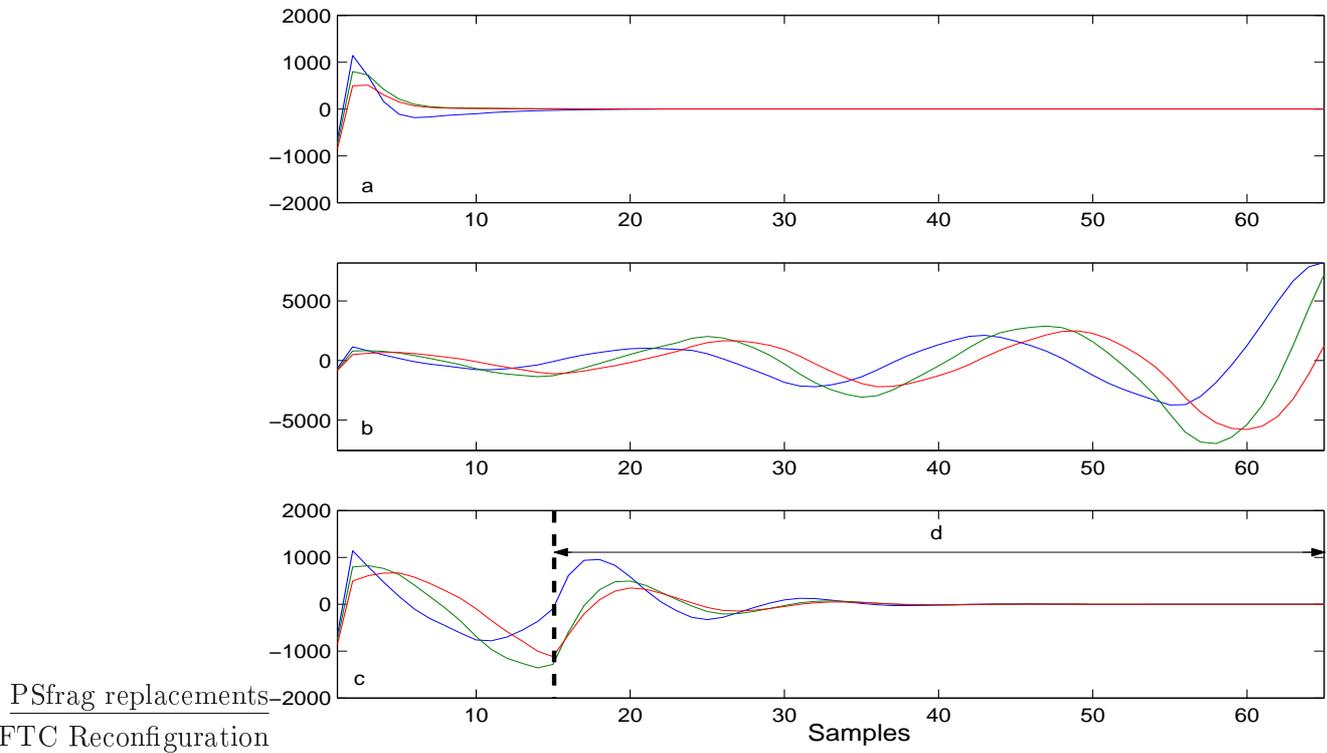


Figure 2: Outputs: (a) Nominal case, (b) Faulty case, (c) Reconfiguration case

at sample  $k = 2$  and finally, *c*) represents the reconfiguration case at time instant  $k = 15$ s. Figure (2).b) illustrates the instability of the closed-loop system in faulty-case and Figure (2).c) illustrates the contribution of the proposed Fault Tolerant Control: the outputs converge toward to their nominal values.

