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Low energy scattering amplitudes for two atoms in one- and two-dimensional atomic wave guides are derived for short range isotropic and resonant interactions in high partial wave channels. Taking into account the finite width of the resonance which was neglected in previous works is shown to have important implications in the properties of the confinement induced resonances. For spin polarized fermions in quasi-1D wave guides it imposes a strong constraint on the atomic density for achieving the Fermi Tonks Girardeau gas. For a planar wave guide, the charateristics of the 2D induced scattering resonances in p- and d-wave are determined as a function of the 3D scattering parameters and of the wave guide frequency.

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Recent experimental progress in degenerate atomic gases make possible accurate studies of quasi-one (1D) [1, 2, 3, 4, 5, 6] and quasi-two dimensionnal (2D) [6, 7, 8] configurations. These systems are interesting in view of future applications involving coherent manipulation of matter waves and can be used also for studying generic phenomenona in low dimensions [1, 8]. One major interest of atomic gases is the precise knowledge and experimental control of the low energy interatomic collisions: the effective two-body interaction can be tuned with a Feshbach resonance [9] by applying an external magnetic field and/or with a Confined Induced Resonance (CIR) by varying the extension of the trap in the tight transverse direction [10, 11, 12]. These techniques open possibilities for achieving new types of strongly correlated quantum systems. For example, thanks to s-wave collisional properties in quasi-1D wave guides [10], it has been possible to observe the so-called Tonks Girardeau (TG) gas [13] where 1D hard-core bosons can be mapped onto a system of non interacting fermions [3, 4]. For quasi-1D spin polarized fermions an analogous regime: the Fermionic Tonks Girardeau (FTG) gas is subject to intensive studies [14, 15, 16]. In this case, as a consequence of the Pauli exclusion principle, atoms interact predominantly in the p-wave channel and it has been predicted that the strongly interacting 1D polarized Fermi gas can be mapped onto a non interacting Bose gas.

In this letter, it is shown that the finite width of the high partial waves resonances is an essential feature in the properties of the corresponding CIR. As a consequence, the resonant fully polarized Fermi atomic gas in quasi-1D traps can reach the FTG regime in a very dilute limit only (which means that for realistic trap parameters, only few particles systems can undergo this regime) and is more generally described by a narrow BEC-BCS crossover. Violation of the fermion-boson mapping theorem [17] opens relevant issues on the quasi-1D resonant polarized Fermi gas such as the equation of state and the shift and damping of the collective modes which are only known at the FTG limit [16]. Motivated by the exciting predictions of exotic superfluid phases [18, 19] with non-Abelian statisitics, the p-wave scattering amplitude in quasi-2D atomic wave guide induced by a resonant interaction is derived. The effective range parameter which were neglected in Ref.[20] appears essential for the determination of the 2D low energy scattering parameters. Due to the possibility of achieving d-wave resonances [21] in 3D systems, the quasi-2D d-wave scattering amplitude is also derived and the characteristics of this new CIR are depicted.

In the following, the true interatomic forces are supposed to be short range and isotropic. Moreover they are considered in the neighborhood of a resonance. The 3D low energy collisional properties of two atoms are then parameterized in each partial wave ($l \geq 0$) by the following phase shift $\delta_l(q)$:

$$q^{2l+1} \cot \delta_l(q) = -\frac{1}{w_l} - \alpha_l q^2 + O(q^4), \tag{1}$$

in Eq.(1) q is the relative momentum of the two colliding atoms, and the resonant regime in the l-partial wave is achieved for $|w_l| \gg R^{2l+1}$, where R is the characteristic radius of the pairwise potential (w_0 is the usual scattering length and w_1 is the scattering volume). The effective range parameter α_l is linked to the width of the resonance: a large (small) value of $\alpha_l R^{2l-1}$ corresponds to a narrow (relatively broad) resonance. In the neighborhood of the resonance and for $l \geq 1$, it has been shown recently that for a finite range potential [22, 23, 24]:

$$\alpha_l R^{2l-1} \gtrsim (2l-3)!!(2l-1)!! \ . \tag{2}$$

Thus $\alpha_l |w_l| \gg R^2$, and $-\hbar^2/(2\mu\alpha_l w_l)$ is a low energy scale (μ is the reduced mass) which is for large and positive values of w_l, nothing but the shallow bound state energy (and the quasi-bound state energy for large and negative w_l). Hence, unlike s-wave broad resonances where $\alpha_0 \equiv O(R)$ can be neglected in the low energy limit, for $l > 0$ the effective range parameter α_l which depends on the specific resonance considered, is an essential parameter involved in the low energy properties. In experiments the resonant regime can be achieved by using a Feshbach resonance [9] on the spin degree of freedom. An external magnetic field B modifies the detuning between an open and a closed channel of the two-body system, and in the vicinity of the resonance in the partial wave l where $B \sim B_0$, $w_l^{-1} \propto -(B - B_0)$ while α_l is almost constant.

