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A novel approach to the heat build-up problem of rubber

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Rubber is well-known to have a hysteretic behavior and a low thermal conductivity. The main consequences are the heat generation and heat build-up phenomena which occur in a rubber structure when submitted to repeated deformations. The purpose of our work is to estimate the steady-state temperature reached in elastomer structures when submitted to cyclic loadings.

A decoupled strategy of the cyclic thermomechanical problem resolution is adopted. First a mechanical analysis is performed with a hyperelastic stabilized behavior of rubber. Then an estimation of the energy converted into heat is given. It is based on an energetic measure of a mechanical cycle and links the hysteretic loss energy to the elastic mechanical variables. It accounts for mean strains. An experimental investigation has been carried out in order to identify the estimation of the heat sources at various temperatures and mean strains. The influence of the Mullins effect on the dissipation is discussed. Finally, the thermal problem is solved and the influence of the geometry deformation on the heat equation in finite strains is taken into account with an approximation based on average values over a cycle.

The heat build-up law is validated: heat build-up experiments have been carried out and comparisons with FE calculations have shown the ability of the law to estimate the transient and stationary temperature of an inhomogeneous structure submitted to cyclic loadings.

1 INTRODUCTION

Heat generation is the conversion of applied mechanical energy into heat due to the hysteretic behaviour of rubber. The causes and consequences of heat generation have been largely explored in the literature (Medalia 1991; Meinecke 1991). It is generally admitted that it is caused by molecular friction, but some recent works (Toki et al. 2000; Trabelsi et al. 2003) attribute the hysteresis in large strain to strain-induced crystallisation.

Heat build-up is the rise of temperature in a structure submitted to repeated deformation caused by heat generation. It is characterized by a slow increase of the mean temperature until the steady-state temperature field is reached. A quick temperature oscillation is superimposed to the mean temperature. This oscillation is caused by thermoelastic effects (Chadwick 1974).

Our motivation in the present work is to develop a

heat build-up law that provides a robust approach to calculate the temperature rise in a non-homogeneous structure. That requires to solve a thermomechanical problem, defined by both mechanical and thermal equations. Coupled or uncoupled methods are available to solve the thermomechanical problem:

1. Fully-coupled algorithm (Reese & Govindjee 1998): both thermal and mechanical variables are calculated at the same time;
2. Fractional step methods (Armero & Simo 1992) that split the thermomechanical problem in two simpler problems;
3. Uncoupling the thermal and mechanical problems under the assumption of a cyclic loading (Sarkar et al. 1987): first the mechanical problem is solved for one cycle and then the thermal problem is solved for many cycles on one fixed

geometry until the rise of temperature is significant to update the mechanical problem with the actual temperature.

The first two methods are exact, and are indicated for example to solve the thermoelastic problem on a few number of cycles (Holzapfel & Simo 1996). The last method is often used in the literature for heat build-up problems since it offers a quick solving.

The outstanding difficulty of the thermomechanical problem lies in the couplings between the thermal and mechanical variables. These thermomechanical couplings can be classified into four categories:

1. The temperature dependence on stress;
2. The thermoelastic coupling (Chadwick 1974);
3. The heat generation induced by the hysteretic behaviour of rubber;
4. The coupling between the geometry deformation and the heat equation.

This last coupling arises from the resolution of the thermal problem on a moving domain due to the cyclic deformation of the body. This coupling is neglected by the uncoupled cyclic solving since the heat equation is solved on one fixed geometry and not on the actual geometry. If this coupling is negligible under small strains, it becomes essential in large strains. The temperature dependence of stress allows a fine analysis of stress, but is not essential when the main matter is the determination of the heat build-up temperature. The contribution of the thermomechanical coupling to the dissipation over a stabilized cycle is actually zero (Lion 1997); it does not affect the steady-state temperature.

The determination of heat generation is the heart of the matter. It is admitted that nearly all the loss energy is converted into heat. Thus determining the heat generation means determining the loss energy. It can be directly calculated from a dissipative model (Bérardi et al. 1996) or from an estimation based on the hyperelastic behaviour. For example, linear viscoelastic models are often used for small strain (Sarkar et al. 1987), and non-linear models for large strain analysis (Gurvich et al. 2002).