Moreover, the corresponding actuator signals are depicted on Figures (3-4). Figure (3).a) and 4.a) present the actuators in nominal case and Figure (3).b) illustrates the lost of the 1st actuator. Figure (4.b) illustrates the instability of the 2nd actuator in faulty-case and Figure (4.c) the reconfigured control law with the 2nd actuator. In order to simulate a time delay of the FDI block, the new control law is only applied at sample  $k = 15$ , see Figures (2.c) and (4.c). [21] discusses issues with a time delay in FTC reconfiguration. The reader could refer to this NASA/NIA Report for more information on time delay in reconfiguration. We do not deal more with time delay on reconfiguration because we assume that a perfect FDI is available. We observe that the outputs and the control laws converge to zero. The system is stabilized with the Fault Tolerant Control law in spite of these actuator fault and failure. Figure (5)

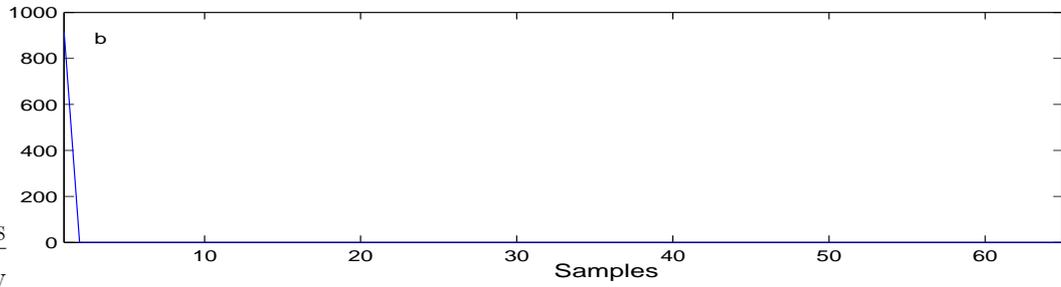
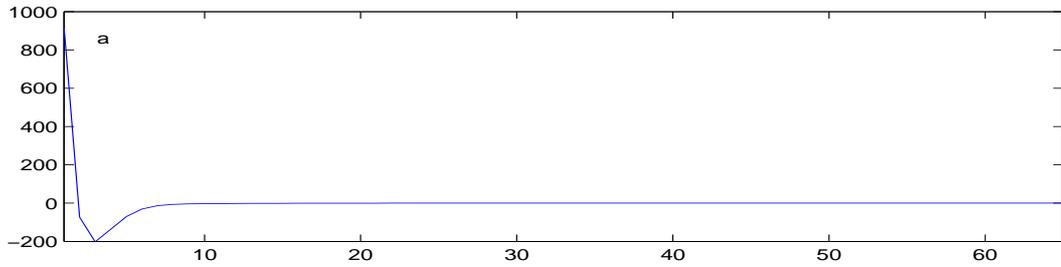


Figure 3: 1st Actuator:(a)Nominal case, (b)Faulty and reconfiguration case

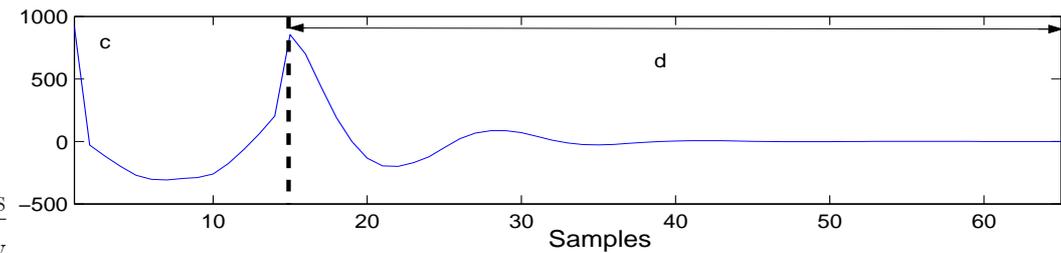
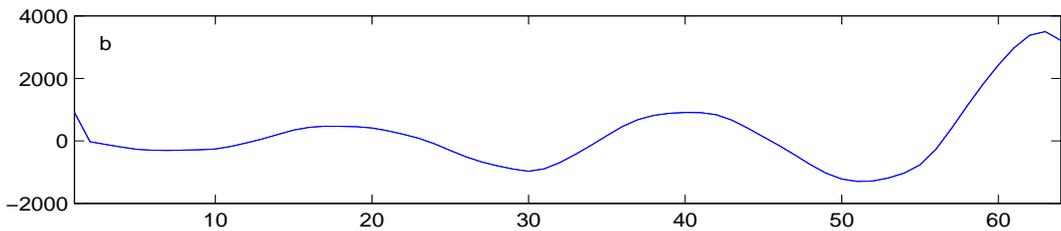
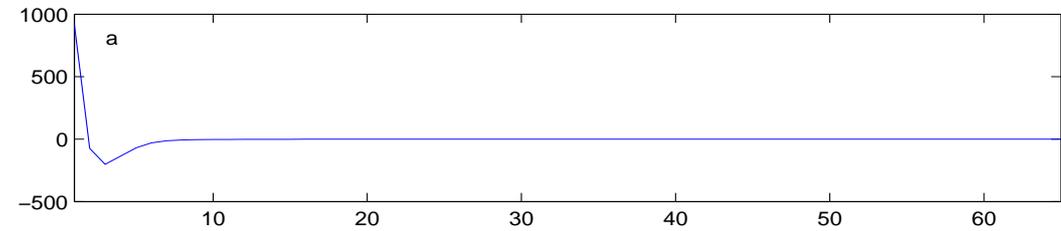