Due to the short range character of interatomic forces, we use the so-called zero range approximation. Hence, the pairwise interaction between particles is replaced by a source term of vanishing range $\epsilon \to 0$ in the Schrödinger's equation (it is assumed that the characteristic lengths of the trapping potential are $\gg R$). Moreover, the trapping potentials considered in this letter are always harmonic, so that the center of mass and relative coordinates are decoupled each from the other. For a binary collision in the center of mass frame at energy $E = \hbar^2 q^2/\mu$, the Schrödinger's equation can be transformed into an integral equation for the wave function $|\Psi\rangle$:

$$|\Psi\rangle = |\Psi_0\rangle + \frac{2\pi\hbar^2}{\mu}\sum_{l\geq 0}\int \frac{d^3\mathbf{k}}{(2\pi)^3}\,\frac{k^l\delta_\epsilon(k)\,(\mathcal{R}_l\cdot\mathcal{S}_{l,\mathbf{k}})}{\mathcal{H}_0 - E - i0^+}|\mathbf{k}\rangle, \quad (3)$$

where $\mathcal{H}_0$ is the free Hamiltonian which includes the external potential, and $|\Psi_0\rangle$ belongs to the kernel of $\mathcal{H}_0 - E$. In Eq.(3), the source term is introduced in the momentum representation: $\delta_\epsilon(k) = \exp(-k^2\epsilon^2/4)$ is the Fourier transform of a normalized Gaussian weight having a vanishing range ϵ [25], $\mathcal{R}_l$ and $\mathcal{S}_{l,\mathbf{k}}$ are Symmetric Trace Free (STF) tensors of rank l, and the notation $(\mathcal{R}_l\cdot\mathcal{S}_{l,\mathbf{k}})$ means a contraction between the two tensors. The tensors $\mathcal{S}_{l,\mathbf{k}}$ are eigenfunctions of the momentum operator and they appear in a standard multipolar expansion in Cartesians coordinates [22]: $\mathcal{S}_{l,\mathbf{k}}^{[\alpha\beta\dots]} = (-1)^l k^{l+1}\left(\partial_{k_\alpha}\partial_{k_\beta}\dots\right)k^{-1}/(2l-1)!!$, where $\{k_\alpha\}$ are the Cartesian components of the vector $\mathbf{k}$ ($\alpha \in \{x, y, z\}$). Eq.(3) shows explicitly that the interacting wave function $|\Psi\rangle$ is the superposition of a regular solution $|\Psi_0\rangle$ and of an irregular part generated by the source term [27]. The correct asymptotic behavior of the wave function (for relative coordinates $|\mathbf{r}| \gg R$) is obtained by the determination of the tensors $\mathcal{R}_l$ which fixes the balance between the regular and irregular solutions. For this purpose, contact conditions (for $\mathbf{r} \to 0$) are imposed in each partial wave and are such that without external potential the phase shifts in a scattering process coincide *exactly* with the first two terms in the right hand side of Eq.(1) [22]. In the momentum representation, the contact conditions can be written as:

$$\underset{\epsilon\to 0}{\text{Reg}}\int \frac{d^3\mathbf{k}}{(2\pi)^3}k^l\langle\mathbf{k}|\Psi\rangle\mathcal{S}_{l,\mathbf{k}} = -\frac{l!\,\mathcal{R}_l}{a_l(q)(2l+1)!!}, \quad (4)$$

where $a_l^{-1}(q) = (w_l^{-1} + \alpha_l q^2)$. In Eq.(4), $\text{Reg}_{\epsilon\to 0}$ means the regular part of the integral obtained when the formal range ϵ is set to zero. This way, the source term in Eq.(3) is itself a functional of $|\Psi\rangle$, and a closed equation for $\mathcal{R}_l$ is obtained by combination of Eqs.(3,4). In the next parts, the scattering problem for p- or d-wave channels is solved successively in 1D and 2D harmonic wave guides.