When the mechanical modeling represents any dissipation when a body is subjected to deformation, an estimation of the heat sources is necessary to perform a thermal analysis. This estimation is composed of three points:

1. A method for defining and counting cycles;
2. A stored-energy-related quantity (SERQ);
3. An energy dissipation function linking the SERQ to the dissipation.

For large strain analysis, non-linearities cannot be ignored and hysteretic loops are not ellipses. We aim at developing a novel estimation of the heat sources based on a specific experimental campaign. Uniaxial tests have been performed on pure shear test pieces. The effects of strain history, mean strain and temperature have been studied and modeled.

With a view to validating the heat build-up law, an experimental study has been carried out. Uniaxial cyclic tests with various mean strain and amplitude loadings have been performed on axisymmetric dumbbell specimens named diabolos (Ostoja-Kuczynski et al. 2003). The experimental results are compared to FE calculations.

The paper is organized as follows: in section two the cyclic uncoupled thermomechanical approach is presented. In section three an estimation of the heat sources is given. The fourth section deals with experimental dissipation results: the effects of strain history, mean strain and temperature are discussed and the phenomenological energy dissipation function is identified. The fifth section presents the heat build-up experiments and validates the heat build-up law. Our conclusions are summarized in the final section.

2 CYCLIC UNCOUPLED THERMOMECHANICAL APPROACH

2.1 *Cyclic uncoupled solving*

Assume a cyclic mechanical loading. Our analysis is based on the difference between mechanical and thermal characteristic times in heat build-up problems: the temperature increases slowly and many mechanical cycles are needed to reach the steady-state temperature. In fact, there are two thermal characteristic times. The first comes from the thermoelastic coupling: the temperature oscillates with the same frequency as the mechanical loading (Lion 1997; Reese 2003). This oscillation is about a few degrees and a mean temperature over a cycle is defined. The second is due to the hysteretic dissipation that makes the temperature increase slowly from the initial field to the steady-state temperature distribution.

In heat build-up problems, the temperature oscillation has no interest and the main matter is the determination of the mean temperature. We neglect the thermoelastic coupling, and then we can use the cyclic uncoupled resolution of the thermomechanical problem. The uncoupled cyclic algorithm (Sarkar et al. 1987) is adopted: the mechanical problem is first solved, then the loss energy is estimated and put into the heat equation as a thermal source. A loop is then created with updating material data with the temperature.

2.2 *Mechanical modeling*

Two possibilities are available to model the mechanical behaviour: a hyperelastic or a dissipative model.

The advantage of the dissipative model is the direct access to the dissipation but it requires to compute each cycle. On the contrary, the main advantage of the hyperelastic model is the estimation of the loss energy for many different mechanical cycles: only one mechanical calculation is needed. For this reason an isothermal hyperelastic model is adopted. This assumption implies an estimation of the heat sources. This feature is developed in the next section.

2.3 Taking into account the geometry deformation

The cyclic uncoupled algorithm is accurate for small or medium strain. The body is not very deformed and the boundary value problem of heat equation can be solved on a fixed geometry and totally uncoupled from the mechanical analysis. But if the strain is large enough to modify the geometry of the structure inside the mechanical cycle, the solution of the thermal analysis will strongly depend on the choice of the fixed geometry, independently of the configuration.

In the case of periodic deformation, a method to solve the heat equation with taking into account the cyclic geometry deformation has been developed and should be soon published. The scheme of the solving is the following: first write the heat equation in reference configuration, then define mean variables such as temperature or thermal conductivity and finally integrate on time and approximate the heat equation and the boundary conditions.

3 ESTIMATION OF THE HEAT SOURCES

An estimation of the loss energy for a deformation cycle is proposed in this section. It should be pointed out that this estimation must be based on local variables and not on global variables in order to account for complex structure. As already mentioned in the introduction, the three points defining an estimation are developed in the following.

3.1 Cyclic assumption

The deformation is supposed periodic and one and only one cycle is defined for one period. This cycle is described by the two instants representative of its extremal values.