Figure 4: 2nd Actuator:(a)Nominal case, (b)Faulty case, (c) Reconfiguration case

represents the evolution of closed-loop system eigenvalues which still remain into unit circle both in fault-free case (in blue with 'o') and faulty case (in red with '\*') with FTC strategy. The  $\mathcal{LMI}$ -region is represented by a dashed circle. Figure (6) represents the evolution of closed-loop system eigenvalues in faulty case without FTC: we can see the closed-loop system is unstable. So,

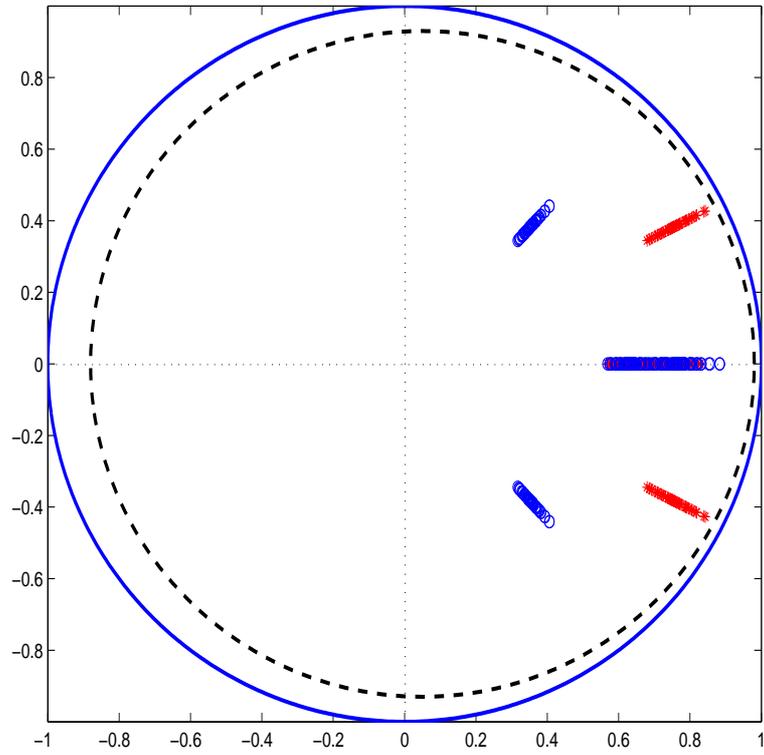


Figure 5: Domain of the closed-loop system eigenvalues in fault-free case (in blue with 'o') and with FTC strategy (in red with '\*')