Linear atomic wave guide. — Two atoms are confined in a two-dimensional harmonic trap while they move freely along the third direction (z). In the center of mass frame, the non interacting Hamiltonian is:

$$\mathcal{H}_0 = -\frac{\hbar^2}{2\mu}\Delta_{\mathbf{r}} + \frac{1}{2}\mu\omega_\perp^2\rho^2 - \hbar\omega_\perp, \quad (5)$$

where $\mathbf{r} = z\,\hat{\mathbf{e}}_z + \boldsymbol{\rho}$ are the relative coordinates. For a scattering process at energy $E = \hbar^2q^2/2\mu$, the state $|\Psi_0\rangle$ in Eq.(3) is: $\langle\mathbf{k}|\Psi_0\rangle = (2\pi)\delta(k_z - q)\,\langle\mathbf{k}_{2D}|\phi_{00}\rangle$, where $|\phi_{00}\rangle$ is the ground state of the 2D harmonic oscillator in Eq.(5), and $\mathbf{k} = \mathbf{k}_{2D} + k_z\hat{\mathbf{e}}_z$. Hereafter, the system is in the monomode regime ($E < 2\hbar\omega_\perp$). At large distances $|z| \gg a_\perp$ (where $a_\perp = \sqrt{\hbar/\mu\omega_\perp} \gg R$), the wave function factorizes as $\langle\mathbf{r}|\Psi\rangle \simeq \langle\boldsymbol{\rho}|\phi_{00}\rangle\psi_{1D}(z)$, and for $E > 0$:

$$\psi_{1D}(z) = \exp(iqz) + \exp(iq|z|)\left[f^{\text{even}} + \text{sign}(z)f^{\text{odd}}\right]. (6)$$

In this situation, the system which is frozen along the transverse direction, can be considered as quasi-1D. The even (odd) scattering amplitude f^{even} (f^{odd}) results from the asymptotic contributions ($z \gg a_\perp$) in the different even (odd) 3D partial waves. Quasi-1D scattering in the s-wave channel has been thoroughly studied [10] and in the following we consider quasi-1D scattering of two spin polarized fermions in the vicinity of a p-wave resonance. Thus, one can restrict the source term in Eq.(3) to the $l = 1$ contribution, where $\mathcal{S}_{1,\mathbf{k}} = \mathbf{k}/k$, and the choice $|\Psi_0\rangle$ imposes that $\mathcal{R}_1 = P_{1D}\hat{\mathbf{e}}_z$. Computation of P_{1D} from Eq.(4) is much simpler in the domain of negative energy $E < 0$ and using Eq.(3) one can show that:

$$P_{1D} = \frac{-3q\phi_{00}(0)}{\dfrac{1}{a_1(q)} + \dfrac{6}{\sqrt{\pi}a_\perp^3}\text{P.f.}\displaystyle\int_0^\infty \frac{du}{u^{3/2}}\frac{\exp(\tau u)}{1-\exp(-u)}}. \quad (7)$$

where $\tau = E/2\hbar\omega_\perp < 0$ and P.f. denotes the "partie finie de Hadamard" of the integral [28]. Interestingly, one recognizes the Riemann-Hurwitz Zeta function $\zeta_H(-1/2, -\tau)$ in the regularized integral of Eq.(7). Hence, by analytic continuation in the domain $\tau > 0$, one obtains a simple expression of the scattering amplitude, and in the low energy limit $qa_\perp \ll 1$:

$$f_p^{\text{odd}} = 2i\pi P_{1D}\phi_{00}^*(0) \simeq -iq\left(\frac{1}{l_p} + iq + q^2\xi_p\right)^{-1}, \quad (8)$$

with the odd-wave scattering length l_p and the effective range ξ_p given respectively by:

$$l_p = 6a_\perp\left[\frac{a_\perp^3}{w_1} - 12\,\zeta(-\frac{1}{2})\right]^{-1} \quad \text{and} \quad \xi_p = \frac{\alpha_1 a_\perp^2}{6}. \quad (9)$$