3.2 SERQ

The research of a SERQ must respect two principles:

1. Positivity;
2. Objectivity (Truesdell & Noll 1992) (invariance under a change of frame).

The positivity is obviously necessary since a dissipation cannot be negative. The objectivity guarantees that a quantity transforms as an objective quantity under change of the observer.

Our motivation in the present work is also to develop a SERQ that account for mean strains. For instance, the strain energy density is a positive and objective quantity, but it is not adapted to mean strains: the strain energy is not representative of the dissipation because the strain energy amplitude of a cycle depends on the mean strain but not the dissipation (the experimental results are presented below).

A global SERQ was firstly built on the force-displacement curve for a homogeneous structure under a cyclic loading. An arbitrary quantity is subtracted from the strain energy amplitude to remove the effect of the mean strain. In the uniaxial case (the force is parallel to the displacement), the Elastic Energy of Dissipation E_{ed} (it has the dimension of an energy, is based on elastic variables and is a measure of the dissipation) is defined by

$$E_{ed} = \int_{l_{min}}^{l_{max}} (F - F_{min}) dl \quad (1)$$

where F and F_{min} are the force at instant t and t_{min} , and l the displacement. It is shown in the experimental part that this quantity is relevant to estimate the dissipation with various mean strains.

This global SERQ is extended for local loadings. The associated stress and strain tensors are the first Piola-Kirchhoff (or Boussinesq) stress tensor and the deformation gradient. Unfortunately, the direct transposition of the equation (1) is not possible since the quantity would not be objective by the use of mixed tensor at different instants. The solution is to use the conjugate tensors (Hill 1968). The strain and stress tensors are then

$$\underline{\underline{E}}^{(1)} = \underline{\underline{U}} - \underline{\underline{I}}, \quad \underline{\underline{T}}^{(1)} = \frac{1}{2} (\underline{\underline{T}}^{(2)} \cdot \underline{\underline{U}} + \underline{\underline{U}} \cdot \underline{\underline{T}}^{(2)}) \quad (2)$$

where $\underline{\underline{U}}$ is the right stretch tensor coming from the unique polar decomposition of the deformation gradient $\underline{\underline{F}} = \underline{\underline{R}} \cdot \underline{\underline{U}}$ and $\underline{\underline{T}}^{(2)}$ is the second Piola-Kirchhoff stress tensor. The Lagrangian description of these tensors allows to write an objective quantity as:

$$\frac{dE_{ed}}{d\Omega_0} = \int_{\underline{\underline{E}}_{min}^{(1)}}^{\underline{\underline{E}}_{max}^{(1)}} (\underline{\underline{T}}^{(1)} - \underline{\underline{T}}_{min}^{(1)}) : d\underline{\underline{E}}^{(1)}. \quad (3)$$

The minimum and the maximum of the cycle are defined by the minimum and the maximum of the strain energy over the cycle, and guaranty the positivity of the E_{ed} .

For a periodic loading, the dissipation law is finally given by the identification of a whole cycle with its extreme values by the Elastic Energy of Dissipation E_{ed} . The identification of the energy dissipation function that links the E_{ed} to the loss energy is presented in the next section.

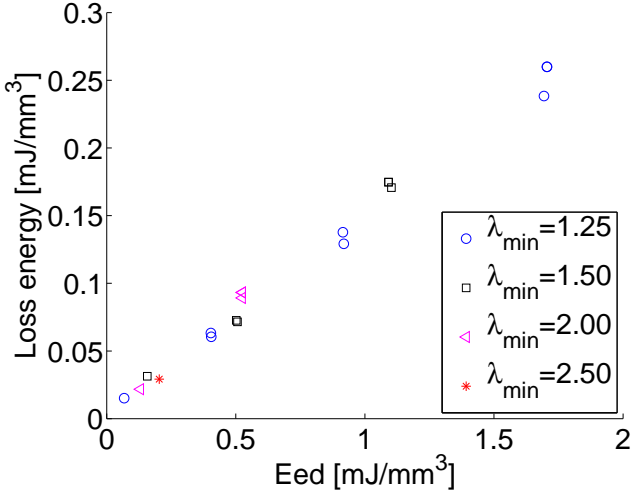


Figure 1: Effect of the minimal stretch on the dissipation.