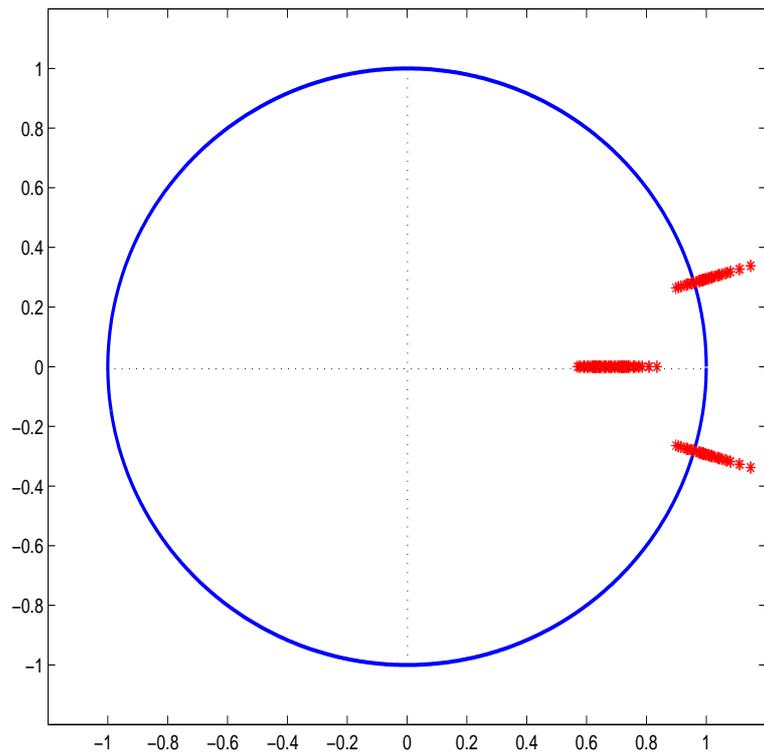


Figure 6: Domain of the closed-loop system eigenvalues in faulty case without FTC

the developed FTC strategy allows the system to continue to operate safely in spite of actuator failures.

## 5 Conclusion

The FTC method presented in this paper underlines the importance of the Fault Tolerant Control on polytopic LPV systems. Controllers are designed for each separate actuator through an  $\mathcal{LMI}$  pole placement in fault-free and faulty cases. The system continues to operate safely and ensures closed-loop stability in spite of the presence of actuator failures. The main contribution is the design of a Static Output Feedback that takes into account the information provided by a FDI scheme. The proposed SOF solution is sufficient and place eigenvalues of the closed-loop system into a predetermined  $\mathcal{LMI}$  region inside the unit circle. From investigating a new algorithm point of view in FTC, it may be a first step to develop a more practical active FTC for nonlinear systems based on polytopic LPV representation. An example on a polytopic LPV system has been presented to illustrate the effectiveness of the scheme.

## Acknowledgments

We are grateful to anonymous referees for their constructive comments that have helped us to improve the paper.

## References

- [1] G. Z. Angelis. *System Analysis, Modelling and Control with Polytopic Linear Models*. Phd thesis, University of Eindhoven, The Netherlands, 2001.
- [2] M. Blanke, M. Kinnaert, J. Lunze, and M. Staroswiecki. *Diagnosis and Fault-Tolerant Control*. Edts Springer-Verlag, 2003.

- [3] M. H. Bouazizi, A. Kochbati, and M. Ksouri. Hinf control of LPV systems with dynamic output feedback. In *Proc. of the 9th Mediterranean Conference on Control and Automation (MED'01), Dubrovnik, Croatia, 2001*.
- [4] A. Casavola, D. Famularo, and G. Franzè. Predictive control of constrained nonlinear systems via LPV linear embeddings. *International Journal of Robust and Nonlinear Control*, 13(3-4):281–294, 2003.
- [5] M. Chadli, D. Maquin, and J. Ragot. A lmi formulation for output feedback stabilization in multiple model approach. In *Proc. of the 41'st IEEE Conf. on Decision and Control, Las Vegas, USA*, pages 311–316, 2002.
- [6] M. Chilali and P. Gahinet.  $H_\infty$  design with pole placement constraints: an LMI approach. *IEEE Trans. on Automatic Control*, 41(3):358–367, 1996.
- [7] J. S. Eterno, D. P. Looze, J. L. Weiss, and A. S. Willsky. Design issues for fault-tolerant restructurable aircraft control. In *Proc. of the 24th IEEE Conference on Decision and Control, Fort Lauderdale, 1985*.
- [8] J.C. Geromel, C.C DeSouza, and R.E. Skelton. Static Output Feedback Controllers: Stability and Convexity. *IEEE Transactions on Automatic Control*, 43:120–125, 1998.
- [9] S. F. Glover. *Modeling and Stability Analysis of Power Electronics based Systems*. Phd thesis, Purdue University, USA, 2003.
- [10] F. Jabbari. Output Feedback Controllers for Systems with Structured Uncertainty. *IEEE Transactions on Automatic Control*, pages 715–719, 1997.
- [11] S. Kanev. *Robust Fault-Tolerant Control*. Phd thesis, University of Twente, The Netherlands, 2004.