Expression of l_p in Eq.(9) coincides with the result in Ref.[12], while the effective range ξ_p and its physical implications was (up to our knowledge) not studied in previous works. However, ξ_p is in general not negligible for all $q \ll a_\perp^{-1}$ and crucially depends on the width of the 3D resonance and on $a_\perp$. Indeed, the inequality $\xi_p \gg a_\perp$ is likely to occur as a consequence of Eq.(2) ($\alpha_1 R \gtrsim 1$) and also from the condition $a_\perp \gg R$ which is needed from the hypothesis that scattering processes are studied at collisional energies $\ll \hbar^2/\mu R^2$. The regime $\xi_p \sim a_\perp$ can be reached only for an extreme transverse confinement ($a_\perp \sim 10R$) and for the broadest p-wave resonances with $\alpha_1 R \sim 1$. In actual experiments, two p-wave resonances are used: the resonance for ^{6}Li atoms ($R \simeq 3$ nm) at $B_0 \simeq 215$ G with $\alpha_1 R \simeq 5$ [29] and the one at $B_0 \simeq 198.8$ G for ^{40}K atoms ($R \simeq 7$ nm) where $\alpha_1 R \simeq 3$ [30]. For $l_p > 0$ the scattering amplitude in Eq.(8) has a pole at $q = i\kappa$ with $\kappa = (-1 + \sqrt{1 + 4\xi_p/l_p})/2\xi_p > 0$ giving a shallow bound state energy at $(-\hbar^2\kappa^2/2\mu)$. For large and positive values of $l_p \gg \xi_p$, $\kappa = l_p^{-1}$ and the bound state has a vanishing energy (*i.e.* $\ll \hbar\omega_\perp$) [31]. The scattering amplitude in Eq.(8) can be obtained from a 1D effective theory where the wave function ψ_{1D} in Eq.(6) solves the non interacting Schrödinger equation and satisfies the following contact condition:

$$\lim_{z\to 0^+} \left(\frac{1}{l_p} + \partial_z - \xi_p \partial_z^2 \right) \psi_{1D}^{\rm odd}(z) = 0, \qquad (10)$$

where $\psi_{1D}^{\rm odd}(z) = [\psi_{1D}(z) - \psi_{1D}(-z)]/2$ is the projection of the wave function onto its odd component [32]. This approach can be generalized for few- and many-body systems by imposing the contact condition in Eq.(10) for each pair of interacting particles. Without performing these calculations which are clearly beyond the scope of this letter, it is of importance to determine the conditions such that the fully polarized fermionic gas can reach the FTG regime or equivalently can be mapped onto non interacting 1D bosons [14]. Eq.(10) implies that this mapping is possible only at resonance ($|l_p| = \infty$) and also if the effective range is negligible. The resonant condition is easily obtained by using a Feshbach resonance. However, the momentum distribution of the FTG gas has a large tail given by a Lorentzian of width $4n$, where n is the 1D atomic density [15], thus ξ_p can be neglected only in the dilute limit: $n\xi_p \ll 1$. For N atoms trapped in a strongly anisotropic trap with a weak harmonic confinement along the z-direction (atomic frequency $\omega_z \ll \omega_\perp$ and axial length $a_z = \sqrt{\hbar/\mu\omega_z} \gg a_\perp$), in the FTG regime one has $n \sim N/a_z$ and the condition $n\xi_p \ll 1$ gives $N \ll 6a_z/(\alpha_1 a_\perp^2)$. For example, considering ^{40}K atoms in a highly anisotropic trap with $\omega_\perp = 2\pi \times 70$ kHz and $\omega_z = 2\pi \times 10$ Hz, the FTG regime is obtained for $N \ll 14$, which makes sense only for few-body configurations (for ^{6}Li atoms with the same trap parameters, ξ_p is of the order of a_z and the mapping to a non interacting Bose system is a poor approximation even for the two-body ground state). Interestingly, if the condition $n\xi_p \ll 1$ is not satisfied, the scale invariance in the linear wave guide is broken at low energy, hence the time-dependent many-body ansatz in Ref.[16] is no more an exact solution. To conclude this part, excepted specific configurations, the criterium $n\xi_p \ll 1$ is in general not verified and the density corrections to the FTG properties are important issues for understanding the resonant gas. Moreover, following the reasoning of the box model in Refs.[23, 34], one expects that by varying l_p from large and positive values to large and negative values, the system experiences a narrow p-wave BEC-BCS cross-over, where the composite bosons in the dilute BEC phase ($l_p > 0$) are dimers populating the quasi-1D shallow two-body bound state.