4 EXPERIMENTAL ENERGY DISSIPATION FUNCTION

This section is devoted to the uniaxial dissipation experiments. They have been carried out on a pure shear testpiece ($2 \times 35 \times 300$ mm). The rubber used in the experiments is a carbon-black-reinforced natural rubber. Its recipe and mechanical characteristics are given in (Ostojka-Kuczynski et al. 2003). Cyclic tests are used for the estimation of the heat sources. A cycle is defined by the minimal stretch λ_{min} , the maximal stretch λ_{max} and the previous maximal stretch λ^M . We assume that the stabilized behavior is reached after a thousand cycles at the maximal stretch. Displacement controlled tests and residual strains make us choose the minimal elongation greater than 1, in order to avoid the collapse of the pure shear sample.

4.1 E_{ed} as a measure of the dissipation - effect of a mean strain

The loss energy versus E_{ed} is plotted for various minimal stretches. The results presented in figure 1 shows that there is no effect of the mean strain and that the loss energy is highly correlated to the E_{ed} . The same plot but with the strain energy amplitude (not presented here) shows a high dispersion in the plot leading to the conclusion that the strain energy is not representative of the loss energy and that the E_{ed} is a relevant measure of the dissipation.

4.2 Effect of strain history

The effect of strain history or the Mullins effect (Mullins & Tobin 1957) is studied on one testpiece. A succession of sinusoidal loadings constitutes the complete loading. We have validated the fact that a great number of cycles at lower strain has not any influence on the behaviour at greater strain as described by the Mullins effect. It is then possible to apply many cycles

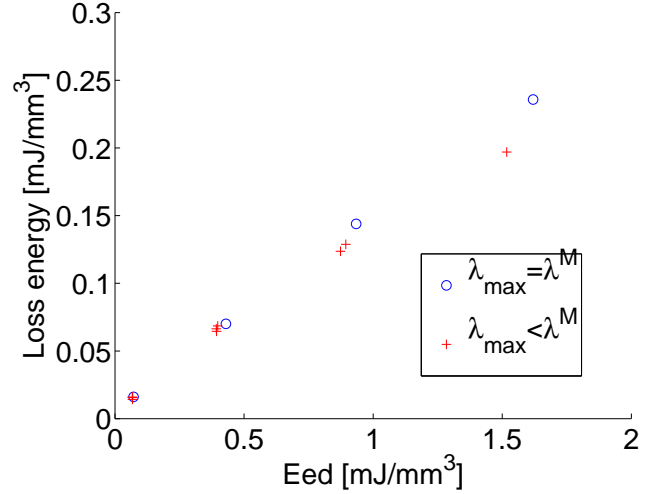


Figure 2: Loss energy versus E_{ed} for two conditions: at maximal extension and under the previous maximal extension.

at various stretches on the same sample.

Given a fixed extension, the effect of strain history on the dissipation is analyzed. The experimental results plotted in figure 2 show that the relation between the loss energy and the E_{ed} is nearly independent of the strain history. It can be inferred that although the dissipation depends on the strain history, the dissipation law based on E_{ed} is nearly independent of the strain history. It is then possible to describe the influence of the Mullins effect on the dissipation in a FE calculation with an accurate constitutive law.

4.3 The energy dissipation function

The phenomenological energy dissipation function is identified on the stabilized behavior. A power law is suitable to fit the data:

$$E_{loss} = k E_{ed}^\alpha \quad (4)$$

The temperature is an important parameter: it has a strong influence on the dissipation and the temperature of a structure is often non-homogeneous. This power law is estimated in the range of temperature 5, 23, 40, 60 and 80°C. It has been concluded that the power α is independent of the temperature, and that the parameter k decreases with the temperature. The relation between k and the temperature T is approximated by a logarithmic function, such as

$$k = a \ln T + b. \quad (5)$$

5 HEAT BUILD-UP EXPERIMENTS

A heat build-up experiment consists in applying a mechanical loading to a structure and measuring the temperature. An experimental campaign has been carried out to validate the heat build-up law through a comparison with FE analysis.