- [12] M. Maki, J. Jiang, and K. Hagino. A stability guaranteed active fault-tolerant control system against actuator failures. In *Proc. of the 40th IEEE Conference on Decision and Control, Orlando, Florida, 2001*.
- [13] H. Noura, D. Sauter, F. Hamelin, and D. Theilliol. Fault-tolerant control in dynamic systems: Application to a winding machine. *IEEE Control Systems Magazine*, pages 33–49, 2000.
- [14] R.J. Patton. Fault-tolerant control: the 1997 situation. In *Proc. IFAC Symposium Safeprocess, Kingston Upon Hull, U.K*, volume 2, pages 1033–1055, 1997.
- [15] L. Reberga, D. Henrion, J. Bernussou, and F. Vary. LPV modeling of a turbofan engine. In *Proc. 16th IFAC World Congress, Prague, Czech Republic, 2005*.
- [16] M. Rodrigues. *Diagnostic et commande active tolérante aux défauts appliqués aux systèmes décrits par des multi-modèles linéaires*. Phd thesis, Centre de Recherche en Automatique de Nancy, UHP, Nancy, France, 2005.
- [17] M. Rodrigues, D. Theilliol, M. Adam-Medina, and D. Sauter. A Fault Detection and Isolation Scheme for Industrial Systems based on Multiple Operating Models. *Control Engineering Practice, To appear*, 2006.
- [18] M. Rodrigues, D. Theilliol, and D. Sauter. Design of an Active Fault Tolerant Control and Polytopic Unknown Input Observer for Systems described by a Multi-Model Representation. In *Proc. 44th IEEE Conference on Decision and Control and European Control Conference ECC, Sevilla, Spain, 2005*.
- [19] M. Rodrigues, D. Theilliol, and D. Sauter. Fault Tolerant Control Design of Nonlinear Systems using LMI gain Synthesis. In *Proc. 16th IFAC World Congress, Prague, Czech Republic, 2005*.

- [20] D. Rosinova and V. Vesely. Robust Static Output Feedback for Discrete Time Systems LMI Approach. *Periodica Polytechnica*, 48(3-4):151–163, 2004.
- [21] J-Y. Shin. Parameter transient behavior analysis on fault tolerant control system. Technical Report NASA-CR-2003-212682-NIA Report No. 2003-05, National Institute of Aerospace, Hampton, Virginia, USA, december 2003.
- [22] S. Da Silva, V. Lopes Junior, and E. Assuncao. Robust control to parametric uncertainties in smart structures using linear matrix inequalities. *Journal of the Braz. Soc. of Mech. Sci. Eng.*, 26(4):430–437, 2004.
- [23] D. Theilliol, H. Noura, and J.C. Ponsart. Fault diagnosis and accommodation of three-tank system bsaed on analytical redundancy. *ISA Transactions*, 41:365–382, 2002.
- [24] D. Theilliol, D. Sauter, and J.C. Ponsart. A multiple model based approach for Fault Tolerant Control in nonlinear systems. In *Proc. IFAC Symposium Safeprocess, Washington .D.C, USA, CD-Rom*, 2003.
- [25] R. Veillette. Design of reliable control systems. *IEEE Transactions on Automatic Control*, 37:290–304, 2002.
- [26] Z. Wan and M.V. Kothare. Efficient scheduled stabilizing output feedback model predictive control for constrained nonlinear systems. *IEEE Transactions on Automatic Control*, 2004.
- [27] N. E. Wu, Y. Zhang, and K. Zhou. Detection, estimation and accommodation of loss of control effectiveness. *Int. J. of Adaptive Control and Signal Processing*, 14(7):775–795, 2000.
- [28] Y. Zhang and J. Jiang. Integrated active Fault-Tolerant Control using IMM approach. *IEEE Transactions on Aerospace and Electronics Systems*, 37(4):1221–1235, 2001.
- [29] Y. Zhang and J. Jiang. Bibliographical review on reconfigurable Fault-Tolerant Control systems. In *Proc. IFAC Symposium Safeprocess, Washington. D.C, USA, CD-Rom*, 2003.

- [30] Y. Zhang, J. Jiang, Z. Yang, and A. Hussain. Managing performance degradation in Fault Tolerant Control Systems. In *Proc. 16th IFAC World Congress, Prague, Czech Republic, 2005*.