Planar atomic wave guide. — High partial wave superfluidity in quasi-2D geometries is interesting for its links with condensed matter physics like for example high-T_c superconductivity and the possible applications for quantum computing [18, 19]. These studies motivate a close investigation of quasi-2D scattering properties. In this geometry, the two colliding atoms are confined in a planar harmonic trap along the z-direction and move freely in the (xy) plane. The free Hamiltonian reads:

$$\mathcal{H}_0 = -\frac{\hbar^2}{2\mu}\Delta_{\mathbf{r}} + \frac{1}{2} m\omega_z^2 z^2 - \frac{\hbar\omega_z}{2}. \qquad (11)$$

The homogeneous solution corresponding to a scattering process at energy $E = \hbar^2 q^2/2\mu$ is: $\langle \mathbf{k}|\Psi_0\rangle = (2\pi)^2\delta(\mathbf{k}_{2D} - \mathbf{q})\,\langle k_z|\phi_0\rangle$, where $|\phi_0\rangle$ is the ground state of the 1D harmonic oscillator in Eq.(11). In the monomode regime ($E < \hbar\omega_z$), the system can be considered as quasi-2D, and for large interparticle separation ($\rho \gg a_z$), the wave function factorizes: $\Psi(\mathbf{r}) = \phi_0(z)\psi_{2D}(\rho)$. The 2D partial scattering amplitudes $f^{[m]} = f^{[-m]}$ can be defined by the following expansion:

$$\psi_{2D}(\boldsymbol{\rho}) \underset{\rho \gg a_z}{=} e^{(i\mathbf{q}\cdot\boldsymbol{\rho})} - \frac{i}{4} \sum_{m=-\infty}^{m=\infty} f^{[m]} H_m^{(1)}(q\rho) e^{(im\theta)}, \qquad (12)$$

where $\theta = \pi/2 + \angle(\boldsymbol{\rho}, \mathbf{q})$ and $H_m^{(1)}$ is the Hankel's function. Using Eqs.(3,4,12) the scattering amplitude for two atoms interacting in the s-wave channel is: $f^{[0]} = 4\pi[a_z\sqrt{\pi}/a_0(q) + J_0(\tau + i0^+)]^{-1}$, with $\tau = E/2\hbar\omega_z$, $J_0(\tau) = \ln(-B/(2\pi\tau)) + \sum_{n=1}^{\infty} \ln(n/(n-\tau))(2n-1)!!/(2n)!!$ [35], and $B \simeq 0.9049$ [36]. In the p-wave channel, Eq.(2) implies the existence of a small parameter $\eta_1 = (\alpha_1 a_z)^{-1} \ll 1$ which plays a central role in the scattering properties [37]. For $\mathbf{q} = q\hat{\mathbf{e}}_x$, the p-wave regular part in Eq.(3) is related to the 2D p-wave scattering amplitude $f^{[1]}$ by $\mathcal{R}_1 = -f^{[1]}(q)\hat{\mathbf{e}}_x/[2\pi q\phi_0^*(0)]$. Using the contact condition in Eq.(4) where $\mathcal{S}_{1,\mathbf{k}} = \mathbf{k}/k$ gives:

$$f^{[1]}(q) = 6\pi q^2 |\phi_0(0)|^2 \left[\frac{1}{a_1(q)} + \frac{6J_1(\tau + i0^+)}{\sqrt{\pi} a_z^3} \right]^{-1}, \qquad (13)$$

where $|\phi_0(0)|^2 = 1/(a_z\sqrt{\pi})$. In the low energy limit $|\tau| \to 0$ then $J_1(\tau) = J_1(0) + \tau \ln(-eB/2\pi\tau) + O(\tau^2)$, where $J_1(0) \simeq -5.4722 \times 10^{-2}$ [35], and the logarithmic term ensures the unitarity condition in Eq.(13). A similar calculation can be done for atoms interacting resonantly in the 3D d-wave channel. In this case, the small parameter related to the resonance width is