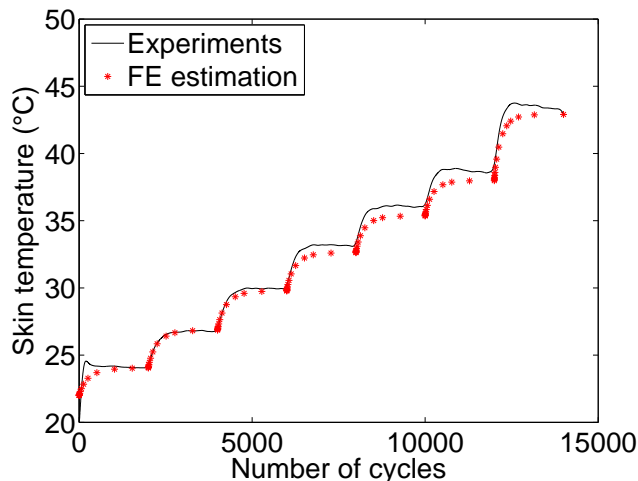


Figure 3: Maximal skin temperature versus time for the experiment and the FE estimation.

5.1 FE modeling

As previously mentioned, a hyperelastic law representing the stabilized behaviour allows to account for the Mullins effect on the loss energy. The constitutive model proposed by Raoult (Raoult et al. 2005) is adopted. The estimation of the heat sources is based on the theoretical E_{ed} from Raoult's law and gives the experimental loss energy. The heat sources are updated with the temperature since the energy dissipation function depends on the temperature. The FE calculations are performed with ABAQUS.

5.2 Experiments

The dumbbell specimen or diabolo (Ostoja-Kuczynski et al. 2003) has been chosen since its geometry is representative of the automotive components for heat build-up. The mechanical loading is composed of 7 blocks of 2000 cycles at increasing maximal displacement. The frequency is 1 Hz. The maximal skin temperature is measured with an infrared thermometer.

Two types of comparison allow to discuss about the heat build-up law. The first is naturally the temperature. The experimental and the FE-estimated maximal skin temperature is plotted versus time in figure 3. A very good agreement is obtained. The second is the estimation of the loss energy. The total loss energy (measured and estimated) is plotted versus time in figure 4. Two numerical estimations are given: at 1 and 2000 cycles for each loading. The difference between these two estimations arises from the temperature dependence of the dissipation law. Again, a very good agreement is obtained.

In view of the results, we come to the conclusion that the heat build-up law is accurate for estimating the loss energy, the transient and steady-state temperature of a structure submitted to a cyclic deformation.

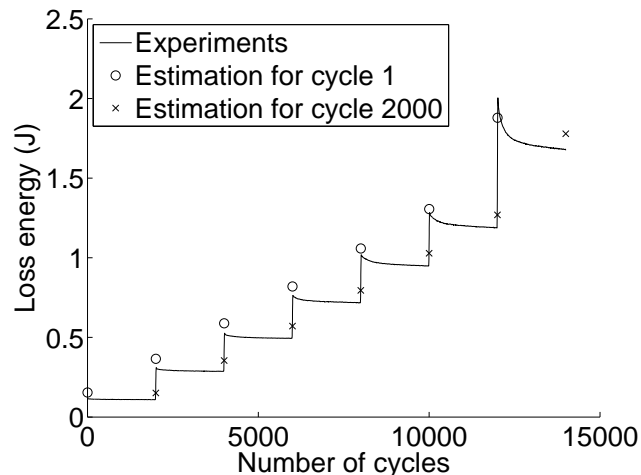


Figure 4: Experimental and FE-estimated loss energy versus time.

6 CONCLUSIONS

A robust approach to the heat build-up problem of rubber has been developed. It is based on a cyclic uncoupled thermomechanical analysis. A hyperelastic mechanical analysis describing the Mullins effect is first performed. An estimation of the heat sources accounting for mean strains has been identified on the stabilized behavior. It estimates from the mechanical variables the heat sources which are put into the simplified thermal analysis. This latter accounts for the variation of the geometry along the mechanical loading.

A comparison between experimental results and FE-analysis has been conducted. The conclusion to be drawn from these results is that this novel approach estimates with a good accuracy the heat build-up in a non-homogeneous structure.

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