$\eta_2 = \alpha_2^{-1} a_z^{-3} \ll 1$. For $\mathbf{q} = q\hat{\mathbf{e}}_x$, the $l = 2$ regular part is given by: $k^2\,(\mathcal{R}_2 \cdot \mathcal{S}_{2,\mathbf{k}}) = S_{2D}(2k_z^2 - k_{2D}^2) + D_{2D}(k_x^2 - k_y^2)$, where S_{2D} contributes to the $m = 0$ 2D partial wave channel and D_{2D} contributes to the $|m| = 2$ channel (2D-d-wave). The scattering amplitude in the $|m| = 2$ channel is $f^{[2]}(q) = -2\pi q^2 \phi_0^*(0) D_{2D}$, and using Eq.(4):

$$f^{[2]}(q) = \frac{15\pi q^4 |\phi_0(0)|^2}{2} \left[\frac{1}{a_2(q)} + \frac{60 J_2(\tau + i0^+)}{a_z^5 \sqrt{\pi}} \right]^{-1}, \tag{14}$$

where $J_2(\tau) = J_2(0) + \tau J_1(0) + O(\tau^2)$ for $\tau \to 0$, and $J_2(0) \simeq -2.2752 \times 10^{-2}$ [35]. From Eqs.(13,14), several conclusions can be drawn on the quasi-2D scattering in $m = 1$ and $m = 2$ partial wave channels: i) At small collisional energy ($q \to 0$), $f^{[m]}(q) \propto q^{2m}/(w_m^{-1} - w_m^{\star -1})$ where $w_1^\star \simeq 5.39 \times a_z^3$ and $w_2^\star \simeq 1.3 \times a_z^5$. Hence, the 2D confinement induces a shift $w_m^\star$ in the generalized scattering length w_m, which grows as the confinement increases. ii) By modifying the trap frequency and/or the external magnetic field it is possible to drive the system from a regime with a 2D m-wave quasi-bound state ($w_m^{-1} < w_m^{\star -1}$) to a regime with a shallow bound state ($0 < w_m \lesssim w_m^\star$), in both cases the (quasi-) bound state energy is $E_b \sim -\hbar^2(w_m^{-1} - w_m^{\star -1})/(2\mu\alpha_m)$ [38], thus having an expression similar to the 3D case: $E_b^{3D} \sim -\hbar^2/(2\mu w_l \alpha_l)$. iii) Resonance in the scattering cross section $\sigma = |f_m|^2/4q$ occurs at a collisional energy $E \sim E_b$, that is only for *positive* values of E_b: in presence of a quasi-bound state. The resonance width is given by $\Delta E/E_b \propto \eta_m (E_b/\hbar\omega_z)^{m-1} \ll 1$. iv) Consequently, α_l appears as a crucial parameter for the low energy scattering properties. For example, by neglecting α_1 ($\eta_1 = \infty$) in Ref.[20], the resonant collisional energy and resonance width of the p-wave CIR are found with other order of magnitudes. v) Following the reasoning in Ref.[34], these scattering properties open the possibility of observing high partial waves BEC-BCS transitions in quasi-2D fermionic gases by varying ($w_m^{-1} - w_m^{\star -1}$) from positive to negative values.

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[32] Eq.(10) can be obtained also by use of a Λ-potential: $\langle z|V_\Lambda|\phi\rangle = -\hbar^2 l_p \delta'(z) \lim_{\epsilon\to 0^+} (\Lambda + \partial_\epsilon - \xi_p \partial_\epsilon^2)\phi^{\rm odd}(\epsilon)/[\mu(1 - \Lambda l_p)]$, where Λ is a free parameter [22, 23, 33].
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[35] The functions $J_m(\tau)$ are defined in the domain $\tau < 0$ by:

$J_m(\tau) = \text{P.f.}\int_0^\infty du\, u^{-(m+1)} \exp(\tau u)[1 - \exp(-u)]^{-1/2}$, and appear in the computation of $f^{[m]}$. The expression of $J_m(\tau+i0^+)$ are deduced using an analytical continuation.

[36] The constant B is obtained from: $\ln(B/2\pi) = -\gamma + \int_0^\infty du \left\{u^{-1}[1-\exp(-u)]^{-1/2} - u^{-3/2} - (1+u)^{-1}\right\}$, where γ is the Euler's constant. This value slightly differs from the one in Ref.[11]. However, a close investigation shows that it coincides with the evaluation of the slowly converging series in Ref.[11].

[37] For the resonances considered previously and a wave guide with $\omega_z = 2\pi \times 70$ kHz, $\eta_1 \simeq 3 \times 10^{-2}$ for ^{40}K atoms and $\eta_1 \simeq 7 \times 10^{-3}$ for ^{6}Li atoms.

[38] For $m = 1$, this expression of E_b neglects the logarithmic correction which is vanishingly small if $\eta_1 \left|\ln\left|a_z^3(w_1^{-1} - w_1^{\star -1})\right|\right| \ll 1$.